

THE IMPACT OF 20 YEARS OF RESEARCH

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In 1998, when I was in the process of applying for permanent residency in the United States, I was required to make a submission on my professional accomplishments to the then Department of Immigration and Naturalization. One of the major criteria I had to satisfy in this submission was as follows:

Demonstrate that your research over the last 20 years has made an impact in at least five different countries. Provide supporting letters from people in these countries.

I remember pondering at the time about the word *impact* and even more strenuously about how I would find people who were willing to convince the immigration authorities that my research had made an impact in their country. What kinds of people should I ask to write letters of support? How should their supporting evidence be organized to demonstrate that my research had made an impact?

In some sense the notion of *impact* has come back to haunt me in this paper because it is an intimidating task to survey an extensive range of research that has undergone changes in philosophy, purpose, and design since my involvement in the first review of Australian research in mathematics education (Jones, 1984). Before returning to the 1984 review and the meaning of impact let me look briefly at why it is important to look back over extended periods of research activity.

The Need to Revisit Extended Periods of Research

The Mathematics Education Research Group of Australasia (MERGA) has been very faithful in undertaking 4-year reviews ever since the inaugural 1984 review; consequently, one might question the need to look at a 20-year period. What else could be ascertained that could not be found in the aggregation of the 4-year reviews? First, it seems to me that a longer review has a better opportunity to identify Australasian *research themes* that are enduring and areas that are new or emerging. Second, the longer review should be able to differentiate between sustainable changes and passing aberrations in the *research process*. Third, given the global influences on MERGA itself,

the longer review should be more sensitive to world trends and the effect they have had on Australasian research. Let me discuss each of these briefly.

Determinations concerning what research themes are enduring and what themes are emerging don't usually occur over a period of 4 years or even 8 years. Such themes are usually determined by individuals, organizations or governments and as such there is not always a quick or impartial response to diverse forces and issues. Conflicting views in research take time to resolve as is nicely illustrated in Gage's (1989) presidential address to the American Education Research Association where he debated the pros and cons of scientific positivism, interpretist, and critical-theory orientations to research.

With respect to the process of mathematics education research there have been major shifts in the last 20-years that have only become evident over time. For example, it is only in recent years that leaders in our field have made claim to mathematics education research as a recognizable domain with its own mode of enquiry and problematic (Grouws, 1992; Sierpiska & Kilpatrick, 1998). Reflective of this claim, there has been a major growth in the number of international journals devoted to mathematics education research and, in concert with other forms of educational enquiry, there have been substantial changes over time in philosophical orientation with the emergence of postmodernism and poststructuralism and in methodology with the growth of qualitative methods and even mixed designs (Tashakkori & Teddlie, 1998)

The global influences on MERGA itself are significant but are often too elusive to capture over a short period. There is no doubt that Australasian research has become more international in the past 20years: witness the substantial increase in the number of Australasian articles published in the *Journal for Research in Mathematics Education* (JRME) or papers presented at *International Conferences of the Psychology for Mathematics Education* (PME) over the 90s decade compared with the 80s. In addition, both Australia, New Zealand and other Pacific countries now contribute to the work of MERGA and its publications receive manuscripts from all over the world.

All of these changes not only bring new perspectives, they have the potential to change the landscape of mathematics education research in our region. For these reasons it is timely to take a longer look at where we have been in the last two decades.

Revisiting the 1984 Review

The 1984 review (Briggs) covered the period 1977-1983 and incorporated three major components: a review of MERGA research (Jones), a review of other Australian research (Conroy), and an annotated bibliography of gender research in Australia (Leder). The key research themes identified by Jones were curriculum; attitudes to mathematics, the learning and teaching of mathematics; problem solving; learning and cognitive development; and calculators and computers. Conroy's themes were similar except that he included a category on tertiary and teacher education but did not establish special categories for problem solving and calculators and computers. Leder's section on gender included both MERGA and non-MERGA research.

In identifying the themes, we certainly did not have in mind the notion of international impact as I have for this paper. In fact, Conroy explicitly states, "no attempt has been made to trace Australian research reported in overseas journals." All of us recognized that Australian research in mathematics education was in its infancy and our intent was to synthesize the relatively small corpus of research into a coherent form for our own members and for overseas visitors at ICME V in Adelaide. Hence we used categories that were significant at the time and where there was substantial research. For example, following the "modern math" revolution, the formation of the Australian Curriculum Commission, and movements away from external examinations in some states, there was considerable interest in curriculum research especially in historical development, evaluation of curriculum projects, and assessment.

As we move to this current review, it is appropriate to ask, "which of these themes have continued to be important and what new directions have emerged?" However, in the interests of brevity and sanity, I will restrict the current review to Australasian research themes that have made an impact in the international scene.

Making an Impact

As noted earlier a longer review of Australasian research should not be just an accumulation of all the 4-year reviews that MERGA has carried out over the last two decades; rather it should have a character of its own and a means by which to deal with such a vast amount of research. I have attempted to create that character by asking a similar question to the one that the former U.S. Department of Immigration and

Naturalization asked me. In its recontextualized form my question became, “What themes of Australasian research have made an international impact over the last 20 years? Such a question immediately raises baffling questions of its own like the meaning of “making an impact.”

What is meant by “making an impact”?

Making an impact could refer to making an impact on the worldwide mathematics education research community or even the broader global educational research community. It could refer to making an impact on mathematics curricula and classrooms nationally and internationally. Although it was tempting to use impact on mathematics curricula and classrooms as a measure of impact, I decided that it was not feasible to work with that important criterion. Hence, I used “influence on the international research community” as the predominant source for establishing criteria to identify research themes that have made an impact.

That brought me back to one of the personal questions I had faced in trying to demonstrate that my own research had made an impact in other countries: “What kinds of people should I ask to write letters of support?” Not surprisingly my answer was to ask recognized leaders and authorities in the international community. When we attempt to refine the notion of international research community, we immediately become aware of the importance of editors and members of the editorial boards of major research journals and publications. In essence, these are the research leaders in mathematics education or the group that Brown and Dowling (1998) call “the authorities within the theoretical field” (p. 139). As Brown and Dowling add, these researchers have often attained authority status by regularly publishing and monitoring the work of others in the most prestigious journals. Hence my *first* criterion for judging impact was to identify those research themes that have caught the eye of editors and reviewers of journals like Educational Studies in Mathematics (ESM) and the Journal for Research in Mathematics Education (JRME). Additional supporting evidence used under this first criterion was recognition of the research by other authorities in the field; for example, those who edit or write research handbooks and monographs for major international publishers like Kluwer and Erlbaum. The *second* criterion dealt with what Brown and Dowling call the “readership” (p. 139) in the field of mathematics education. Readers and consumers of

research such as mathematics educators, graduate students, policy makers, curriculum developers, and teachers all have the potential to evaluate, use, and disseminate research. Although it is impossible to identify the number of readers of particular research, it is possible to examine the number and scope of citations of the research and more importantly to gauge the extent to which the research has seeded other research. In essence, the second criterion deals with the extent to which research activity has been *seminal* or has been linked to seminal research in the field of mathematics education. The *third* criterion focused on the link between theory and practice. That is, it asked how well did the research and researchers “engage with practitioners’ knowledge, perspectives, work, and activity situation” (Bishop, 1998a, p. 36). This criterion was also concerned with the extent to which the research activity supported curriculum development: effective resources, understanding of students’ thinking, and research-based change in mathematics curriculum (Clements, 2002). In particular, I found it salutary to identify what Australasian research had been cited in the National Council of Teachers of Mathematics (NCTM) document, “A Research Companion to Principles and Standards for School Mathematics (Kilpatrick, Martin, & Shifter, 2003). The *fourth* criterion I used was entirely idiosyncratic: the “reviewer” factor. Notwithstanding my attempts to be up front with the criteria used to identify Australasian research that has made an impact, the choice ultimately lies in the “eyes of the beholder.” The reviewer factor even comes into identifying the journals and other documents where Australasian mathematics educators report their research. Hence I will identify the sources where I looked for Australasian research and will also expose “my pecking order.”

Where do Australasian Mathematics Educators Report their Research?

My review of the literature indicates that the majority of Australasian mathematics educators report their research in sources close to home: *Mathematics Education Research Journal* (MERJ), *Mathematics Teacher Education and Development* (MTED), *Australian Educational Researcher* (AER), *Science and Mathematics Education Paper* (SAME), and *The Mathematics Educator*, the *Curriculum Journal*, and the *Proceedings of MERGA*. As an example, I observed that about 200 research papers in MERJ have been authored by researchers in our region.

There is also extensive publishing by Australasian researchers in international journals like ESM, JRME, *International Journal of Mathematics Education in Science and Technology*, *Journal of Mathematical Behavior*, *Journal of Mathematics Teachers Education* (JMTE), *Mathematical Thinking and Learning* (MTL) and *School Science and Mathematics*. In addition there are also numerous short research reports in the proceedings of the International Group for the Psychology of Mathematics Education, the International Congress in Mathematical Education and in more specific proceedings like the International Conference on Teaching Statistics. As a region we are increasingly being invited to contribute to handbooks in mathematics education (e.g., Bishop, Clements, Keitel, Kilpatrick, & Laborde, 1996) and to books and yearbooks that focus on specific aspects of mathematics education research (e.g., Sierpiska & Kilpatrick, 1998).

