GENERALISED AMBIENT REFLECTION MODELS FOR LAMBERTIAN AND PHONG SURFACES

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ABSTRACT

Ambient reflection is widely present in many applications of computer graphics and image processing, which is traditionally modelled as a constant free from environmental factors. This paper reconsiders ambient reflection modelling of Lambertian and Phong surfaces and calculates it as reflection integrations of infinitesimal incident beams from the environment. It reveals that ambient reflection exists in variable forms for Lambertian surfaces of non-convex objects and Phong surfaces of all objects. For convex objects with Lambertian surfaces, ambient reflectance coefficient is actually the diffuse reflectance coefficient. Generalised ambient reflection models are proposed to calculate ambient reflection using the same reflection model as used in calculation of other reflections. Based on this analysis, new ambient reflection formulations of Lambertian and Phong surfaces are derived to enable efficient computations in computer graphics and image processing.

Index Terms— Ambient reflection, reflection analysis, Lambertian model, Phong model.

1. INTRODUCTION

Apart from directional illumination, an object is often considered to be uniformly illuminated to account for weak light sources and multiple reflections from the environment. This non-directional light source is often modelled as evenly distributed and is known as ambient light. To calculate the reflection of ambient light, termed as ambient reflection, a constant ambient reflectance coefficient is introduced to model the ratio of reflected ambient light to incident ambient light. This traditional modelling of ambient reflection, however, doesn't follow any reflecting mechanism in optics [6]. This paper examines the mechanism of ambient reflection by treating it as integrations of reflections of infinitesimal incident lights from the environment. It reveals that the ambient reflections under different assumptions are actually different from the traditional modelling result. To better model ambient reflection, this paper proposes different formulations for Lambertian surfaces and Phong surfaces.

To create a realistic scene in computer graphics or to solve reverse problems in image processing, reflection models are widely used, such as the Lambertian model and the Phong model. These reflection models mainly deal with slim light rays, while area lights are considered as integrations of slim rays. Light sources can also be modelled using spherical harmonics [11]. However, the use of high order spherical harmonics is often limited due to lack of information about the lighting environment and to insufficient computational power. These researches are all limited to Lambertian surface assumption and didn't consider specular reflections. Sloan et al. proposed a pre-computed radiance transfer method to achieve efficient rendering of scenes under spherical harmonic lighting modelling [13]. Single or few point light sources are often considered while weak light sources such as background lights and environmental inter-reflections are grouped to form ambient light. In many cases, these weak lighting and reflections provide critical effects and information. For instance, face recognition is often performed on face images under ambient illumination only. These lighting conditions help face recognition algorithms to extract intrinsic surface characteristics better than directional illumination. Careful consideration of those "less significant" illumination and reflection is then required in these applications. In most cases, ambient light means area room lights [12]. It is also used and considered in medical imaging [4]Basri and Jacobs explained ambient light as the DC component of light spherical harmonics which is uniformly distributed around an object whose irradiance is constant in all directions [1].

Ambient reflection is traditionally considered an independent reflection detached from diffuse and specular reflections [3]. The reflected ambient intensity from an object's surface is expressed as

$$I = I_a k_a , \tag{1}$$

where I_a is the intensity of ambient light and k_a is the ambient reflectance coefficient of surface. Cook and Torrance used the diffuse reflectance coefficient to directly substitute the ambient reflectance coefficient [5]. The ambient light is usually modelled as an evenly distributed

incident light from the hemisphere above each surface point. Under this assumption, part of the hemisphere could be occluded by nearby geometry such as a nearby object. Ambient occlusion techniques extended this approximation to ambient reflection by multiplying an occlusion factor to the traditional ambient reflection [9]. In image processing, many researchers neglect ambient illumination and limit their algorithms within the scenarios of a single incident light, such as in [7]. Other researchers substitute the ambient reflectance coefficient with other reflectance coefficients. Blanz and Vetter [2] substituted the ambient coefficient with the diffuse coefficient when using the Phong model.

Given a model of ambient light distribution, it is possible to calculate the corresponding ambient reflection just as calculating other using a reflection model. This research intends to examine whether and how the traditional ambient modelling and ambient reflection integrations are different for various surface materials. For non-convex objects, the focus of this chapter is the recalculation of the traditional ambient reflection intensity so that the strategy is different from that of ambient occlusion, because ambient occlusion assumes the ambient reflection intensity to be constant and computes the un-occluded solid angle. This research also explains the rationales of substituting ambient reflectance coefficients with diffuse reflectance coefficients as used in [2] and the conditions of this substitution. The popular Lambertian and Phong reflection models are chosen to generalise ambient reflection models. Pre-integrated formulations of ambient reflection are presented for faster computation in computer graphics and for solving reverse problems in image processing.

