

RELATIONALITY AND MATHEMATICAL KNOWING

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Current conceptualizations of knowing and learning tend to separate the knower from others, the world they know, and themselves (Radford, 2008). In this article, we offer *relationality* as an alternative to such conceptualizations of mathematical knowing. We begin with the perspective that Maturana and Varela (*e.g.*, 1998) outlined and move on to articulate some of its problems and our alternative. We then illustrate relationality with the help of a short classroom episode in which geometrical knowing arises from and is constitutive of a threefold relationality between (a) the subject and the material world, (b) the subject and others, and (c) the subject and itself. Rethinking knowing in terms of relationality enables us to think of the learning of geometry as knowing *anew*.

The Maturana/Varela position

In the biological theory of cognition, often referred to as enactivism or enaction, humans are thought about in relational terms (Maturana & Verden-Zöller, 2008). Human beings are theorized as complex biological “learning systems” that coordinate with the co-emerging environment that they “bring forth” (Maturana & Varela, 1998). Like all other living organisms, humans and their environment are in mutual determination because “organism and medium are operationally interdependent systems in their dynamics of states, each following its independent structural specification” (Maturana, 1978, p. 41). Figure 1, adapted from the work of Maturana and Varela, is a good way to succinctly illustrate this situation.

In the illustration, each circle represents an organism (for example, an individual student) as an ongoing process of being and becoming (simple black arrows). As both a condition for and a result of this process, the organism is also in constant relation to itself (the circular double arrows), in relation with other organisms (the horizontal double

arrows), and in relation with the material human world (the vertical double arrows). Furthermore, *knowing* in this perspective has to be taken as inseparable from *being* and *doing* because cognition, as a feature of all living systems, is to be found in the actions by which the organism coordinates with its surrounding conditions: systems come to know through their transactions with their environment. These transactions define the organism, affecting it and everything with which it is in relation. Being, doing and knowing thus concern, simultaneously and without any specific order, a relation to the world, to others and to oneself.

Many scholars have drawn on one or another of these ideas to help us rethink what it means to know mathematically. For example, rather than observing mathematical cognition as answer-generating or problem-solving, mathematical knowing exists in the “personal action of bringing forth a world of mathematical significance with others in a sphere of behavioural possibilities” (Kieren & Smmit, 2009, p. 23). In this view, students participate in creating a world while effecting their own structures: they co-emerge with the mathematical world they bring forth *with* and *for* one another. In knowing mathematically, subject and object (observer/observed) are not independent entities. They co-evolve together and, as a result of a subject-object relationality, “understanding [and consequent knowing] of self is not abstracted from the world which contains it but, rather, is the world” (Davis, Sumara & Kieren, 1996, p. 154). In action, knowing mathematically is co-extensive with the emergence of embodied thought, social relationships and cultural practices. It is not simply the result of a rational activity: knowing is centered in *living* and actions correlate with what others do and consider as being mathematical (Lozano, 2005). This account breaks with epistemologies in which mathematical knowing is thought of and theorized in terms of mental representations that monadic individual knowers construct in their minds.

This change in how we think about knowing mathematically comes with a change in what we look at as we pursue an interest in students’ learning. Thus, enactivism may prompt scholars “to pay attention to the relationship between things in a mathematical environment (ideas, fragments of dialogue, gestures, silences, diagrams, *etc.*) rather than to what each of those things might mean or represent in their own right and for the individual generating them” (Glanfield, Martin, Murphy & Towers, in Proulx, Simmt & Towers, 2009, p. 259). If mathematical cognition emerges from “listening-with” (rather than listening *for* or *to*), it is really the “inter-action” we need to attend to as the locus of

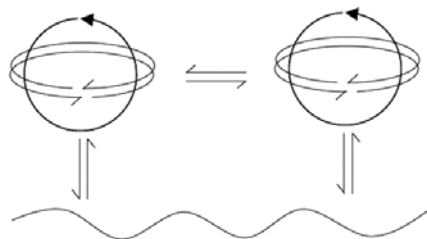


Figure 1. *Being and becoming in/as relations with oneself, another, and the world (adapted from Maturana & Varela, 1998, p. 180).*

mathematical cognition (Kieren & Smmit, 2009, p. 23). Students' actions are fully implicated in each individual's cognition; they change the world they all live in: it is through these non-linear, recursive, self-organizing processes that one contributes in a mathematical world (with words, gestures, intonation, *etc.*) and to which we turn our attention.