Returning to my comments on the “reviewer factor,” I make no excuse for establishing an internal pecking order for research publications in mathematics education. For example, the flagship journals in our field, ESM and JRME, rate much higher than any other mathematics education journals. Not only has research published in these journals undergone exacting scrutiny and survived low acceptance rates, it also has the potential for much greater impact because of the extensive dissemination capabilities of the journals. In the same way proceedings from international groups like PME International rate much higher than MERGA proceedings because they have undergone more rigorous jurying and enjoy better dissemination.

Geared with these criteria and more than a little trepidation I am ready to return to my central question, “What themes of Australasian research have made an impact in the international mathematics education research community during the last 20 years?” As an aside, I wish to remark that my admiration for the ingenuity of the former U.S. Department of Immigration and Naturalization has grown immensely as has my eternal gratitude for the colleagues who managed to convince them that my research had made an impact in their various countries.

Themes in Mathematics Education Since 1984

I have selected six themes where I believe that Australasian research has made significant impact during the last two decades. These themes are as follows: cognitive aspects of mathematical learning, gender and mathematics, language and mathematics, sociocultural

aspects of mathematics teaching and learning, early childhood mathematics, and teacher education and the teaching of mathematics. In dealing with each theme and subtheme, I will make a case for its inclusion, identify the critical issues, recognize the leaders, and discuss their contribution to the theme or subtheme. Space limitations demand that I present only the barest details of researchers' contributions and limit citations unreasonably.

Cognitive Aspects of Mathematical Learning

During my interactions with mathematics educators at international conferences and on editorial panels like JRME, I have been impressed by the number of times leaders in the field have referred to the impact made by Australasian research in the field of cognition and learning. The extent of this impact not only lies in the research on cognitive psychology but also in cognitive research on the various mathematical domains. As an example of the impact of our region's research in this theme, I noted recently that there are 20 citations of Australasian research in the NCTM *Research Companion to Principles and Standards for School Mathematics* (Kilpatrick, et al., 2003). These citations occur in most mathematical domains: whole numbers (1); rational numbers (1), algebra (1), geometry and measurement (9), and probability and statistics (7). While this may not seem a large number, one needs to remember that the research is unashamedly biased in the direction of the US.

Cognitive development models. With respect to cognitive development, the formulation of models has been a major research endeavour since Piaget's monumental works. In cognitive models the research of Biggs and Collis and Halford has had a pervasive and enduring effect both nationally and internationally for two decades. Not only has their research been extensively applied in the field of mathematics, it has also been extended and enhanced by other researchers. Biggs and Collis's SOLO taxonomy (1982) is a general model of intellectual development postulating that all learning occurs within five modes of functioning and that within each mode there is a cycle of five levels of response that represent an increasing sophistication in mental functioning. The SOLO model has been enhanced by later research (Biggs & Collis, 1991; Pegg, 1992) that has emphasized the multimodal nature of mental functioning and has argued the need for multiple cycles within a single mode of representation. Its sustained impact in mathematics education has

undoubtedly been due to extensive applications to domains like geometry, probability, and statistics.

Halford's analogical reasoning model (1992) and structure mapping theory of cognitive development (1993) account for cognitive development in terms of structural mapping levels, processing capacity, and knowledge factors. Each higher level of mapping enables more abstract ideas to be represented; however, higher levels impose high processing demands. Recent work by English and Halford (1995) has provided a broadened perspective of the models' applicability to mathematics education and it has been used in mathematical domains like number and place value.

Given the applicability of these cognitive models to various domains of mathematical learning, it is not surprising that Australasian research has made a pervasive impact in mathematical domains. I will discuss this impact below.

Whole numbers and their operations. Wright and Bobis have made a significant impact in research on children's *counting and number development* and both have generated large-scale teacher development projects associated with their research. The *Count Me in Too* project (Bobis, 1997) has a vast national profile and *Mathematics Recovery* (Wright, Martland, & Stafford, 2000) has enviable visibility nationally and internationally. Both of these projects have been developed to improve students' learning through a school-focused teacher development program. Wright's research (Wright & Gould, 2002) on the classification of children's counting and number development has been used to inform classroom practice in both *Mathematics Recovery* and *Count Me in Too*. Irwin's work on young children's number reasoning has also impacted the international scene with presentations at ICME (Irwin, 1992) and publications in JRME (Irwin, 1996a). Her key work was the formulation of a framework for understanding how young children build number knowledge using schemas of comparison, and part-whole relationships.

The fact that Mulligan's research (Mulligan & Mitchelmore, 1997; Mulligan & Watson, 1998) on children's intuitive models of *multiplication and division* of whole numbers is cited in the NCTM *Research Companion* (2003) is indicative of its impact in the global scene. Her research generated a framework incorporating six developmental

levels of students' thinking and, as an added strength, linked assessment tasks to each level. Later work with Watson connected the levels to the SOLO model.

Boulton-Lewis and Jones working independently have impacted the vast research base in *numeration and place value* with international publications and citations. Boulton-Lewis's research (Boulton-Lewis, 1993; Boulton-Lewis, 1998) used Halford's structure mapping theory to explain the processing load implicit in the concrete materials teachers use in place value activities. By observing children's thinking, she concluded that teachers can unwittingly increase children's processing loads if they use unfamiliar concrete representations. Jones and his colleagues (Jones, Thornton, & Putt, 1994; Jones et al., 1996) validated a framework that described children's thinking on four key constructs: counting, partitioning, grouping, and number relationships. Their framework incorporated five levels of thinking from pre-place value to essential place value. Thomas and Mulligan (1999) used the Jones framework in building the model of children's understanding of the base-ten system and Higgins (1999) used it in analyzing teachers responses to a survey on number and place value.

Mental computation and number sense. Mental computation and number sense constitute relatively new areas of mathematics education research even though they have long been part of mathematics. Mental computation relates to inventing procedures that are idiosyncratic but appropriate for a particular problem while number sense deals with quantitative intuition, a feel for quantities represented by numbers (Sowder, 1992, pp. 380-81).

Alastair McIntosh has been one the world leaders and prime movers in this field and his publications with other leaders (McIntosh, Nohda, B.J. Reys, R.E. Reys. 1995; McIntosh, Reys, & Reys, 1997) provide a clear warrant for the impact of his own work. His work is showcased in two international comparative studies one involving Australia, Japan, and the United States, and the other Australia, United States, Sweden and Taiwan. In these studies McIntosh and his colleagues assessed elementary and middle school students' performance on mental computation and generated a characterization of the kinds of computation children prefer to do mentally. They also demonstrated that number sense could be assessed using written tests based on the number sense framework formulated in the publication with Reys and Reys. His book (McIntosh, De Nardi, &

Swan, 1994) is a cogent demonstration of how his work has contributed to classroom practice.

Rational numbers. When graduate students throughout the world begin to examine research on rational numbers it is not long before their search leads inevitably to the prodigious work of Robert Hunting. Hunting and his colleagues, especially Gary Davis, have not only produced an impressive trail of research on partitioning, sharing, and fractions they have also set the agenda for research in this field through their reviews. As an illustration, I highlight their edited book (1991) on *Early Fraction Learning* published by the prestigious Springer-Verlag.

Hunting and Davis have consistently focused on prefraction knowledge such as partitioning, equivalence, and forming dividable units. Hunting's research demonstrated that children exhibit a variety of meanings and responses to tasks involving the subdivision of physical quantities such as food and other materials. Moreover, the meanings they exhibit are inextricably bound up with the social practices they experience and the politics of sharing commodities like food and toys. Hunting concluded that preschoolers know how to deal in discrete quantity before they are able to use rational counting (Hunting & Sharpley, 1988) and he claims, contrary to extant practice, that children's first experiences of rational numbers should be based on discrete quantities rather than continuous quantities (Bigelow, Davis, & Hunting, 1989).

Hunting and Davis (1991) also discuss the various meanings that children (aged 4-6 years) have for one-half. Based on this research they formulated a framework for teaching basic fraction concepts that uses discrete quantity (Hunting, D. M. Clarke, Lovitt, & Pepper, 1991). In addition they have produced a computer-based tool, Copy Cat, that enabled children to see a metarelation like one-half across particular relational instances like 4 in 2 out, 6 in 3 out, 10 in 5 out. Contrary to competing claims in the literature (Streefland, 1991), they claim that their research with Copy Cat shows that children's whole number understanding need not interfere with fractional understanding (Hunting, Davis, & Pearn, 1996). It is impossible to do justice to their work in this brief synopsis; however, it is enough to say that I will certainly ensure that my grandchildren experience the kind of tasks and learning environments that Hunting and his colleagues suggest for prefraction and fraction learning.

A number of Australasian researchers have also looked at decimal representations of rational number. Stacey and Steinle's work and that of Irwin dominates the field with all of them active in international arenas. Stacey and Steinle (Steinle & Stacey, 1998; Stacey & Steinle, 1999) carried out a series of longitudinal studies on students' misconceptions (Years 4-10) when making decimal comparisons or what Resnick and her colleagues (1989) refer to as "buggy algorithms." Their key study categorized students as belonging to one of four categories: apparent experts, longer-is-larger, shorter-is-larger, or no classification. Following several survey studies on students' understanding of decimals (Irwin, 1996b), Irwin (2001) produced an experimental study that focused on the role of students' everyday knowledge in enhancing their understanding of decimals. She found that students who worked on contextual problems made significantly more progress with decimals than those who worked on noncontextual problems. She claimed that greater reciprocity existed within pairs who worked on contextualized problems, partly because less able students were able to exploit their everyday knowledge of decimals.

