

SELF-CONSISTENT IMPEDANCE METHOD FOR THE SOLUTION OF ELECTROMAGNETIC PROBLEMS

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A two-dimensional, self-consistent, impedance method for modelling electromagnetic problems has been derived from Faraday's and Ampere's Laws. The result is a single matrix equation for the magnetic field scaled by the constitutive electrical parameters of the media. From this a complete solution of the magnetic field is calculated for every cell in the solution space. The source field is introduced into the model as a fixed magnetic field value on the right hand side of this matrix equation. This extends previous formulations of the impedance method to cover a wider variety of electromagnetic problems across the complete electromagnetic spectrum, including all types of materials. An absorbing boundary consisting of a single cell backed by a PEC was optimised to give a reflection coefficient of less than -55 dB for normal incidence. The method has been applied to various models including CW scattering from a dielectric filament at 10 GHz to determine permittivity changes, wave guides containing lossy media, and VLF surface impedance calculations.

1 Introduction

The impedance method is a numerical modelling technique based on Faraday's Law [1-2]. In two dimensions, the solution area is discretized into pixels bounded by impedance elements. The applied magnetic field perpendicular to the pixel generates a circulating current. In three dimensions, each voxel is bounded by impedance elements and the circulating currents are calculated in 3-D. The properties of the impedance elements are directly related to the complex permittivity of the local media and the size of the cell. In this way, any complex object can be modelled. From the circulating current values, the electric field can be determined. The advantages of the method include its intuitive simplicity, the use of cells of any shape or size [7] and the use of various excitation sources.

Previous forms of the impedance method have been used to solve low frequency (i.e., quasi-static) electromagnetic problems in both two and three dimensions [1-7]. One common target problem was the calculation of currents in biological tissue at 60 Hz [1-3], and at UHF [6]. Recently, the technique was adapted to solve eddy currents in metal plates [4-5], and in the earth for the calculation of VLF surface impedance [5-6]. In all cases, it has been assumed that: (a) the applied magnetic field is known throughout the solution space; (b) the magnetic permeability of all media in the model is that of free space; and (c) the self and mutual inductance between the elements is ignored.

In the self-consistent formulation developed in this paper, only the source field is known, and the total magnetic field throughout the solution space is calculated. From this, the distribution of the current and the electric field can be calculated.

2 Formulation

Consider the inhomogeneous model shown in Figure 1, subjected to an oscillating magnetic field H_0 with angular frequency ω . For convenience we will use a rectangular mesh of impedance elements throughout the solution space. Applying Faraday's Law to the $(i,k)^{\text{th}}$ cell we can write

$$(I_{i,k} - I_{i,k-1})Z_{1ik} + (I_{i,k} - I_{i+1,k})Z_{2ik} + (I_{i,k} - I_{i,k+1})Z_{3ik} + (I_{i,k} - I_{i-1,k})Z_{4ik} = -j\omega\mu_{ik}H_{i,k}\Delta x_{i,k}\Delta z_{i,k} \quad (1)$$

where $I_{i,k}$ is the circulating current in the $(i,k)^{\text{th}}$ cell, $H_{i,k}$ is the magnetic component perpendicular to the $(i,k)^{\text{th}}$ cell, $\Delta x_{i,k}$ and $\Delta z_{i,k}$ are the cell dimensions and Z_{mik} for $m=1..4$ are the impedance values given by

$$Z_{mik} = \frac{\Delta x_{i,k}}{(\sigma_{ik} + j\omega\varepsilon_{ik})\Delta y\Delta z_{i,k}} \quad (2)$$

where σ_{ik} , ε_{ik} and μ_{ik} are the conductivity, permittivity and permeability, respectively, of the material located in the $(i,k)^{\text{th}}$ cell.