Problems and an alternative

Although the concept of relationality developed so far has offered a way to overcome the Galilean division of mind and nature and the Cartesian division of mind and body, the very way in which Maturana and Varela framed the issue makes them blind to a deeper concept of relationality. Because biology of cognition focuses on variables and their relations, it misses the very issue it has made its object of study: life and its *inner* unity (Henry, 2000). In phenomenological philosophy, the term "ekstatic" is used to emphasize the "standing" "out" that representation makes possible. Externalities and the relation between them miss the inner relation that is fundamental to the ekstatic forms of knowing mathematics through representations. This inner relation is not one of self-identity (a thing is identical to itself) but one of non-self-identity (a thing is *not* identical to itself). We never perceive the inner relation because it reveals itself only in and through its manifestations. But theorizing the relation via its manifestations does not get us to the unity – just as physicists do not understand the nature of light when they oppose its two manifestations, that is, its appearance as particle or as wave.

Enactivism theorizes an *inter*-action, the relation that binds two entities, such as an organism and its environment. This is where the problem lies, because a relationship and the entities involved will be thought of differently when we begin to think the relation first – *i.e.*, the non-self-identical unity – and then see how this unity manifests itself in different ways: as an individual cognizing organism, on the one hand, and as its relevant environment, on the other hand. The first approach takes the organism as the starting point. But on an evolutionary level, this is implausible, because the universe and life did not begin with the cognizing organism. Our approach, therefore, begins with a unity that itself unfolds into living and non-living because it always already contains both as possibilities.

Phenomenological philosophers have offered several alternative forms of conceptualizing relationality, which is thought to emerge from an unmediated experience of *with* (Nancy, 2000), *proximity* (Levinas, 1971), and *presence* (*Being*) (Heidegger, 1927/1977). This inner relation of a thing to itself is broken when the presence is made present again using representation. These representations are externalizations, ekstatic forms of presence. Relations between them – *e.g.*, between "oneself," "another," and the "world" – are relations between ekstatic terms rather than inner relations.

Relationality, therefore, cannot be thought of as beginning with a perspective that deals in externalities – such as, for example, the cognizing organism and its environment. Such a separation is possible only when life already exists: cognition is the *consequence* of life rather than its antecedence. For example, Maturana and Varela (1998) state that their "starting point to get an explanation that can be scientifically

validated is to characterize cognition as an effective action, an action that will enable a living being to continue its existence in a definite environment as it brings forth its world" (pp. 29–30). Sheets-Johnstone (2009) invokes the inherent "spectre of Cartesianism" that comes with taking the individual and the physical environment as such starting points, conveniently packaging "the mind" and "the body" without explaining "how 'the package' got there in the first place" (p. 215). Nancy shows how, when conceptualized as a collection of separate unities in relation or as a unified communal substance, a "community" (which can be a body, a mind, a society, *etc.*) loses the fundamental "in" of the being-*in*-common that defines it. Being, Nancy argues, needs to be thought of as *being-with* or simply as *with*. In this *with*, relationality (with oneself, others, and the world) is internal rather than external. It is the inner difference of the *with* that unfolds and manifests itself in terms of the externalities that the biology of cognition deals with. There is *with* in being-with, doing-with and knowing-with by means of which we are *relations*. Relations are what we are and what we do: knowing mathematically is not only a means of relating with known objects, it is the relation itself. The *with* of knowing-with is what defines the relation, that in and by which it realizes itself: if being-with "loses the with or the together that defines it [it] yields its being-together to a being of togetherness" whereas "the truth of community, on the contrary, resides in the retreat of such a being" (Nancy, 1991, p. xxxix). The essential idea is that despite the evidence of "being-with" or "knowing-with", we tend to forget the "with" and focus on the "being" or the "knowing." A relational perspective, by contrast, always keeps the "with" at the forefront, even though it might mean "the retreat" of being or knowing. Nancy therefore conceptualizes the relations between self, the world, and any form of cognition, as inner relations of non-self-identity in which being, doing or knowing are different manifestations of an essential *with*.