Algebraic reasoning. Starting with Booth's research (1988), algebraic reasoning has been a strong impact area for Australasian research largely as a result of the work of Macgregor and Stacey. MacGregor and Stacey's research is cited in the NCTM *Research Companion* (2003) and their work has high visibility in journals like JRME and ESM.

In setting the scene for Macgregor and Stacey's impact, we need to recognize that algebra research in the eighties was very much focused on students' conceptions and errors. More specifically, it was postulated that there were two ways to formulate equations from verbal data: direct translation of key words to symbols or by trying to express the meaning of the problem. Herscovics (1989) referred to these two procedures as *syntactic* and *semantic* translation respectively and inter alia attributed students' translation difficulties and reversals (like in the professors' and students' problem) to the fact that students engaged in syntactic translations rather than semantic ones.

Building on this research, Stacey (1989) investigated students' ability to recognize a linear pattern and express it as a rule and Macgregor (1990) reported that the writing of English sentences, to describe graphical relations between two variables, facilitated tertiary students' equation writing. Hence MacGregor and Stacey (1993) were

well poised for their “tour de force” study that questioned the prevailing theory of student translation errors and proposed an alternative theory. This alternative theory suggested that students construct from the natural language statement a cognitive model that simulates the semantic situation but is based on comparison rather than equality. More specifically, MacGregor and Stacey claimed that the model for a situation like “s is 8 more than t,” has the “8 more associated with the s (larger variable) and not with the t (smaller variable). In a nice follow-up (MacGregor & Stacey, 1997), they again examined a prevailing theoretical explanation for students’ inability to interpret algebraic letters as generalized numbers or even as specific unknowns. Macgregor and Stacey claimed that the cognitive levels explanation was insufficient and they pointed to more specific sources such as intuitive assumptions, interference from new learning, and the effects of misleading teaching materials. This kind of theory building over a sustained period is one of the features that sets MacGregor and Stacey’s research apart from some of the other Australasian studies published in national proceedings and journals.

In addition to Macgregor and Stacey’s work, English and her colleagues (e.g., English & Sharry, 1996) have contributed some nice links between analogical reasoning and algebra understanding as have Cooper and Boulton-Lewis (e.g., Cooper et al., 1997) on transitions from arithmetic to algebra and the use of representations in algebraic equations. This is a prodigious area for Australasian research as is evident in all MERGA reviews.

Geometry and measurement. There are nine citations in the NCTM *Research Companion* (2003) on geometry and measurement and this highlights the strength and the international reputation of our research in this theme. Mitchelmore has provided ongoing leadership to this area. His work on children’s conceptions of parallel lines and perpendiculars, Pegg’s investigations of the Van Hiele models, Outhred, Owens, and Mitchelmore’s research on area measurement, and Lawson and Chinnapan’s on geometrical problem solving dominate the field

Following a series of studies on young children’s difficulties in drawing parallel lines, Mitchelmore (1991) claimed that the difficulties were neither perceptual nor due to their failure to notice parallels. Rather he postulated that children form a copy of a figure that is different from adults and for them preservation of length has greater salience than

parallelism. He interpreted this low salience as resulting from the fact that the concept of parallelism is rarely used in everyday life except as two lines perpendicular to a third line. In a similar series (Mitchelmore, 1998; Mitchelmore & White, 1998) he and White surveyed children's conceptions of angle and determined how children interpreted the angles in six realistic models. They concluded that children's angle recognition varied considerably across these six situations as did their recognition of similarities between the situations. In a nice theoretical summation, Mitchelmore & White (2000) proposed that children progressively recognize deeper and deeper similarities between their physical angle experiences and classify them firstly into specific situations, then into more general contexts, and finally into abstract domains. There is a sense of coherence to their research that has not gone unnoticed internationally.

Since the early eighties the van Hiele model has been under constant investigation and debate (Hoffer, 1983). So much so that in 1986, van Hiele proposed an alternative set of geometric levels that he believed provided a more suitable geometrical structure. The positioning of these alternative levels (visual, descriptive, and theoretical) vis-a-vis the original Levels 1 through 5 (visual through rigor) was problematic. Pegg and his colleagues (Pegg & Davey, 1989, 1998; Pegg, 1992) were prominent among the international group that investigated the van Hiele model. In fact, they produced their own adaptation that not only combined the two van Hiele models but forged links between the van Hiele and SOLO models.

Outhred, Owens, and Mitchelmore have also produced some readily discernible international studies on area measurement. Following earlier studies on young children's understanding of tiling areas (Outhred & Mitchelmore, 1996; Owens & Outhred, 1998), their key study, (Outhred & Mitchemore, 2000) examined the strategies young children (Grades 1 through 4) used to solve rectangular covering tasks. Children's solution strategies were classified into five developmental levels that, according to the authors, are learned sequentially. Such classifications of students' thinking are very prevalent in current international literature because they provide background on students' prior knowledge that can be used to inform instruction (Carpenter et al., 1989).

High school geometry research is much in demand and Lawson and Chinnappan's contributions to geometrical problem solving and the processing behaviour of successful

and unsuccessful problem solvers are highly visible. Their key study (2000) investigated content and connectedness indicators of students' knowledge and they found that connectedness indicators were more influential than content indicators as differentiators of problem-solving performance. Higher performers could retrieve knowledge more spontaneously and activate more links between their knowledge schemas.

As Lowrie and Owens (2000) note, in their review of Australasian research on space and measurement, the research in this area is not only of the highest quality it also possesses an enviable diversity. Did I miss it or is it time for a special MERJ issue to showcase the research efforts of our region in this subtheme?

Probability and Statistics. In the field of probabilistic and statistical reasoning, two groups of researchers Jones and his colleagues and Watson and her colleagues have made extensive contributions to the international arena. Both groups have several citations in the *NCTM Research Companion* (2003) and in major research handbooks (Grouws, 1992; Bishop et al., 1996). Although undertaking their research independently, both link their research to the SOLO model (Biggs & Collis, 1991) and both have engaged in clinical and classroom studies.

In both *probability and statistics*, Jones and his colleagues (Jones, Langrall, Thornton & Mogill, 1997, Jones et al., 2000) validated frameworks that described elementary students' (Grades 1 through 5) probabilistic and statistical reasoning across a number of key constructs. In both cases, they identified four hierarchical levels of reasoning that were linked to a single cycle in the SOLO model. Follow-up teaching experiments (Jones et al., 1999; Jones et al., 2001) were informed by their frameworks. In the probability experiment, they claimed that overcoming a misconception in sample space, applying both part-part and part-whole reasoning, and using invented language were key determinants in producing probabilistic reasoning. With respect to statistical reasoning they showed that experiences with data reduced children's idiosyncratic reasoning and enabled them to build conceptual knowledge about center, spread, and variation. They also found that children's contextual knowledge was critical in the analysis and interpretation of data and that technology facilitated different kinds of reasoning. The work of Jones and his colleagues resonates with "cognitively guided

instruction” (Carpenter et al., 1989) and with “teaching experiment” methodologies (Cobb, 2000)

Watson and her colleagues (e.g., Watson, Collis, & Moritz, 1997; Watson & Moritz, 1998, 2003) have investigated students’ conceptions of probability measurement, conditional and conjunction events, and fairness of dice. In relation to probability measurement, they have linked students’ responses (Grades 3, 6, & 9) to two SOLO cycles and have shown that students’ response levels increased by an average of one SOLO level over 4 years. Similar growth seemed to occur for conditional events but not conjunction events. In their most recent study dealing with children’s conceptions about the “fairness of dice,” they found that the association between their beliefs about fairness and strategies for assessing it was not strong for the original or the longitudinal measures.

Watson and her colleagues have published a prodigious amount of research on various aspects of students’ *statistical reasoning*: inference (Watson & Moritz, 1999), sampling (2000a), average (2000b), variation (Toorok & Watson, 2000) and association (Chick & Watson, 2001). Because space will not allow me to discuss all of these, I will choose the one on “sampling” which typically demonstrates the powerful characteristics of Watson’s work: links to the SOLO model, samples covering a wide grade range, and both cross-sectional and longitudinal effects. In the study on students’ conceptions of sample, responses from the 62 students in Grades 3, 6, and 9 revealed six categories of reasoning that were linked to two cycles of the SOLO model. These categories were described according to a hierarchy of development in sampling and this characterization was seen to provide curriculum developers and teachers with insights into the knowledge that students bring to the classroom. The links between Watson’s research and the development of new curriculum guidelines in Tasmania exemplifies the kind of partnership that Clements (2002) espouses for the 21st century.

When the earlier seminal study by Wild & Pfannkuch (1999) on statistical thinking at a more formal level is added to the research of Jones and Watson, it is clear that Oceania has made a significant impact in this area. This impact will become even more evident in a soon to be published volume on statistical reasoning (Garfield & Ben Zvi, in press) that involves all of the above-mentioned researchers and others in our region.