This paper is organised as follows. Section 2 presents a generalised ambient illumination model. In Section 3 and 4, generalised ambient reflection models for Lambertian surfaces and Phong surfaces are proposed respectively. Conclusions are drawn in Section 5.

2. AMBIENT ILLUMINATION

Other than using (1), one can treat ambient light and reflection as an integration of infinitesimal incident lights and reflections using reflection models directly. For a surface point on a convex object, the ambient light from a hemisphere reaches the surface and gets reflected. If the object is not convex, ambient light from some regions in the hemisphere may be blocked by other parts of the object (ambient occlusion) [9, 14]. The solid angle where ambient light reaches the surface point is called unblocked angle and denoted as Π . Given the shape of an object, Π for a surface point can be obtained by examining object geometry. The maximum value of Π is 2π . Ambient light is assumed evenly distributed whose irradiance is denoted as i_a . The ambient intensity is then calculated as

$$I_a = i_a \iint_{\Pi} d\omega_a \ , \tag{2}$$

where

$$d\omega_a = d\alpha \cdot d\beta \sin\alpha \ , \tag{3}$$

is the infinitesimal solid angle within Π . (2) indicates that I_a is spatially varying on the object due to ambient occlusion. This variation affects ambient reflection for non-convex objects. Only when an object is convex, (2) becomes $I_a = 2\pi \cdot i_a$ where I_a is assumed constant as in the traditional ambient reflection approximation.

3. GENERALISED AMBIENT REFLECTION OF LAMBERTIAN SURFACES

Assume the surface reflection of a given object obeys Lambert's law. The reflected intensity of a point light source on a Lambertian surface is expressed as

$$J_L = I_p k_{dL} \max\left((\vec{N} \cdot \vec{L}), 0\right). \tag{4}$$

Considering ambient light as distributed infinitesimal point lights, ambient reflection can thus be integrated as

$$J_{aL} = i_a k_{dL} \iint_{\Pi} (\vec{N} \cdot \vec{L}) d\omega_a \ , \tag{5}$$

which indicates that ambient reflection of a Lambertian surface is view-independent and only dependent on Lambertian diffuse reflectance coefficient and the unblocked solid angle Π .

For convex objects, (5) becomes

$$J_{aL} = i_a k_{dL} \int_{a=0}^{\pi/2} \int_{\beta=0}^{2\pi} \left(\sin\alpha \cos\alpha\right) d\beta \cdot d\alpha = 2\pi i_a k_{dL} \ . \tag{6}$$

which is similar to (1). The traditional modelling then becomes valid if the ambient reflectance coefficient is assigned properly ($k_a = k_{dL}$).

For non-convex objects, ambient light may be occluded by protruded parts of the object and then be attenuated [14]. The reflected ambient light can be modelled as multiplications of the calculated ambient reflection for convex objects with the ambient occlusion attenuation factor. Recently, ambient occlusion has been extensively studied and several acceleration schemes have been proposed [8, 9]. Ambient occlusion can significantly reduce the computation time than full global illumination computation. However, ambient occlusion approximates each unoccluded infinitesimal reflecting beam with equal intensity and consequently is based on traditional ambient reflection model. On the other hand, generalised ambient reflection model of Lambertian surfaces considers the relationships of the incident directions from the illumination situations and normal directions from object geometry. Thus it generates better rendering results as shown later in Fig. 3.

4. GENERALISED AMBIENT REFLECTION OF PHONG SURFACES

In the Phong model [10], the reflected intensity of a point light Ip is expressed as

$$J_P = I_a k_a + I_p k_{dL}(\vec{N}\cdot\vec{L}_p) + I_p k_{sP}(\vec{R}_p\cdot\vec{V})^{n_P}. \quad (7)$$

This model separates surface reflections into a diffuse component and a specular component where the diffuse term is identical to the Lambertian model. To calculate the ambient reflection of a Phong surface, the diffuse component is identical to (5) and the specular component J_{aPs} is calculated as

$$J_{aPs} = i_a k_{sP} \iint_{\Pi} (\vec{R}_a\cdot\vec{V})^{n_P}\, d\omega_a, \quad (8)$$

where $\vec{L}_a = (\sin\alpha\cos\beta, \cos\alpha, \sin\alpha\sin\beta)^T$ is the ambient direction and $\vec{R}_a = 2\vec{N}(\vec{N}\cdot\vec{L}_a) - \vec{L}_a$ is the reflecting direction. The overall ambient reflection is combined as

$$J_{aP} = i_a \iint_{\Pi} [k_{dL}(\vec{N}\cdot\vec{L}_a) + k_{sP}\max\left((\vec{R}_a\cdot\vec{V}), 0\right)^{n_P}] d\omega_a. \quad (9)$$

This equation shows that the reflected intensity of ambient light is variable due to changes of object geometry, reflectance coefficients (k_{dL} , k_{sP} , and n_P), and viewing condition $\vec{V}$.