Knowing mathematically is not the production of an individual making-sense of the world, nor that of a collective in which such an individual comes to be part. This is so because the individual is always already an effect of this plurality and alterity. It is *in* the inner relation of the cultural-historical, socio-genetic and ontogenetic systems, in the togetherness that defines them, that knowing occurs.

Threefold relationality in a geometry task

From this point, we rearticulate the threefold nature of relationality (with the material world, with others, and with oneself) as an *inner* relation, through a case study of the learning of geometry. This analysis will lead us to rethink knowing mathematically as knowing *anew* in the external and internal threefold nature of relationality.

In the tradition of phenomenology, developing new ways of seeing requires attentively focusing on relatively simple, even ordinary phenomena, and training our attentiveness so that the experience of seeing can be renewed (Depraz, Varela & Vermersch, 2003). Similarly, the experience of knowing as relationality can become salient in the most mundane moments of mathematical activity. In the next three sections, we develop an analysis of a short episode from a third grade mathematics classroom. It took place at the very beginning

of a series of activities designed for the learning of geometry. It features two students, Nadia and Nate, who had been invited by their teacher to search the schoolyard for geometrical shapes. The lesson came about from the idea, important in many elementary school mathematics curricula, that students should develop the ability to identify geometrical figures in the environment. Using the episode, our intention is to illustrate the fruitfulness of the concept of relationality. As in phenomenological experiments, the object we attend to is not to be taken as “empirical evidence” to “prove” the truthfulness/validity of our claim, but as a means to develop new ways of seeing that allow students such as Nadia and Nate to make geometrical distinctions in their environment that they have not made before.

First fold: relationality with the material world

What concretely happens so that the students actually make geometrical distinctions in their environment? Nadia is walking, looking at the ground around her. All of a sudden, she runs to a something lying in the grass where she passed by minutes ago. She squats down next to it (Figure 2), and says, while pointing with her pencil:

01 Nadia: I see a circle right here! I’m gonna put down a circle ((she draws a circle on her clipboard)).



Figure 2. Nadia squats, pointing to the “circle” she found.

Nadia describes something geometrically. Some might say that she rationally develops, from the subjective experience of a knower who represents the world for herself and acts on the basis of those representations, intuitive theories about what she sees (e.g., von Glasersfeld, 1995). However, what is visible in the videotape fragment does not resemble a student making *rational* observations and developing theory. If mathematical rationality seems to come about then it is in the form of a student’s relationship with an object, a relationship that already exists as knowing; it develops geometrically, as a recognizable way of knowing.

What is it that makes Nadia’s seeing possible in the first place? The phenomenology of perception teaches us that seeing (e.g., a circle) requires eye movements. These movements are not random but *follow* structures in the world (a circular movement pattern); and they stabilize any circular image against the ground. [1] The seen arises from the invisible (unseen) *through* the ground to become visible (Marion, 1996): the circular thing *is given* as much as it is enacted. It is given because at some point in Nadia’s life, her eyes have formed the patterned movements that make the seeing and seeing-again of a circle possible. Seeing the circle also requires the movement of the eye and an *immanent* knowing of how this movement relates to circles: that is, a

movement arising from a self-affection from which sensory capacities and intentions themselves are borne (Henry, 2000). Our account differs from enactivism, for example, where the circular thing is attributed to the actions of Nadia (cognition as an *effective action*). In our relational account, agency and the fundamental passivity of “being given” (Marion, 2002) that characterize any give-and-take situation are made salient simultaneously.