Problem solving and mathematical modeling. Lester's (1994) review of the problem-solving literature identifies several key areas for future research: cognitive processes, metacognition, group and classroom research, and the teachers' role in promoting problem solving. Lester and others have also recommended further research on mathematical modeling (Lesh, 2000; Schroeder & Lester, 1989).

Interestingly, the problem-solving issues identified by Lester are areas where Australasian researchers have made significant impacts. English has made international contributions in cognitive processes, Goos, Galbraith, Putt and Stillman have added new knowledge and direction to metacognitive research, and Galbraith and English have championed mathematical modeling. Moreover, their studies have not only investigated students' individual thinking they have also investigated collective thinking in group and classroom settings.

In addition to her book with Halford (English & Halford, 1995) that contains a chapter on problem solving, English has also edited a book on mathematical reasoning (1997a) that incorporates some highly visible international authors in problem-solving research. She has also produced an extensive network of studies on cognitive processes involved in combinatoric problems (e.g., English, 1993) and problem posing (English, 1997b, 1998). I will concentrate here on the *problem-posing* studies because they typify her research style in starting from the classroom. Her first study involved a class of Grade 5 students as they participated in an 11-week program of problem posing and model-eliciting activities. She found that students improved in their ability to identify problem structures, became better problem solvers, and demonstrated more divergent and flexible thinking. In a follow-up study she showed that children had more difficulty posing problems in formal contexts (given a number sentence) than in informal contexts (given a picture or piece of literature). Her work provides a valuable addition to the knowledge base in problem solving that is available to teachers.

Research on metacognitive processes focuses on two related components: (a) knowledge of one's own thought processes, and (b) monitoring and regulation of one's activity during problem solving. Goos and Galbraith (1996) identified a new perspective for looking at metacognitive processes--one that involved analyzing verbal protocols from students' think-aloud-problem-solving sessions. They demonstrated that, although

students generally benefited from adopting complementary metacognitive roles, unhelpful social interactions and power relations could impede practice. In an extensive follow-up study (Goos, Galbraith, & Renshaw, 2002), the researchers examined student-student social interaction that mediated metacognitive activity and identified the metacognitive characteristics of *successful* and *unsuccessful* problem solvers. These studies and Goos's earlier work (1994) extended the knowledge base on the individual and collective metacognitive strategies used by secondary school students. They also took research on metacognition in a new direction by reconceptualizing it as a social practice.

Putt (Adibnia & Putt, 1998) and Stillman (Stillman, 1996; Stillman & Galbraith, 1998) have also contributed to metacognitive research. Adibnia and Putt's quantitative study revealed that the experimental classroom, which used Garofalo and Lester's cognitive-metacognitive framework as the lens for instruction showed significantly better problem-solving performance than two control classes who were not introduced to metacognitive activity. Stillman found that secondary students have a store of metacognitive knowledge, but they are more concerned about mechanical aspects of obtaining a solution than in taking time to monitor their thinking and verify their solutions.

Galbraith and English have provided strong leadership in mathematical modeling for our region. Galbraith initiated modeling research in a study with Clatworthy (1990) that dealt with an innovative two-year high school course. They made strong claims about the modeling performance of students, the effectiveness of the small group and large group instruction, and the personal growth perceived by students who took the course. Revisiting this research also suggests how it presaged later work on modeling and metacognition that Galbraith undertook with Goos and Stillman.

English moved modeling research from high school into the middle grades (English, Charles, & Cudmore, 2000; Doerr & English, 2003). Her key study with Doerr examined the characteristics of a sequence of tasks that can be used to elicit students' data modeling processes. They found that students can create generalizable and reusable models for selecting, ranking, and weighting data. Moreover, the range of models used by students suggests that they use multiple paths to develop ideas about ranking data. With even younger children, Diezmann's research (e.g., Diezmann, Watters, & English, 2002)

on mathematical investigations also shows great promise in that it focuses on children's conceptualizing and reasoning as well as teacher behaviours that support or inhibit such reasoning.

The review suggests that our research on problem solving is making an impact in almost all of the topics identified by Lester and others as critical for this area. However, apart from Diezmann's work, there is little research on the role of the teacher in problem-solving instruction. Perhaps this could be approached in the form of teaching experiments bringing together teams with research expertise in the psychological, sociocultural and pedagogical domains.

Gender and Mathematics

Research on gender and mathematics may be an area in which the impact on the international mathematics education community has been as great or even greater than that in cognition aspects of mathematical learning. It probably depends on whom you talk to in the international scene.

Gilah Leder has been a world leader in this field for two decades as attested by her book with Elizabeth Fennema (1990) on *Mathematics and Gender*, her review chapter in the first NCTM *Handbook of Research on Mathematics Teaching and Learning* (Leder, 1992), her editorship of a special ESM issue (1995) on gender in mathematics, and her chapter with Helen Forgasz and Claudie Solar in the *International Handbook of Mathematics Education* (Leder et al., 1996). These major international works, her prodigious research, and her leadership have inspired other Australasian researchers like Helen Forgasz and Margaret Walshaw to make this field their own.

In a plenary address at the 25 th annual conference of PME International (2001), Gilah Leder looked back over the last 30 years of research on mathematics and gender. She noted that the early trends in the 1970s focused on documenting performance and participation differences, exploring likely contributing factors, and assessing the effectiveness of various intervention strategies. This early research carried assumptions that females were to be assimilated into the male domain; that is, they should strive to attain achievements equal to those of males. She observed that when this *assimilationist* and *deficit* model was finally challenged in the tenets of *liberal feminism* and *radical feminism*, the focus of the research shifted to making females more central to

mathematics and using females' experiences to shape the mathematics taught and methods of instruction. These changing theoretical frameworks, highlighting the roles of class and culture and the status of mathematics, have continued to inform gender equity research not only in highly technological societies but also in developing nations. Responding on the extent to which perceptions about gender and mathematics learning have changed, Leder observed that recent studies (Forgasz & Leder, 2001; Leder & Forgasz, 2000) have revealed results that are inconsistent with previous findings and imply that changes have occurred in gendered perceptions related to mathematics education.

In a real sense, this paper is not only a description of the history and major issues in gender and mathematics research, it is also a history of Leder's contribution to it. It also foreshadowed the directions of contemporary research where Leder and other Australian researchers are making a major impact in the international scene.

Recent studies by Forgasz and Leder have revisited the field and have provided updates and extensions of earlier research. Following an earlier paper (Forgasz, Leder, & Gardner, 1996) which argued that the assumptions underlying the Fennema-Sherman *Mathematics as a Male Domain* scale (1976) were no longer valid, Forgasz, Leder and Gardner (1999) reported the development and validation of a new scale, *Mathematics as Gendered Domain*. In a study that used this scale, Leder and Forgasz (2000) used a sample of 860 students in coeducational high schools to survey *stereotypes* related to the learning of mathematics. Their findings revealed that there were only 8 items (out of 30) for which the findings were consistent with previous research findings. They concluded that this data reversed many earlier trends, in that students now perceive that girls are more likely than boys to persist with a challenging problem and also to enjoy mathematics more than boys. The trends in these studies are reinforced by data in the studies by Forgasz and Leder (2001) that reveal not only a changing educational landscape but tacitly new challenges in relation to mathematics learning for both girls and boys.

In discussing Leder's (2001) plenary address, I drew attention to the new frameworks and lenses that were driving gender research. The study on girls in mathematics by Walshaw (2001) reflects that kind of research in that she contends that

Foucault's ideas and those of other poststructuralists disrupt what we have come to know as the practice of gender research. The first section of the paper looks at some recent feminist reconstructionists' proposals developed from the orientation of "different experience," while the second section introduces Foucaultian ideas for a theoretical discussion about the ways in which girls become gendered through available discourses and practices. Building on this discussion, the final section provides an analysis of some moments of classroom life that are focused on a girl called Donna. The analysis calls attention to the unmarked and obscured relational practices and competing discourses that surround Donna in the classroom. It also reminds us that there are still significant inequities for girls in mathematics classrooms.

Hence the strength of the impact made by Australasian research in gender and mathematics continues unabated as it revisits key questions in the literature and charts new waters. On the one hand, changes in the way girls and boys perceive mathematics and its teaching are being corroborated from several sources; on the other hand, new philosophical positions are creating the need for new practice in gender research.

Language and Mathematics

In my earlier review (Jones, 1984), I noted that Australian research (e.g. Newman, 1981) was already beginning to draw attention to the importance of language and mathematics. This momentum continued into the eighties and early nineties when a number of researchers increasingly published in international journals, like *ESM* and *JRME*, on topics such as the following: error analysis (Clements, 1980), language comprehension errors (Clarkson, 1991), language and counting (Bell, 1990), bilingual studies (Clarkson & Galbraith, 1992), and language approaches to instruction (Reuille-Irons & Irons, 1989). Developments also occurred in related areas like writing and mathematics (D. J. Clarke, Waywood, & Stephens, 1993; Shield & Galbraith, 1998; Waywood, 1994).