For a convex object where $\Pi = 2\pi$, (9) becomes

$$J_{aP} = 2\pi i_a \left[k_d + \frac{k_{sP}}{2\pi} \iint_{\substack{0\le\alpha\le\pi/2 \\ 0\le\beta\le 2\pi}} \sin\alpha\cdot(\vec{R}_a\cdot\vec{V})^{n_P}\cdot d\alpha d\beta \right]. \quad (10)$$

Fig. 1 illustrates the specular component of ambient reflection of Phong surfaces with varying Phong specular exponent n_P and viewing angle θ. Ambient reflection is high when n_P and θ are small and reduces as n_P and θ increase.

To facilitate the derivation of ambient reflection formulation based on the Phong model, a polar coordinate system is introduced as shown in Fig. 2. Let α' and β' denote the azimuth and zenith angles between $\vec{V}$ and $\vec{R}_a$ with respect to y axis, respectively. The ambient light is modelled as in (2) and the infinitesimal ambient light corresponding to reflected direction $\vec{R}_a$ is expressed as

$$dI_a = i_a d\alpha' \cdot d\beta' \cos\beta'. \quad (11)$$

In the Phong model, the specularly reflected intensity of $dI_a(\alpha', \beta')$ is calculated as

$$dJ_{aPs} = i_a d\alpha' \cdot d\beta' \cos\beta' \cdot k_{sP}\cos^{n_P}\phi, \quad (12)$$

where ϕ is the angle between reflecting vector $\vec{R}_a$ and viewing direction $\vec{V}$. In Fig. 2, we have

$$\cos\phi = (\vec{R}_a\cdot\vec{V}) = \cos\alpha'\cos\beta'. \quad (13)$$

The specular component of ambient reflection is expressed as

$$J_{aPs} = i_a k_{sP} \int_{-\pi/2+\theta}^{\pi/2} \int_{-\pi/2}^{\pi/2} \left(\cos^{n_P}\alpha' \cos^{n_P+1}\beta'\right) d\beta' d\alpha'. \quad (14)$$

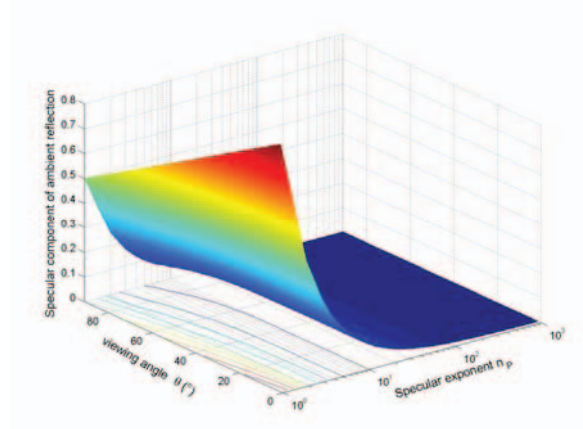

Fig. 1. Specular component of ambient reflection of Phong surfaces with respect to viewing angle θ and specular exponent n_P .

Based on the formulas of integrals of trigonometric functions, (14) becomes

$$J_{aPs} = \begin{cases} \dfrac{2 i_a k_{sP}}{n_P+1}\left(\pi - \theta + \cos\theta \displaystyle\sum_{r=0}^{n_P/2-1} \frac{2^{2r}(r!)^2 \sin^{2r+1}\theta}{(2r+1)!}\right) \\ \qquad n_P = 2m \\ \dfrac{\pi i_a k_{sP}}{n_P+1}\left(1 + \cos\theta \displaystyle\sum_{r=0}^{(n_P-1)/2} \frac{(2r)!}{2^{2r}(r!)^2}\sin^{2r}\theta\right) \\ \qquad n_P = 2m+1 \end{cases}. \quad (15)$$