Observing the circular shape of the object is not a mere reaction to raw stimuli (the physical world). Nadia walks about the school ground and recognizes something particular about a bottle lid on the ground: she “put[s] down a circle.” She has been walking in the area for several minutes without saying anything, although she could have seen the lid. Her sudden observation, “I see a circle right here,” shows that the presence of something does not necessarily correlate with its being there for the observer. It was not there to be constructed; it suddenly appeared, and therefore cannot be an intentional object. But once it has appeared, the bottle lid only exists for Nadia; and it exists geometrically, because of her way of making sense and interpreting the raw material offered to her senses.

Geometrical rationality emerges not by the presence of shapes in the environment on which a (reducing) pre-existing rationality applies. Nadia gives a description of the bottle lid and simultaneously defines herself as an observer making geometrical distinctions about her material world. When Nadia touches the bottle lid with her pen and says, “I see a circle right here!” she identifies the presence of “something” on the ground, and at the same time makes a geometrical observation about it. The world is not something merely known but something that we are in, moving, touching and breathing. Nadia comes to attend to one aspect that manifests itself to her as a result of the relation. What we see is not simply a construing of the material world, but the emergence of an object within its geometrical context. To know geometrically, objects become salient as they are attended to in geometrical ways.

When Nadia “knows” the bottle lid as a circle, there is a mutual determination of the observer and what is observed, but also the ongoing structuring of an event in and as a relation. It is not just “effective action” but also passibility that we need to bring into our account to understand how the agent is structured by the environment as much as the environment is structured by the actions of the organism, as suggested in an enactivist account. Subject and object do not just co-emerge and co-evolve, and knowing geometrically is not simply about rational attending. When Nadia geometrically brings into the realm of attention components of her surroundings, she is already in relationship, dynamically realizing herself with and in (with/in) the material world and with/in a material body. Moving about, looking for shapes and seeing an object are themselves relations to the material world. When Nadia comes to distinguish a shape in her environment, she is in a relationship with the material world from which emerges a thing: she *realizes* it, and *she* is realized simultaneously within this relation. [2] Here, objects in the environment *come to exist* for Nadia as she cognizes them *again*; and they come to exist as the locus of geometrical shapes because Nadia attends to them in a

particular way. In fact, Nadia herself comes to exist as a result of the relation – Nadia as the product as much as the antecedent of the relation. Thus, *recognizing* a circle in a bottle lid is necessarily re-cognizing what is already known, and yet knowing it *anew*. Nadia can learn about the circular nature of the shape of an everyday object, and bring it into the realm of mathematical thinking, only because she is already in dynamical relationship with this material object, because she already knows it, and thus can direct her attention to it. In fact, it is the inner relation that makes possible a separation of Nadia, who sees, and the visible object.

Second fold: relationality with others

Immediately after drawing a circle on her clipboard to record the observation, Nadia takes a few more steps, followed by her partner, Nate, still looking at the ground. Upon stopping again, she places her hands on her hips as she makes the following observations:

- 02 Nadia: Flowers have some sort in it ((points to the ground with her pencil))
 03 Nate: Circle! ((looks down))
 04 Nadia: Circle ((rotates her pencil, still pointing with it))

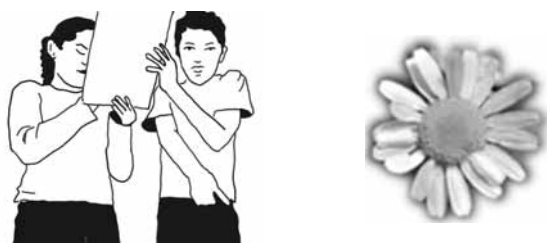


Figure 3. (a) The students find shapes and record their observation (b) a flower similar to the ones they observed.