From this somewhat fragmented frenzy of studies, Ellerton, Clarkson and Clements emerged as international leaders in the field. They began to provide a strong theoretical orientation to the research on language and mathematics that is encapsulated in a summative paper by Ellerton and Clarkson (1996). They look back at two key frameworks (Ellerton, 1989; Gawned, 1990) that subsequently became the theoretical bridges for a number of research studies. The framework by Ellerton with its focus on

language factors in mathematics education was especially influential in giving the research in language and mathematics a broader theoretical base; one that is located “within larger social and cultural domains” (p.991). In fact, the extent of the framework’s influence is seen in the fact that Bauersfeld (1998) and Adler (1998) disputed its comprehensiveness. As a consequence of Ellerton’s framework and other research from overseas (e.g., Voigt, 1995), research on language and mathematics began to move away from studies on error patterns, comprehension in problem solving, and textbook evaluations to areas such as the following: classroom discourse, sociolinguistics including classroom culture, psycholinguistics including semantic and syntactic structures in problem solving, and writing. In fact, Ellerton’s (2002) sociolinguistic study on issues associated with the translations of performance tests from English to other languages is an instance of these broader directions to research on language and mathematics.

Wayward has made a strong impact on *writing* and mathematics. Following a series of longitudinal studies into secondary school students’ writing about mathematics and their journal reflections on mathematics lessons, Waywood (Waywood, 1989; D. J. Clarke, Waywood, & Stephens, 1993; Waywood, 1994) developed a theoretical stance to the *writing-to-learn* phenomenon, a movement that had captured considerable momentum in the United States. In investigating the writing-to-learn phenomenon, Waywood claimed that researchers needed to look beyond the writings of Vygotsky, Piaget, Dewey, Halliday and Bruner if they were to establish an appropriate theoretical base for the movement. Adopting a phenomenological philosophy where the link between language and the inner world is broken, Waywood asserts that the hermeneutic tradition, associated with the interpretation of sacred text, has been extended to accept almost any thing or any act as a *text*. Hence, a writing-to-learn text is a student’s communication about becoming a learner. What one reads in a text does not represent the writer’s inner self, for that “is never expressed but is the silent frame” (Waywood, 1996, p. 611). In essence, the pattern of language used presupposes and formats what can be said. This theoretical position offers a new lens for research on writing and mathematics but as Ellerton, Clarkson and Clements (2000) observe, it does raise a continuing question for

research in this field: “Does thought require language or does language emerge through thought?” (p. 44).

In response to the initiatives of Ellerton, Clarkson, and Clements, the link forged between language and culture has built a bridge for researchers in the field of language and researchers in the area of what I later refer to as the “Sociocultural Aspects of Mathematics Learning and Teaching.” In fact, further reviews may see language and mathematics totally integrated within this broader field.

Although the integration of research on “language factors and mathematics” into the broader area of “sociocultural aspects of mathematics learning and teaching,” is a desirable move, the disappearance of language as a discrete category in its own right brings a touch of sadness. It is an area in which Australasian research has played a key role in the international arena; often acting as a balance and a buffer to the European research with its strong focus on culture and language.

Sociocultural Aspects of Mathematics Learning and Teaching

This is a theme where Australasian research has made a significant and growing impact in the last decade. In addition to the international impact of gender and language research that I have already discussed, Alan Bishop’s arrival in Australia appears to have had a generative effect on Australasian research in this theme. Besides Bishop’s own seminal contribution, I highlight international contributions such as Bill Barton’s clarifying and focusing contributions to ethnomathematics, Robyn Zevenbergen’s critical analyses of assumptions and actions in mathematics education, and Bill Atweh’s prodigious contributions to sociocultural research including recent initiatives in the globalization of mathematics education. As further evidence of our strength in this field, I draw your attention to the special ESM issue (Lerman, 1996) on Sociocultural Approaches to Mathematics Teaching and Learning. This issue contained eight articles --four of them by Australasian authors: Barton, Crawford, Taylor, and Zevenbergen.

Alan Bishop’s seminal work in the field of social and cultural studies (Bishop, 1988a, 1988b, 1994) rests centrally on the tenets that mathematics is part of everyone’s cultural heritage and mathematical knowledge influences every aspect of today’s technologically-based societies. Hence, for Bishop, mathematics needs to be better understood and part of everyone’s curriculum (Bishop, 1991, p.196). Bishop’s

groundbreaking work has generated a far-reaching rubric not only for Australasian research on social and cultural contexts in mathematics education but also for international research in this field.

For this review I will confine my attention to two major issues in the sociocultural theme: ethnomathematics and research that *critically* examines a range of presuppositions associated with contemporary theory and practice in mathematics education. This latter area on critical perspectives includes politics of mathematics education, and microcontexts such as schools, teachers, classrooms, and democratic access to mathematics.

Ethnomathematics. In a key paper that is cited frequently in overseas journals, Barton (1996b) attempts to clarify the nature of ethnomathematics. Acknowledging that there are difficulties with the term ethnomathematics, Barton analysed the work and ethnomathematical directions of the key players, D'Ambrosio, Gerdes, & Ascher. From this analysis, he negotiated a new definition of ethnomathematics that envisages it as a research programme of the way in which cultural groups (ethnic, national, historical, or social) understand, articulate, and use concepts and practices that we describe as mathematical, whether or not the cultural group has a concept of mathematics. (p. 214).

Bishop (1994) and Barton (1996a) have also suggested that ethnomathematics can be informed by an anthropological approach and this approach appears to be evident in the monumental works of Harris (1991) and Lean (1992). Harris's study documented Australian Aboriginal conceptions of time, space and money and compared their remarkable skills with the skills developed by mariners who had access to mathematical computation and technology. Lean collected data on 900 counting systems of Papua New Guinea, Oceania, and Irian Jaya, compared older and newer accounts of the counting systems, and compared neighbouring counting systems from both Austronesian and non-Austronesian languages. Lean's conclusions about the spontaneous developments of these ancient cultures challenged traditional theories describing the spread of number systems from Middle East cultures (For a valuable analysis of Lean's study, see Owens, 2001). With respect to Lean's work it is noteworthy that Bishop (1995) discussed and paid tribute to his work in a paper that focused on the anthropological perspective.

Bishop (1994) and Barton (1996b) definitions of ethnomathematics also incorporate investigations of mathematical knowledge of different groups with the same society. Within this domain Balatti (1995) has examined financial planning in an advanced capitalist society and Peard (1995) compared the ways that two groups, one familiar with out-of-school gambling contexts and one not familiar with these contexts, viewed probability. Although these studies need further international exposure, they are in a relatively emerging area.

Politics of Mathematics Education. Jan Thomas (e.g., Thomas, 2000, 2001; Clements & Thomas, 1996) has led research in this area with several papers dealing with issues that encompass and influence government policy decisions in mathematics education. Barton (1995) and Ellerton and Clements (e.g., 1994) have also contributed to this work especially in relation to the national curricula debate. Much of the work in this area has focused on Australasia and hence the impact is somewhat limited. However, Ellerton and Clements' (1998) paper on curriculum and research certainly raised some ripples in other countries (e.g., Kilpatrick, 1996) and Thomas's finding, that governments are increasingly diminishing the influence of expert opinion in quest of rapid solutions and global conformity, is likely to resonate with the wider international community.

Microcontexts. In the field of microcontexts (schools, teachers, classrooms, and democratic access to mathematics), there appears to be a robust and extended line of research from our region. A number of the studies use frameworks from theoretical perspectives, such as critical theory, discourse theory, sociolinguistics, situated cognition, and poststructuralism, to question what is happening in mathematics classrooms (e.g. Crawford, 1996; Klein, 1994; Zevenbergen, 1996). Moreover, they often examine adequacy issues in relation to factors like social difference, ethnic background, dominant language, and school characteristics.

In a timely paper, Crawford (1996) used a Vygotskian perspective to explore the opportunities and challenges of learning mathematics in the information era. Her insightful treatise analysed the ways in which information technologies have changed the context and nature of mathematical activity in society and proposed changes that will be needed if the full mathematical potential of information technology is to be fulfilled in school mathematics.

Two Australasian papers on microcontexts, Zevenbergen (1996) and Taylor (1996), were amongst the first mathematics education papers to use what Gage (1989) called the “critical theory” paradigm. Both of them argued that constructivism, as an epistemology, had limited value in transforming mathematics classrooms. Zevenbergen claimed that constructivism ignored the social and political contexts in which mathematical knowledge is located, and ipso facto legitimated the marginalisation of mathematics for many social and cultural groups of students. Taylor advocated critical constructivism, based on the social philosophy of Habermas, as an alternative culture of communicative action because of its central concern with discourse ethics and the moral agency of the teacher. Using a similar approach, Zevenbergen (2000, 2001a) has also written some internationally-noted papers on “positioning” of students in school mathematics classroom and issues of reform and justice.

Klein’s (e.g., 1994) earlier studies also dealt with constructivism as a deficit model in teacher education. Her recent research (Klein, 2002) strengthened that position when she used a poststructuralist perspective to examine how mathematical knowledge is produced during classroom interactions. She concluded that it may be possible to revive mathematics teaching when taken-for-granted assumptions of mathematics, as a rational, free choice, “proficient” (NCTM, 2000), and humanist subject, are revised.

Like Zevenbergen, and Klein, Atweh and his colleagues (Atweh, Bliecher, & Cooper, 1998; Atweh & Cooper, 1995) take the position that the social context needs to be critically analyzed in terms of structure, values, and power relationships. In the two studies cited, the researchers used a sociolinguistic frame, developed by Halliday and Hasan (1989), to investigate the social context of two mathematics classrooms that differed in the socioeconomic backgrounds and genders of their students. In their latter study, they concluded that differences in the interactions of the two classrooms were mediated by teachers’ perceptions of students’ needs vis-à-vis ability, gender, and socioeconomic background. In essence, teachers perceived their classes as needing different dialects of mathematics that have differing values in our culture.