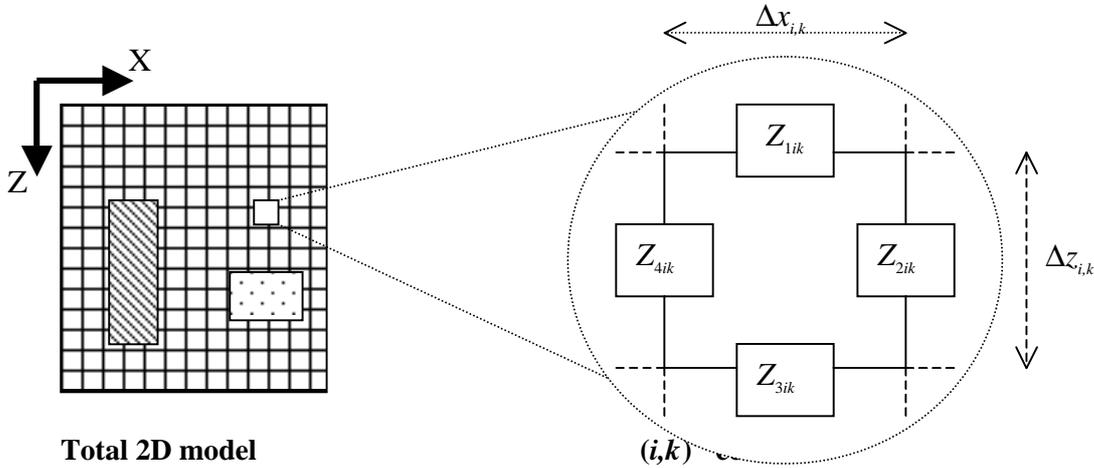


Figure 1: Discretization of the inhomogeneous 2-D solution space

Applying a discrete form of Ampere's Law along a rectangular path in the yz plane around Z_{1ik} , the current through this element I_{1ik} is

$$I_{1ik} = I_{i,k} - I_{i,k-1} = (H_{i,k} - H_{i,k-1})\Delta y \quad (3)$$

Using equation (3), equation (1) can be rewritten as

$$\left(\frac{2}{(\gamma_{ik}\Delta z_{i,k})^2} + \frac{2}{(\gamma_{ik}\Delta x_{i,k})^2} + 1 \right) H_{i,k} - \frac{H_{i,k-1}}{(\gamma_{ik}\Delta z_{i,k})^2} - \frac{H_{i+1,k}}{(\gamma_{ik}\Delta x_{i,k})^2} - \frac{H_{i,k+1}}{(\gamma_{ik}\Delta z_{i,k})^2} - \frac{H_{i-1,k}}{(\gamma_{ik}\Delta x_{i,k})^2} = 0 \quad (4)$$

where $\gamma_{ik} = [\omega^2\mu_{ik}\varepsilon_{ik} - j\omega\mu_{ik}\sigma_i]^{1/2}$ is the complex propagation coefficient in the $(i,k)^{\text{th}}$ cell. Equation (4) is the relationship for the magnetic field components in one cell. If the total number of cells in the model is N , then the matrix equation for all cells in the total solution space can be written

$$[S]_{N \times N} [H]_{1 \times N} = 0 \quad (5)$$

where S is a sparse, square, matrix of size N^2 which, although dimensionless, represents the electrical properties and the physical dimensions of the pixels in the solution space. H is a column vector of size N .

2.1 Source field

The source field is introduced into one or more cells by defining the H field in equation (4). For example, a plane wave propagating in the $+z$ direction is generated by defining $H_{i,k-1} = H_0$ in equation (4), for the top row of cells at $z = 0$ (i.e., $k=1$). We must solve the equation

$$H = S^{-1}H_0 \quad (6)$$

where H_0 is a column vector of length N , containing the source fields. In general, the number of non-zero elements in H_0 will be small. Clearly, the source field can be either located immediately outside, or anywhere inside the solution space.

The current through Z_{1ik} is calculated from equation (3). The horizontal electric field component E_{1ik} is calculated from Ohm's Law, i.e.,

$$E_{1ik} = \frac{(H_{i,k} - H_{i,k-1})}{(\sigma_{ik} + j\omega\epsilon_{ik})\Delta z_{ik}} \quad (7)$$

If the source field is set at the top of the solution space, and the air/material interface is at the top of the $(i,k)^{\text{th}}$ cell, then the electromagnetic surface impedance Z_s can be written as

$$Z_s = \frac{E_x}{H_y} = \frac{(H_{i,k} - H_{i,k-1})}{(\sigma_{ik} + j\omega\epsilon_{ik})H_{i,k-1}\Delta z_{ik}} \quad (8)$$