Pointing to flowers or a bottle lid, using words and gestures, Nadia and Nate speak and gesture for themselves and for each other. Nadia articulates the natural objects as “flowers” and the presence of something in them, and thereby names and categorizes entities *for both Nate and herself*. Nate responds by stating, “circle!” but this, as any statement (Bakhtine [Volochinov], 1977), belongs to them both. In uttering the same word, Nadia confirms Nate’s hearing of her own turn as making the observation of a circle in the flowers. She further affirms and confirms Nate’s hearing by means of a hand gesture. Making their visual observations and conceptualizations (e.g., flowers and the presence of shapes in it) available to one another, the two students make common and one – *i.e.*, *communicate* (from Lat. *com-*, with, + *unus*, one, or *munis*, bound, under obligation) – what can be seen and thought. In doing so, they do not *construct* the world or simply enact it (alone or together). Rather, from a phenomenological perspective (e.g., Levinas, 1971), “Nadia” and “Nate” are the product of a relationality much more originary than any thought about relation and relationality. They hear, talk and listen with/in a consciousness

that is non-identical to itself: a consciousness that is continuously shaping as they bring about knowing geometrically.

By addressing and responding to one another, the students are not only thinking, but thinking and being aware *together*. Representational talk, talk *about*, is a derivative form of talk that arises from the whole moment in which a person is “talking-with-another-for-the-purpose-of” realizing something. In that wholeness, the ekstatic, relational dimension of knowing-with another makes possible the formulation of a geometrical distinction/description *about* a flower in the activity of looking for shapes in the environment. Representational talk is thus this external manifestation of an internal relationality with the other, for consciousness is possible only thanks to a state of *being-with*. Emerging from an undifferentiated and indifferent *being-with*, others are the condition for knowing anything at all. Knowing mathematically is necessarily knowing-with others (Radford, 2008) because mathematical understandings always occur through concrete actions of human beings in a social world. Mobilizing communicative resources (e.g., naming and gesturing the concept of “circle”) presuppose intelligibility, which entirely depends on the relationality with others that precedes and is irreducible to objective knowledge. We see the two students mutually orienting each other and themselves to the flowers and to geometrical shapes. The presence of circles in the flowers is brought into awareness in and through Nate’s confirmation of Nadia’s observation and Nadia’s confirmation of Nate’s interpretation. Both affirm to one another that something is visible and relevant. From the complexity of the visible that has arisen from the invisible (and therefore impossible to be aimed at), knowing about circles and flowers is something they do relationally as what they see together becomes what they see, although is it *never exactly what they see*.

The students make distinctions about aspects of the flower in and through their conversation resulting in a shared visibility of something that is not simply “out there” and also not simply “in here.” About it, the students develop explanations, elaborations or justifications: they are *subject to* and *subjected to* the visibility of the flower as much as they are the subjects (agents) of this visibility. [3] Before anything like knowing geometrically can happen, distinctions are *made*: students bring forth those distinctions in their recursive and responsive articulations. Nate, picking up on Nadia’s utterance, creates with her the experience of allowing geometrical words to accrue to a world always already shot through with (infinite possible) significance, including flowers. Nadia similarly contributes by specifying with an iconic gesture what the index word “circle” denotes. Here, then, the students’ knowing about the circular shape of flowers is entirely intertwined in their coordinated communications. This requires the words they use to be true bridges that constitute the speaker and listener as an irreducible unit. That is, their knowing is not “negotiated” nor “taken-as-shared,” but the commodity itself, resting on the common understanding required by the use of words. In a world that is as much given to Nadia and Nate as it is the result of their active vision and verbal description, knowing the flower geometrically in that moment is knowing-with one another, and with another, knowing anew.