More recently, a book edited by Atweh with Forgasz and Nebres (2001) has given sociocultural research an added boost. Grouping together an international cast of authors, Atweh and his colleagues take a timely look at key issues in the field as they relate to

global, regional, and local contexts. Atweh's own chapter with Clarkson breaks new ground in producing a framework to examine international collaboration in mathematics education reform.

When one combines the studies in sociocultural aspects of mathematics education with those in gender and language, it is clear that the expanded field is a poignant one using rich paradigms that are already common in other disciplines like literature, philosophy and the arts. Although many of these paradigms are still relatively new in mathematics education, the extent and the power of the research in this theme is already challenging the hegemony of psychological research on mathematics learning.

Early Childhood Mathematics

As I have already noted in the section on "Cognitive Aspects of Mathematical Learning," Australasian research has made a pervasive impact on young children's mathematical thinking. I refer particularly to the international books and journal contributions of researchers who have worked in *cognitive models* (Biggs & Collis, Halford), *number* (Bobis, Davis, Hunting, Irwin, McIntosh, Mulligan), *space and measurement* (Mitchelmore, Outhred, & Owens), *data and chance* (Jones, Watson), *problem solving* (English) and language (Ellerton, Clarkson, & Clements)

Our early childhood research has also been showcased through the leadership of two groups, Bob Perry and Sue Dockett and the late Helen Mansfield and Neil Pateman. Perry was the chair of the first Early Childhood Working Party at ICME 5 and he and Dockett (2002) recently wrote a chapter on early childhood research in an international handbook (English et al., 2002). Mansfield also chaired a similar working group at ICME 7 and later served with Pateman and Bednarz as editors for the book "Mathematics for Tomorrow's Young Children." As a tribute to the efforts of these two groups I will briefly summarize the thrust of their work and locate references they made to Australasian early childhood research.

Following a historical overview of early childhood education, Perry and Dockett (2002) focused on the following elements: powerful mathematical ideas that are accessible to young children, mathematical ideas that young children bring to school, and learning and teaching of mathematics for young children. In the sections on *powerful*

mathematical ideas that are accessible to young children and *mathematical ideas that young children bring to school*, Perry and Dockett refer to research on cognition, number, space and data: areas where the Australasian researchers I cited at the beginning of this section are well represented. Their third section examined young children's *learning of mathematics and its teaching* and Australasian researchers figure prominently in areas like social and cultural relationships, play and mathematics, and technology. With respect to *social and cultural relationships*, Perry and Howard (2000) observe that in both prior-to-school and school settings, one powerful way in which mathematics can be connected to children and their knowledge base is through a consideration of cultural aspects of learning mathematics. In the area of play and mathematics, Dockett and Fleer (1999) have noted that warm facilitating relationships can be supported through and within play and that children's play can be complex in terms of theme, content, social interactions and mathematical understandings. With respect to technology, I will make particular mention within the next theme of Groves' work in that it demonstrates how calculators expand children's knowledge of number when calculators are used in a variety of ways. Dockett (2000) also found that computers have great value in encouraging children to explore mathematical ideas in ways not otherwise possible with other materials.

The book edited by Mansfield, Pateman, and Bednarz (1996) was conceived from the working party at ICME 7 and was part of the prestigious Kluwer mathematics education series. It examined four key issues related to the mathematics curriculum for young children: what elementary mathematical concepts should be included and how are they formed; the influence of social and cultural environments on the formation of elementary concepts; the role of language in the formation of elementary concepts; and the development of classroom strategies for optimal development of elementary concepts. The authorship included powerful researchers from overseas (e.g. Cobb, Fischbein, & Steffe) and, in addition to Mansfield and Pateman, it included Australasian researchers like Irwin, Renshaw and Wright whose work is referred to elsewhere in this chapter.

These works ensure our international standing in early childhood research. In fact, it can reasonably be said that Australasian researchers have enriched both theory and practice in early childhood mathematics during the past 20 years.

Teacher Education and the Teaching Mathematics

The impetus of the Mathematics Education Lecturer's Association (MELA) in the eighties and nineties provided a valuable forum for mathematics educators to discuss and formulate research that related to teacher education, professional development of teachers and processes of classroom teaching. In some sense these developments in teacher education were consummated at a more significant level with the advent of *Mathematics Teacher Education and Development* in 1999.

When I looked for ways to provide a warrant for an Australasian impact in this theme, the activities of several people came sharply into focus: David Clark's major international books and papers; Peter Sullivan's appointment as an editor of JMTE and ipso facto his research record; Susie Groves' and Patricia Foster's well recognized work in technology; Judith Mousley's and Bill Atweh's international activities in action research. Add to these contributions, the fact that Australasia has its own teacher education research journal (MTED) and recently published in MERJ a focus issue on technology, and the evidence for impact is difficult to refute.

What is the nature of the research problematic in teacher education and teaching processes? My response is as follows: methodologies to capture classroom teaching processes, prospective teachers and quality mathematics teaching, teacher development, teacher knowledge and beliefs, instructional processes, the role of technology, and action research.

Methodologies. In the area of research design, David Clark's "complementary accounts methodology" (1998) has made a major contribution to research that focuses on classroom teaching and learning processes. The methodology combines videotape data with participants' (researchers, teachers, students) reconstruction of classroom events. From this integrated data set, complementary accounts are constructed by the research team in such a way that consensus is rejected in favor of accounts that must be consistent with the data but not necessarily consistent with each other. This design process is ideally suited to using a research team with diverse theoretical perspectives and Clarke claims, "... it is only through the accumulation of complementary accounts that our portrayal of classroom learning will approach the complexity of the process we seek to model" (Clarke, 1998, p. 111). Given the huge impact of TIMSS video study (Stigler et al.,

1999), it is highly possible that Clarke's methodology could be used in future international studies on classroom learning environments.

The monograph edited by David Clarke (2001) provides a large-scale application of his complementary accounts methodology. Researchers from three nations observed and analyzed mathematics and science lessons from different perspectives such as negotiation and intersubjectivity; learning in the zone of proximal development; scaffolding and value positions of the teacher and the students. Space will not let me describe the results of this global study; however, the multifaceted perspective that the results provide on the complexity of practices and meanings in classrooms is particularly impressive. Further elucidation of this methodology and the kind of multiple conclusions it generates can be found in other Australasian studies (D. J. Clarke & Helme, 1997; Helme & D. J. Clarke, 2001).

Prospective Teachers and Quality Mathematics Teaching. Following some early work by Sullivan (1989) on the characteristics of beginning teachers and factors inhibiting change in their teaching, he and Mousley (Mousley & Sullivan, 1997; Sullivan & Mousley, 1998,) generated some important studies that looked at pedagogical reflections of prospective teachers of mathematics during teacher education courses that incorporated a multimedia approach. They claimed that use of "pedagogical dilemmas" and reflections facilitated prospective teachers' understandings of mathematics classrooms and the roles teachers play in them. A U.S. study involving Jones and Malone (Langrall, Thornton, Jones, & Malone, 1996) also investigated the effect of reflective analysis on prospective teachers' beliefs and teaching practice. These researchers demonstrated that an intervention program which enhanced students' knowledge of the NCTM *Standards* (1989) and provided opportunities for reflective analysis significantly enhanced their beliefs and instructional practice.

In the area of preservice teacher beliefs, Nisbet advanced the methodology and Schuck produced some striking findings about their beliefs. Nisbet's (1992) beliefs scale focused on teaching mathematics and incorporated four subscales: anxiety, confidence, desire for recognition and pressure to conform to peer standards. Schuck (1996, 1999) examined prospective teachers' beliefs about the learning and teaching of mathematics. She claimed that prospective teachers were fettered by various chains

(beliefs) including a perception that being knowledgeable about mathematics was a disadvantage for a primary teacher. This is a fascinating result and given overseas research on the relationship between teacher beliefs and classroom practice (e.g., Fennema & Nelson, 2000), it would be interesting to follow some of Schuck's students in their novice years.

In a study that focuses on prospective teachers but could have been included in the section on action research, Zevenbergen (2001b) discussed a four-year project that involved prospective teachers in peer assessment. Zevenbergen claimed that peer assessment is an effective tool in preservice teacher education but cannot be seen as an alternative to lecturer-based assessment due to the variability in marks between and within student cohorts.

Australasian research on *quality mathematics teaching* has attracted a lot of international interest. Because much of the research was grounded in one of several forms of constructivism, it is appropriate to underscore the leadership role that John Malone and his colleagues (e.g., Malone & Taylor, 1994) played at two ICME conferences in elucidating and clarifying constructivist epistemology. Frid and Malone (1995) also used constructivism, in combination with Frid's (1992) work on sources of conviction and D. J. Clarke's work on classroom consensual processes (1986), to study the relationship between students' classroom experiences and their construction of mathematical meaning. The fact that the teacher emerged as playing the key role in mathematical sense making highlights the need for further research on quality teaching.