Combined with the diffuse reflection of ambient light, the generalised ambient reflection of Phong surfaces becomes $J_a = I_a k_{dL} + I_a k_{sP}\eta$, where

$$\eta = \begin{cases} \dfrac{1}{n_P+1}\left(1 - \dfrac{\theta}{\pi} + \dfrac{\cos\theta}{\pi} \displaystyle\sum_{r=0}^{n_P/2-1} \frac{2^{2r}(r!)^2 \sin^{2r+1}\theta}{(2r+1)!}\right) \\ \qquad n_P = 2m \\ \dfrac{1}{2(n_P+1)}\left(1 + \cos\theta \displaystyle\sum_{r=0}^{(n_P-1)/2} \frac{(2r)!}{2^{2r}(r!)^2}\sin^{2r}\theta\right) \\ \qquad n_P = 2m+1 \end{cases}. \quad (16)$$

The pre-computation of ambient reflection of Phong surfaces is limited to convex objects. For non-convex objects, both diffuse and specular components of the ambient reflection will be attenuated since the ambient light sources are occluded. The diffuse component can be treated using ambient occlusion as previously discussed. The calculation of specular reflection of ambient light is a global illumination problem and a case-by-case computation.

In Fig. 3, the plane and platform was rendered under ambient illumination only based on traditional ambient reflection model (Fig. 3a) and generalised ambient reflection models (Fig. 3b and 3c). Using the traditional model, the reflection of each unoccluded infinitesimal incident light is simply modelled as a constant. Using the

generalised ambient reflection models, on the other hand, the reflection of an unoccluded infinitesimal light is modelled dependently on object geometry and viewing conditions. Especially for Phong surfaces which reflect specularly, the rendering effect is greatly improved from that of the traditional ambient occlusion.

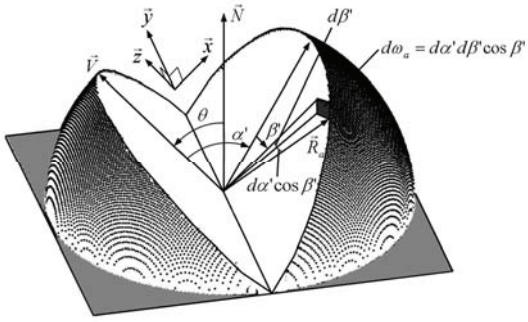

Fig. 2. Polar coordinate system.

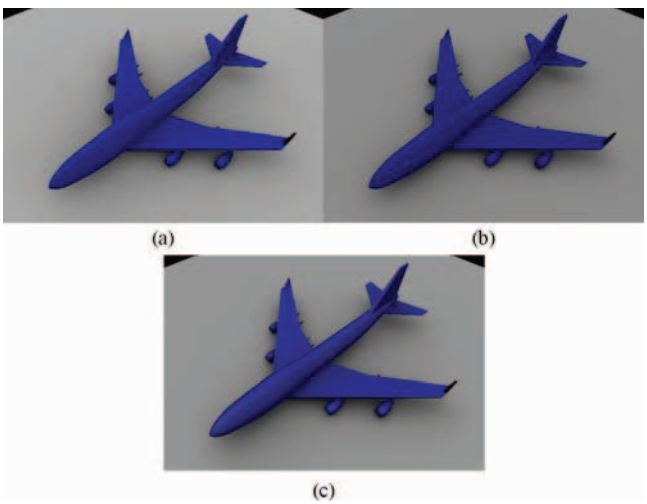

Fig. 3. Boeing 747 in ambient illumination only rendered using (a) the traditional model (ambient occlusion), (b) the generalised ambient reflection model under Lambertian assumption, and (c) the generalised ambient reflection under Phong assumption.

5. CONCLUSIONS

This paper examined the mechanism of ambient reflection of object surfaces characterised by Lambertian and Phong models and analysed ambient reflection as reflection integrations of infinitesimal incident beams from the environment. These ambient reflection approximations were compared to the process of the traditional ambient reflection model. Substantial differences were found which suggested that ambient reflection is variable and dependent on the selection of reflection models. Hence, ambient reflection cannot be modelled traditionally as a constant regardless of the choice of reflection models. It should be calculated using the same reflection model as used in calculation of other reflections.

Generalised ambient reflection models have then been proposed for Lambertian and Phong surfaces with considerations of object convexity. Pre-calculated ambient reflection formulations have been derived for convex objects for these two surfaces to facilitate fast computation. The future work will be to model ambient reflection under even more complex reflection assumptions, such as considering Torrance-Sparrow model.

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