Third fold: relationality with oneself

From the phenomenological perspective we work out here, the visibility of the world is the result of an inner relation from which the visible object (noema) and visual process (noesis) emerge as two irreducible moments or manifestations. The *with-others* is one condition for the ekstatic nature of the things that surround us, which, to be known, essentially require the possibility to make visible the invisible in something already seen. This capacity of making present again an “absent presence,” of knowing anew, not only demands relationality with the material world and with others, but also the mediated relation of the self to itself. To this relationality with oneself, a “thought” is the external relation of knowing to the very process of knowing, a manifestation of an inner relation of an undifferentiated *with* (others, world).

Having come to see the presence of circles in the flowers together, Nadia and Nate continue observing them. In the last part of the episode, we see them expanding their recognition of a shape, noticing other geometrical aspects in the flowers, and producing a model:

- 05 Nadia: And then they have the little roly things petals ((gestures an oblong in the air, then moves to the clipboard and starts drawing))
06 Nate: There are little circles in it ((looks down at the flower, then joins Nadia in completing the drawing, see Figure 4))

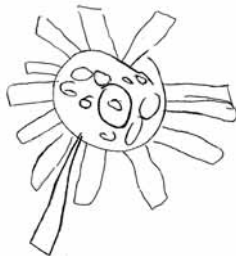


Figure 4. The students' model of a flower.

Nadia and Nate now make new observations, calling the petals “little roly things” (Nadia later explains them to be “cylinders”) and mentioning “little circles” (the anthers in the flower head). Both students then join in the production of a drawing (Figure 4), attending to the flowers in a way that connects everyday experiences with geometry. Their search for shapes in the environment now looks like a systematic inquiry in which their “thinking” transforms itself in the process of objectification (by means of speaking, gesturing, drawing) into material entities and in the concurrent reverse process of subjectification of these material entities (Hegel, 1979; Radford, 2008). It is precisely in this concurrent double movement that thinking and conceptual development take place. By finding shapes that are given to them as much as they enact (bring forth) them, the two processes are enabled to occur and their thinking to develop. Nadia finds a circle (bottle lid) and then observes that flowers too “have some sort in it,” and now notices cylindrical shapes she calls “little roly things.” Through Nadia’s history of relation with her own thinking, a certain form of awareness develops. Looking for shapes that are then given to her in perception, she identifies one, and then another. In the never-ending

flow of her experiences, a pattern of thought and communicative action emerges. In the recursive pattern of making geometrical observations, the inner relation of awareness to itself is once again *knowing anew*. That is, the relationality of knowing-with-onself is externalized in the expression of a thought embodying the contradictions of a restless mind that nevertheless always seems to know.

Nadia and Nate do not simply make observations but relate these observations and produce a model that expands their experience of the flowers. Producing their sketch (see Figure 4), the modeling activity not only continuously determines what counts as relevant in the (geometrical) observation of the flower, but also contributes to what can be counted on: geometry itself in the eyes of the student. That is, because knowing is an internal relation with oneself, the students’ model is not only that of a flower, but also makes present their own “knowing geometrically.” Not only “circles” are relevant here, but also the “little roly things,” the locus and proportions of these shapes. These elements are not necessarily externalized in the form of a thought, but emergently brought to the students’ knowing with/in knowing (geometrically). Nate and Nadia’s thinking develops as it relates recursively with itself through the mediation of communication. Knowing provides itself the conditions and the means by which further knowings emerge. Hence, we do not consider students’ knowing to rationally organize itself while progressively organizing the world. Rather, we articulate students’ relationship with their own selves developing in the form of (more or less) organized and rational knowings, according to the experienced contingencies of the material world and others. Through Nadia’s and Nate’s history of transactions with this social-material world, they develop their ability to *relate* geometrical ideas and experiences, make relevant distinctions to the purpose at hand, and delimit the object of their observation. A mathematical/geometrical form of rationality then is what actually results in the form of a consistent pattern of thought and communicative actions. The “organization” of Self stands externally and comes after the fact: knowing as the relationality with oneself.

