

## **Risk Distribution Profile for Differential Column Shortening Using a Possibility Theory Approach**

Tsu-Te Huang

*PhD Student, Griffith School of Engineering, Griffith University, Gold Coast, Australia*

Rodney A. Stewart

*Senior Lecturer, Griffith School of Engineering, Griffith University, Gold Coast, Australia*

Jeung-Hwan Doh

*Lecturer, Griffith School of Engineering, Griffith University, Gold Coast, Australia*

Dennis Song

*Director, FOSTA Pte. Ltd., Singapore*

### **Abstract**

As buildings inevitably increase in height, vertical support elements (e.g. columns and shear walls) in tall buildings are required to carry vertical load increments from a number of floors. Therefore, axial shortening of vertical elements due to long term creep and shrinkage effects is inevitable in reinforced concrete buildings. However, the calculation of reliable values for axial shortening is not a straight forward task. All parameters may be uncertain or may not be available at the design stage. Recently, engineers have also become concerned with differential shortening of adjacent vertical elements, particularly in the lower and basement levels of super high rise structures. Largely varied values and rates for axial shortening of adjacent support elements is not only a concern for vertical deformation, but can critically impact on the performance of horizontal structural elements such as beams and slabs. This research aims to develop a robust possibility-based differential shortening prediction framework, and associated risk distribution profiles, which overcomes the deficiencies in the current models for predicting axial column shortening in reinforced concrete high rise buildings.

### **Keywords**

Axial Shortening, Differential Shortening, Possibility Distribution Theory, Risk Distribution Profile

### **1. Introduction**

The rapid growth of population and relatively limited residential and commercial space has been the driver for the development of high rise buildings in major cities around the world. It is an inevitable trend and also implies competitive aspects of civilization between countries. As a buildings height increases, its vertical support elements will be subjected to a number of load increments. In practice, vertical elements will not shorten by the same amount since they have different loading conditions and design parameters such as percentage of reinforcement and surface-to-volume ratios. Nonetheless, column shortening of individual members is rarely a problem when they are considered in isolation. However, potential structural and non-structural problems associated with axial shortening occur when supporting elements shorten at different rates and amounts causing damage to the horizontal elements such as beams

and slabs, building façade and finish, interior partitions, and rigid services such as lift guide rails, etc. The cumulative differential shortening of columns causes the slabs to tilt with resulting rotations of partitions as can be seen in Figure 1. Engineers working in fields such as design, construction, maintenance and even asset management are beginning to pay more attention to the effects of differential shortening of vertical support elements when designing reinforced concrete super high-rise buildings. Thus, to avoid problems related to the differential axial shortening of these vertical elements, their elastic and inelastic shortening should be accurately predicted and properly compensated (Park, 2003).

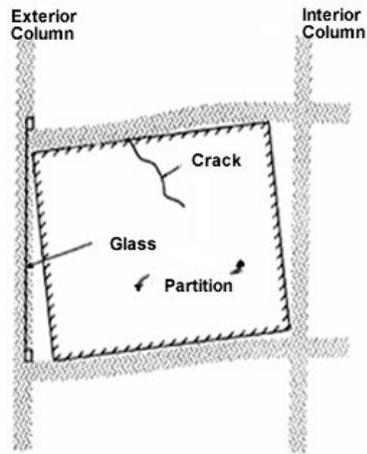
As buildings and infrastructure are substantial investments for government and the private sector, continuous structural health monitoring (SHM) becomes an important process for determining imminent structural failure due to serviceability issues (long term effects), poor construction or natural disasters. This has resulted in an increasing interest in developing more reliable prediction methods which utilize advanced sensor monitoring technology and risk evaluation approaches.

Risk assessments have also become an important aid in the decision making process related to the management of infrastructure and building assets (Guyonnet, *et al.*, 2003). The most common way to address uncertainty due to randomness is to collect data and perform a statistical analysis. When information is incomplete, or statistical data are not available, field observations and human experts can provide information on parameter values. Such subjective information can be represented in various methods such as probability theory, possibility theory and uncertainty intervals, etc.