Mousley and Sullivan with Clements used a constructivist base to inform their research on quality mathematics teaching. Their early studies (e.g., Mousley, Sullivan, & Clements, 1991) examined prospective teachers' perceptions of mathematics teaching observed during their practicum. As these perceptions turned out to be the antithesis of what is suggested in current pedagogical theories, like constructivist learning, Sullivan and Mousley (1994) developed a more informed model of quality mathematics teaching for prospective teachers. In constructing their model, they claimed that *building understanding* was the focus of the network of categories they identified. In a related study, Van Zoest, Jones, and Thornton (1994) compared beliefs about quality mathematics teaching for a group that had engaged in a mentoring experience (oriented

towards socioconstructivist learning) and a control group. Although both groups were taking a methods course focusing on contemporary pedagogy, the mentor group showed significantly stronger beliefs in socioconstructivist approaches to learning.

Teacher Development. Research on quality mathematics teaching and reform documents (e.g., AEC, 1991) call for a very different role for the contemporary mathematics teacher. Teachers are expected to create supportive learning environments, to utilize worthwhile mathematical tasks, to manage students' mathematical discourse, and to promote mathematical sense making. Teachers are also expected to help students make mathematical connections and to use technology to enhance students' mathematical conceptions. In the wake of these reform visions there has been strong pressure for professional development programs and for research activities to monitor teacher change.

Australasian mathematics educators were ahead of their time in initiating major professional development programs. The Reality in Mathematics Education (RIME) project for secondary teachers (Lowe & Lovitt, 1984) preceded the Australian reform document in establishing a network for teachers to develop and share exemplary lessons and to collaborate with researchers. In fact, I suspect that RIME provided a strong impetus for subsequent teacher enhancement research (e.g., Stephens, Lovitt, D. M. Clarke, & Romberg, 1989) as did the link that was forged between Australasian researchers and the University of Wisconsin.

Doug Clarke's research in the nineties seems to have emerged from all of these links. In a series of well-cited studies (D. M. Clarke, 1997, 1999), he established not only a viable theoretical framework for monitoring the role of the teacher in a reformed classroom, he also developed a potent design for tracing teacher development and factors that influenced it over an extended period. The design involved case-study analysis, participant observation and clinical interviews. Observing teachers initially over a 7 month period and briefly 5 years later, Clarke observed that the greatest changes in the teachers' role related to increased comfort with posing non-routine problems, allowing students to struggle together, and providing opportunities for student reflection.

Jones also benefited from U.S collaboration and financial support in a series of studies on two professional teacher enhancement projects (Jones, Thornton, et al., 2000; Swafford, Jones, Thornton, 1999). These studies focused on the professional

enhancement of middle school teachers and examined changes in teachers' classroom practice that resulted from enhanced teacher knowledge, reflection, and regular classroom support. Although the teachers in both projects showed a marked increase in the extent to which they used student discourse and student collaboration, one group of teachers demonstrated greater growth in engaging their students in mathematical sense making. In a related study (Swafford, Jones, Thornton, 1997) the researchers demonstrated that increased knowledge (subject matter knowledge and knowledge of student cognitions) led to a significant shift in student expectations, and in teacher characteristics like risk taking, time spent in teaching geometry, and trying new instructional approaches.

Classroom Instructional Processes. The impact of new theoretical designs, like teaching experiments (Cobb, 2000), instructional design experiments (Wittmann, 1998), and educational development and developmental research (Gravemeijer, 1998), has brought instructional processes into sharp relief. Apart from the work of D.J. Clarke with the "complementary accounts methodology," there does seem to have been much Australasian research that is grounded in these new designs. However, Australasian researchers have made an impact in the following areas of: curriculum perspectives, use of open-ended mathematical tasks, assessment, and the role of technology.

The nature of classroom instruction is closely to the role that the teacher plays in the curriculum process. Barbara Clarke, Doug Clarke, and Peter Sullivan (1996) focused on the *curriculum and the teacher* in a chapter that was well positioned for international exposure. Their comprehensive review started by revisiting some of the influences and trends that have affected the mathematics curriculum development over the last 30 years. The main thrust of the chapter examined issues associated with teacher-driven curricula versus externally imposed curricula and on illustrations like the Japanese Lesson Study (teacher-driven) and Mathematics in Context (externally imposed). I found their discussion of the teachers' role and status in the curriculum process insightful and provoking for researchers engaged in small and large-scale research.

Recent curriculum movements (AEC, 1991), have generated considerable interest in "learning via problem solving" (Schroeder & Lester, 1989) and the use of *mathematical tasks*. Sullivan and his colleagues have made key contributions to research on content-specific open-ended tasks from the perspective of both design and theory

building. After some early studies (e.g., Sullivan and D. J. Clarke, 1991) comparing the difficulty of open-ended tasks and closed tasks, Sullivan, Bourke, and Scott (1997), concluded that interventions, characterized by a predominance of open-ended tasks and few teacher explanations, generated effective learning environments for both confident and less confident students of mathematics. In a later study, Sullivan, Warren, and White (2000), generated a framework for analyzing open-ended and closed tasks and used it to examine the effects of context and degree of difficulty. One of the strengths of the research is the judicious use of both qualitative and quantitative designs; the other is its practical value for teachers.

Assessment activities are central to classroom instruction. In this area, the research of David Clarke and his colleagues (D. J. Clarke, 1996; D. J. Clarke & Stephens, 1996; Lesh & D. J. Clarke, 2000;) has made a strong international impact not only in research but also in the practice of classroom teachers. I will try to focus his work by highlighting key ideas in his insightful chapter in his 1996 chapter. Exhorting educators to exploit assessment as a powerful means of developing mathematical competence rather than as an end in itself, Clarke examined developments in the field according to policy, practice, practicalities, and praxis. *Policy* was concerned with the politics of assessment, devolution and responsibility for assessment, groups disadvantaged by assessment, and social perspectives on mathematical competence. In *Practice* he focused on the new face of assessment and issues like student-constructed responses (rather than memorized responses), technology, quality mathematical performance, and inferences from student performance. *Practicalities* is concerned with the place of traditional methods, principles by which students are graded, tensions between local and systemic issues, and the consequences of assessment. Within *Praxis* Clarke deals with what is happening in assessment in various parts of the world. The Australasian exemplars highlighted impressive assessment trends in our region. Mind you like other international countries, we will continue to face the political, social, and educational ramifications of international assessment comparisons like TIMSS.

Technology. With respect to the role and use of technology in the mathematics classroom, Jones et al. (2002), Perry (2002), and Ruthven (1996) make strong references to Groves' calculator research with elementary students over the last decade. A key

finding of her work (e.g., 1993, 1994) is that elementary children who use calculators identify new insights into modes of calculation, build earlier conceptions of large numbers, and develop different perspectives on checking calculations. Groves (1994) also noted that primary children who took part in projects emphasizing mental computation and the use of calculators did not make more use of their calculator; rather they made more important choices of methods of calculation.

Forster's extensive research on graphics calculators has made a much-needed contribution over a relatively short time period. As a personal note three of my U.S. doctoral students independently identified her work as a key reference in relation to secondary students' mathematical thinking with graphics calculators. Using constructivism as a rationale for much of her early work Forster (e.g., Forster, 2000) has looked at the successes and difficulties encountered by students using technology in classrooms and examinations. More recently, it was heartening to see she and Taylor (2000, 2003) publish in *ESM*. Combining their research strengths, they used multiple psychological and social lenses to examine an individual student's conceptual development of the ARG function and also to explain the rationale for favourable and unfavourable classroom outcomes within a technology environment. Their first study underscored the importance of reflection on prior activities as a key to long-term mathematics advancement, with or without technology; their second study highlighted the importance of teachers' mode of questioning, instructional sequence and the cooperative endeavour of the students.

In a similar manner the MERJ special issue on "Technology in Mathematics Learning and Teaching" (Chinnappan & Thomas, 2000) has accentuated the magnitude and scope of technology research in our region. While it is not appropriate to review all the articles in this issue, I will emphasize the range of topics covered by Australasian researchers: MATLAB in undergraduate mathematics (Cretchley, Harman, Ellerton, & Fogarty); understanding and representation of fractions in a *JavaBars* environment (Chinnappan); student difficulties in operating a graphics calculator (Mitchelmore & Cavanagh); teacher and student roles in technology-enriched classroom (Goos, Galbraith, Renshaw, & Geiger); and supercalculators and university entrance examinations in calculus (Hong, Thomas & Kiernan). Thomas has also coauthored an *ESM* paper, with

Graham (2000), that analysed a learning environment where students used the letters of a graphics calculator as “static labeled stores with changeable values (p. 279)” to gain an appreciation of algebraic variable. They claimed that the graphics calculator enhanced learning of the concept in a significant way compared with previous learning approaches they had used.

As a final fanfare to the growing impact of our research in technology, I make mention of Frid’s (2002) study where students worked “in the manner of mathematicians” in a virtual classroom and offered their solutions in the form of correspondence on the internet. While it is still early days with technology research it seems to me that our region is demonstrating a high level of energy and versatility in the area. By way of caveat, I would highlight the need for Australasian researchers to mount further studies that use microworld environments and situated cognition (e.g., Noss and Hoyles, 1996).

Action Research. After a period of inactivity in the international scene, action research seems to be regaining momentum. Crawford and Adler (1996) make a strong argument for having teachers engaged in action research as a medium for professional development and ipso facto improvement in the processes and quality of mathematics teaching. Crawford also illustrates this notion of teacher enhancement by outlining some interesting work that she has done in involving preservice teachers in action research.