Knowing-with-in: an enfolding relationality

In this article, we have articulated *relationality* as a different way of thinking about knowing. That is, we propose to begin theorizing cognition on the basis of a relation that precedes cognition and action, thereby escaping the theoretical problems that arise when cognition and learning are thought of – as they are in embodiment and enactivist theories – in terms of agency alone. This relationality contains three folds, three forms of relations that are integrally related. This threefold nature is captured in our second concept: *knowing-with-in*. In the moment of describing flowers or a lid, knowing mathematically is something Nadia and Nate do themselves, together, and with the material world. Recursive (cognitive) coupling takes place between these three dimensions, but it is enabled by a much deeper coupling in an originary relationality that is the condition for the subject, the material world, and the others. Today, at this place of time and history, we do not come into existence, and come to know, out of chaos. Knowing is also, and always, culturally-historically situated as a continuation of the

world, others, and oneself. When Nadia and Nate are looking for shapes in the schoolyard, they realize material, collective and personal possibilities whose origins make it so that knowing is always knowing *in* a situation (of self, others, the world). Not all children can engage in a search for shapes, second graders cannot everywhere safely walk a “school yard” to find junk and flowers, and the two students we followed could have been doing something very different on that day. *Unfolding* and *enfolding*, knowing is knowing-with-in: knowing anew from with/in forms of knowing that are embodied in the world, others, and oneself.

Critical and culturally sensitive approaches to mathematics, its teaching and its learning have already raised questions about what we term as “rationality” and its relation with mathematical thinking. Indeed, over the past 15 years or so, mathematics education has consistently moved toward a broader understanding of what it means to know mathematically. Theoretically, key to these new perspectives is the inseparability of knowing, doing and being taken in the enfolded (implicated) nature of the cultural-historical, socio-genetic, and ontogenetic levels. To this perspective, relationality offers a new, holistic, approach. Beyond situatedness and commonality, we think individual and collective location and cooperation in terms of what Nancy (1991) calls “co-proprietation”: the essential conjunction by which individual and collective are united and mutually define one another. In other words, the relationality of knowing appears to us to be conditional on any individual and collective “rational” deliberation. From our perspective, rationality is neither a condition for, nor a result of, mathematical thinking and knowing. Both, however, originate from an irreducible threefold relationship with/in which knowing is always already knowing anew. Rational observations, negotiations or self-organization in and as mathematical activity are themselves relations. They are what we are and what we do as relational beings: external manifestations of the ekstastic internal relations by means of which all knowing, doing, and being is possible at all.

At a more practical level, the threefold nature of relationality has important methodological repercussions for mathematics education researchers and for teachers and students. It means, for example, giving attention to examining what the students themselves articulate as the resources in and for their mathematical knowing. It also means considering mathematical communication as co-production of mathematical understandings. Relationality demands giving attention to the *process* by means of which certain forms of knowing are produced, and to how students and teachers recursively delimit the field of their experiences, as opposed to trying to make assumptions regarding what students or teachers know or do not know.

In this article, we have offered *relationality* and *knowing-with-in* as alternatives to perspectives that tend to focus on knowledge as something personally “constructed” by rational, autonomous and culture-free individuals. Relationality and knowing-with-in express the situated and situating nature of knowing in action. These ideas articulate knowing as an essential condition for any kind of knowing to take place. In contrast to deficit perspectives theorizing the absence or the inadequacy of student’s knowing, it emphasizes learning as knowing anew.

Notes

- [1] Detailed phenomenological analyses of the perception of mathematical objects can be found in Roth (2011a).
- [2] Phenomenologically speaking, it is in this instant of realization that the object seen (noema) and the work of seeing (noesis) split: the former becomes *objectively* present separate from the subject’s perception (Roth, 2011b).
- [3] Again, readers are alerted to attend to the agential *and* passive formulation, which does not exist in the enactivist formulation of cognition, where cognition is completely attributed to and understood in terms of *effective action* on the part of the organism (or the observer).

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