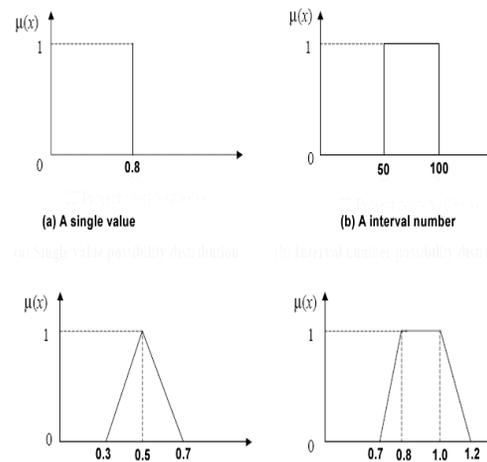
## 2. Literature Review

Fintel and Khan (1969) developed a method for analyzing the differential shortening of columns. Their method was compared with field observations and was found to have acceptable agreement. They observed that buildings up to thirty stories high with flexible slab systems, such as flat plate and average spans, or long span joist systems are usually not adversely affected structurally by differential shortening of supports. They reported that thermal effects became significant for buildings taller than twenty stories with exposed exterior columns. Pan, *et al.*, (1993) described a method for the prediction of long term axial deformation of reinforced concrete columns of tall buildings. They compared predicted and measured values of column shortening in tall buildings and reported good agreement between them. Song and Cho (2002) developed a prediction model for analyzing the differential shortening of columns based on deterministic theory and probability analysis using the Monte Carlo simulation technique. The comparison result between deterministic and probability values were found to be within an acceptable range. Through field and case studies, Jayasinghe and Jayasena (2004) developed a user friendly means for calculating axial shortening of a concrete member due to elastic, shrinkage and creep strains. A spreadsheet using Microsoft Excel was developed. In their study, the effects of construction sequence, rate of construction and grade of concrete on axial shortening were determined based on a number of case studies covering a 10-40 storey range.

This study investigates opportunities for using possibility theory as a modeling tool for predicting ranges of differential column shortening. This technique involves defining each factor as a possibility distribution. The type of possibility distribution is determined by its membership function,  $\mu(x)$ . When the factor's value is possible, it has a membership value of 1. On the other hand, when the value is impossible, it has a membership value of zero. The factor can have a possibility distribution between these values. For the purpose of the differential column shortening prediction framework, it is assumed that the distributions will be one of the following four types: single value; interval number; triangular distribution; and trapezoidal distribution (see Figure 2).



**Figure 1: Effect of Differential Shortening (Linjawi, 1994)**



**Figure 2: Possibility Distribution Types**

### 3. Research Method

The research activities described in this paper mainly included conceptual framework development, selection of appropriate variability modeling techniques, risk distribution profiles application software development, framework refinement and validation, and guideline development. Each of these major research activities are described in the following sections.

#### 3.1 Conceptual Framework Development

The first stage was to develop an innovative conceptual framework for determining differential column shortening in vertical support members of reinforced concrete buildings. The review of existing prediction models uncovered a gap in the research. Reported research on column shortening prediction approaches appeared to only consider this structural phenomena in a deterministic manner and rarely considered the more critical aspects related to differential column shortening; a major concern when designing super high-rise structures. Thus, this study sought to develop an improved approach for measuring differential column shortening which accounted for the inherent variability of creep and shrinkage parameters and encapsulates the risk distribution profiles of various differential column shortening scenarios. Thus, a more advanced conceptual framework is provided in this paper. The proposed framework not only takes into account prediction values and influencing factors, but also will include a companion prediction software tool which automatically generates relevant possibility distribution plots of axial shortening for each vertical element and risk distribution profiles for various scenarios.