Mousley has impacted the international scene with two standout papers. The first paper (Mousley, 1992) provided a valuable expose on “teachers as researchers” by clarifying the context in which teachers operate and by presenting an agenda of the potential research interests for teachers: “what works for us,” partnerships in classroom research, curriculum development, and school improvement programs. Mousley’s other contribution has made even more impact. She and her colleagues (Zack, Mousley, & Breen, 2000) aggregated a collection of action research studies carried out by teachers and by teachers in collaboration with researchers. This book met a real lacuna in the field of action research because there are all kinds of useful books describing action research but almost nothing that displays *reports* of action research. The Mousley et al. collection addresses this need and as a result is ideally suited for workshops and courses involving teachers and researchers.

Atweh and his group of researchers and teachers have been active for more than 6 years in the pursuit of action research. Much of their work (Atweh & Heirdsfeld, 1998) relates to novice female teachers in transition and focuses on their roles and challenges as they become teachers of mathematics. In spite of support from the researchers, the novice teachers still report a sense of isolation and a need to involve more experienced teachers. In fact, given the shortage of research in the area of becoming a teacher (Brown & Borko, 1992, Mewborn, 1999), such an undertaking has the potential for ongoing global impact.

In discussing the theme, *Teacher Education and Teaching Mathematics*, I have not incorporated to any extent research contained in the first four volumes of *Mathematics Teacher Education and Development*. Although I believe that this journal has great potential and am heartened by the growing number of overseas researchers who publish in it, there has not been sufficient time for it to impact the international scene except through the work of researchers whose work is already well known. In passing I note that a significant number of such researchers are represented in the first four volumes in fields that have been recognized in other parts of this chapter: Frid (constructivism); Goos (scaffolds for learning); Groves (numeracy); Helme and Stacey (decimal understanding); Klein (poststructuralist analysis); Nisbet & Warren (teacher beliefs); Schuck and Foley (electronic learning communities); Walshaw (poststructuralism); and Watson (statistical intuition)

Impacts on Mathematics Education Research Per Se.

In addition to the major themes, there have been three significant publications that have impacted the nature and directions of mathematics education research: *Handbook of International Research in Mathematics Education* (English et al., 2002); *Research and Supervision in Mathematics and Science Education* (Malone, Atweh, Northfield, 1998), and *Mathematics Education Research: Past Present and Future* (Clements & Ellerton, 1996). All three have been the brainchild of Australasian researchers and the fact that they look at key issues like, “where are we at in mathematics education research?” is indicative of our growing status in the international arena. In dealing with each publication, I will briefly summarize the intent of the work and add my own comments or the comments of a reviewer.

The handbook, edited by Lyn English, brings together authors with rich cultural diversity in examining international research priorities, democratic access to powerful mathematical ideas, methodologies, and the influences of advanced technologies. As the topics suggest, this is not a traditional survey of research on familiar topics in the teaching and learning of mathematics; rather it adopts a critical stance in gauging how elements of mathematics education research like democratic access, methodologies, and technology have and should interact with learners, teachers, and learning contexts.

Malone, Atweh, and Northfield's insightful book focuses on mathematics education as a research enterprise and is especially concerned with supervision of research, and ipso facto the building of future researchers in the field. Milton (1999, p. 76) encapsulates the core value of work when he states, "...this publication makes an important addition to the literature relating to research activity, and deserves the attention of the community of mathematics educators in general, and of researchers and research students in particular."

The publication by Clements and Ellerton (1996) questions whether research in mathematics education has lost sight of its crucial role. As Bishop (1998, p. 82) notes, Clements and Ellerton take issue not only with the frameworks and the methodologies of our research but also the role and generalizability of theory. In essence, they question whether the kind of research I have identified as making an important impact in the international field, really does enhance our knowledge of both theory and practice in mathematics education.

These are books with both a retrospective and prospective viewpoint. As such they make us think about the enterprise that is mathematics education research: the need for it to be critical and cognizant of the democratic ideals we serve and the means by which we intend to develop research and researchers for the future.

Conclusion

Before attempting to presage future directions in Australasian mathematics education research, let me make a brief connection between the key themes of this review and those of 1984 review. There were six themes in my 1984 review and the current review contains only three of these: cognitive aspects of mathematics learning, gender and mathematics, and teacher education and teaching mathematics. The other 1984

themes are now subthemes of these. The new themes on “language factors and mathematics” and “early childhood mathematics” were certainly emerging areas in 1984 and a number of studies in each of these categories were previously reported under learning and cognition. Research on “sociocultural aspects of mathematics learning” was not represented in 1984, and, as I will discuss below, this theme may be even more encompassing in future reviews.

Future Directions in Mathematics Education Research

Suppose we could crystal ball what happens to mathematics education research in the next twenty years. How will the directions and key themes change in that period? Space will not allow me to do a careful exposition of such changes but I would like to adumbrate the big picture.

The manifest tensions of our troubled world demand that everyone from world leaders to researchers in mathematics education play a role in promoting equity at all levels of education. In spite of our robust efforts in the last 20 years to provide equitable mathematical access to all children irrespective of culture, ethnicity, gender, economic and social position, mathematics is still seen by large groups including both the powerful and the marginalized as a great social and international divide. Success in mathematics guarantees reasonable prosperity if not incredible wealth.

Consequently, I believe that research dealing with sociocultural aspects of mathematics education will assume far greater importance in the next two decades. At present, research on social and cultural aspects of mathematics education tends to be a loose aggregate with at best limited links between researchers in its various areas. However, this is beginning to change as integrated and international perspectives like those of Atweh, Forgasz, and Nebres (2001) begin to emerge. Moreover, I envisage that sociocultural research on mathematics education will not only encompass issues like equitable access to powerful mathematical ideas, ethnomathematics, and critical perspectives mathematics classrooms, it will of necessity begin to incorporate the powerful theme I have referred to as “cognitive aspects of mathematical learning.” Let me foreshadow some signs of this merger. First, situated cognition is already one of the key components of sociocultural research and its role as a grounding theory in studies involving technology and microworlds (e.g. Pratt, 2000; Noss & Hoyles, 1996) shows

refreshing potential for individual and collective learning. Second, English et al., (2002) note that one of the major problems that mathematics educators face is how to restructure the overcrowding in existing curricula so that students from diverse backgrounds have access to the more sophisticated and complex mathematical ideas that society requires (p. 791). It goes without saying that this kind of problem is not going to be solved by psychological studies that often ignore student diversity, context, and cultural differences. Moreover, although researchers like Lakoff and Nunez (2000) have focused recently on *mind-based mathematics*, other researchers like Sfard (1997) and Goldin (2002) are quick to point out that processes of conceptualization require social mediation and occur with some social context.

What I am saying is that mathematics education research in the next two decades will take a more multifaceted yet integrated approach to learning and teaching; one that uses multiple lenses to look at teachers and learners from cultural, social, political and psychological perspectives. The research problematic will increasingly incorporate objects of research like equity, access, and context in concert with extant objects like learning and teaching,

Future Methodologies and Tools in Mathematics Education Research

In line with my proposal that research in mathematics education will take on a more multifaceted and integrated approach to teaching and learning, I suggest that research designs will need to accommodate theoretical frameworks that examine equity and access and incorporate cultural, social, political, and psychological perspectives. There are already designs that have the potential to accommodate this more complex research.

Teaching experiment methodologies (Cobb, 2000; Gravemeijer, 1998; Wittmann, 1998) have moved in this direction with Cobb's focus on the emergent perspective giving a central role to both the sociological and psychological, that is to both individual and collective learning. With respect to equity, Cobb and Yackel (1996) claim that if such issues are to be addressed during a teaching experiment, learning activities need to be complemented by a strong sociocultural perspective; one that is grounded in the social norms of the home and the school. The designs used by Klein (2002), Walshaw (2001), and Zevenbergen (2000) also have the potential to critically evaluate issues such as equity and access within methodologies like a teaching experiment.

David Clarke's (1998) *complementary accounts methodology*, described earlier in this chapter, is another contemporary methodology that enables multiple perspectives of teaching and learning. As Clarke notes, this methodology embodies multiple perspectives by utilizing a team of researchers whose interests may range from teacher values to student metacognition. At a more micro level the *principled design experiment* (Hawkins, 1997) is an exciting methodology that explores novel opportunities for learning in what are high-risk situations for software designers. In a real sense, principled design experiments are the technological counterparts of the teaching experiment in that they provide opportunities to observe students' mathematical thinking at the brink.

In proposing these directions and designs for the next two decades, I am aware that they require a high level of collaboration between researchers, teachers, and policy makers. Teams of researchers and teachers with different perspectives need to work together developing new research questions, enhancing extant methodologies, and complementing their analyses of data rather than engaging in a consensual reduction that eliminates the potency of the data. Researchers who have moved in these directions (Cobb, 2000) see this collaboration as one of the striking outcomes not only for themselves personally but for the teachers and students that become part of such projects.

Let me finish by saying that this kind of research requires large injections of external funds from both private and government sources. The record of both groups in the Australasian scene, during the last 25 years, has been patronizing at best and will continue to be that way until our research persuades the voters and the consumers that mathematical empowerment for all must be a nationally funded goal rather than political rhetoric.

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