2.2 Boundary conditions

Unless otherwise specified, the magnetic field is not calculated at the boundaries of the model, and is assumed to be zero there instead. Thus the current in boundary cells must be directed parallel to the boundary implying a perfect magnetic boundary condition, i.e., a PMC termination. If we write $\partial H_y / \partial x = 0$ at the left and right boundaries, and $\partial H_y / \partial z = 0$ at the top and bottom boundaries, then the current parallel to the boundary cells is zero. This implies a perfect electric conductor (PEC) type of termination. An absorbing boundary condition requires losses in the boundary layer condition. A single cell wide boundary layer (thickness $0.07\lambda_0$) optimised for minimum reflection coefficient by varying σ , ϵ and μ , resulted in a reflection coefficient for normal incidence of less than -55 dB at the chosen frequency.

3 Examples

The impedance method has been applied to a variety of two-dimensional, inhomogeneous problems. For example, one can measure variations in the relative permittivity of a thin filament using CW plane wave backscatter at microwave frequencies. Figure 2 shows the experimental arrangement where the filament is drawn through a parallel plate wave-guide terminated at one end by an absorbing layer. The radiation is incident in the $+z$ direction. The 10 GHz reflection coefficient was calculated using the impedance method and results are plotted in Figure 3. The two lines illustrate the very small difference in scattering over the conductivity range $0 < \sigma < 0.001$ S/m. The separation of the plates and the source configuration ensure a TM plane wave is incident on the filament. The reflection coefficient is most sensitive changes in the relative

permittivity over the range $1.05 < \epsilon_r < 2.4$.

Other problems investigated include parallel plate wave guides terminated in lossy media, the reflection coefficient from metallic and dielectric beams, and VLF surface impedance over various earth structures including faults, intrusions and buried pipes.

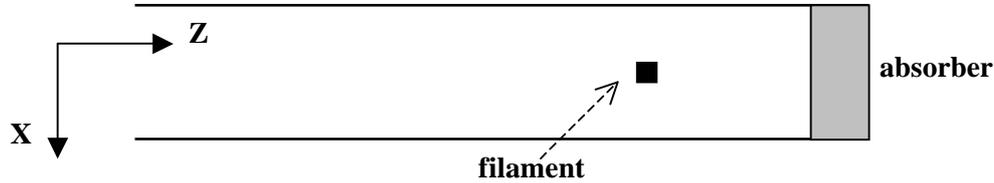


Figure 2: Parallel plate wave-guide technique used to measure the permittivity of a thin dielectric filament. The reflection coefficient of the 10 GHz, TM wave is measured.

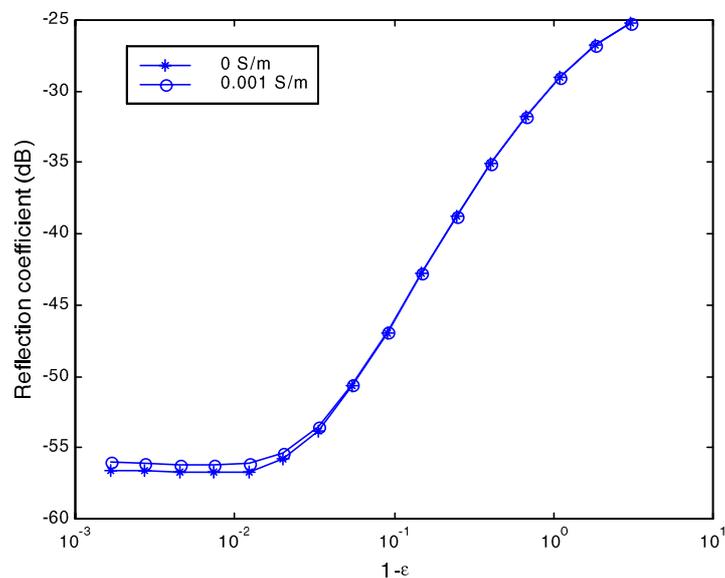


Figure 2: Variation in the TM reflection coefficient at 10 GHz, of a dielectric filament (cross-section 2mm x 2mm) as a function of the relative permittivity.

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