#### 3.2 Selection of Appropriate Techniques

One of the critical parts of this research was to select an appropriate method that gives a more accurate representation of uncertainty parameters. There are a number of approaches that can be applied to this problem, including, probability theory, Monte Carlo simulation, possibility theory, to name a few. Possibility theory (fuzzy sets) was selected as the most appropriate modeling tool for this study than more accurate techniques such as probability theory because it does not require huge data sets that are not currently available for column shortening measurement but is based on the concept that all values are considered possible within a certain range that can be established by experienced professionals. Thus, the possibility theory approach is time and resource efficient and does not require large amounts of data to develop probability distributions. By utilizing this technique, internal and external influencing parameters

of concrete structures can be defined and represented as single values, intervals, triangular and trapezoidal or even more rounded distributions using fuzzy ranges. Consequently, possibility distribution plots can be developed for each vertical element and through the comparison of two vertical elements, the possibility distribution plot of differential column shortening can be produced.

### **3.3 Companion Software Tool Development**

Once the theoretical aspects of the conceptual framework have been determined and proven a companion software tool will be developed. The proposed software tool will consist of a number of modules, including: (1) input data module; (2) output data module; (3) floor plan input module; (4) axial column shortening possibility distribution output module; and (5) differential column shortening risk analysis profile module. After providing the range of values (fuzzy set) for the various parameters which determine column creep and shrinkage the software will automatically calculate possibility distributions for each respective column in a particular level of a high rise building (i.e. usually consider basement and lower levels). Utilizing the horizontal distance between vertical support members from the input floor plan and the determined element shortening distributions for each column and shear wall the software will be able to generate the differential column shortening profile for the various combinations of support elements. Following this step, the software will utilize the developed threshold damage index chart (i.e. acceptable range of differential column shortening against horizontal distance between supporting members) to determine the threshold level for structural damage of horizontal support elements (e.g. beams) and position this value on the differential column shortening possibility distribution. This threshold level (e.g. 6 mm differential shortening) can be utilized to determine the risk profile for possible differential shortening outcomes (i.e. calculate area under possibility distribution plot). Specifically, the software will determine the likelihood of differential shortening being above a threshold value. For example, high percentage chances (e.g. 70%) would represent a high risk that would need to be mitigated by structural engineers designing the building.

### **3.4 Framework Refinement and Validation**

Through the necessary procedures of refinement and validation, the framework can be improved and made fit for purpose. The refinement and validation procedures utilized in this research investigation will include the following three individual stages:

- (1) *Industrial trials*: the framework will be tested by related design and construction companies and engineers for pilot trial. The expected recommendations and opinions will be considered as important constructive references for the improvement of the framework.
- (2) *Case studies*: the implementation of around five case studies (mainly derived from previous field measurements of axial shortening on vertical elements in tall buildings from different countries, such as Q1 Tower, Australia and Lake Point Tower in Chicago, USA, etc.) will be conducted to verify the reasonable agreement between the proposed prediction framework and the selected case studies.
- (3) *Structural engineering experts review*: field experts will be invited to review the proposed prediction framework and the structural performance relationship between the horizontal distance between vertical support elements and their differential shortening magnitude (i.e. closer columns and shear walls can endure a lower value of differential shortening than those that are spread further apart).

### **3.5 Guideline Development**

Finally, guidelines will be developed, which can be used by design engineers to adjust their particular structural design for a building to accommodate differential shortening of vertical elements. The potential guideline will focus on: (1) the effect of compressive strength, column cross section and environmental coefficients on column shortening; (2) the differential shortening between columns and shear walls; and (3) the effect of the construction rates on column shortening.

#### 4. Hypothetical Application

The primary purpose of this hypothetical application is to help users understand the prediction process and the required parameters which eventually produce the risk distribution profiles for possible differential shortening scenarios for the various vertical support elements in high rise buildings. Therefore, possibility information needs to be identified and a description of the range and likelihood are essential for selecting influencing parameters. The flow chart for determining the risk distribution profiles is shown in Figure 3. First, users are required to enter data in the appropriate cells in the input data sheet and the most likely and least likely ranges of selected factors (See Figure 4). For this brief example, only the compressive strength ( $f_c'$ ) and maturity coefficient ( $k_3$ ) are selected as influencing factors for column No. 10, whereas the compressive strength and creep coefficient ( $k_2$ ) are selected for column No. 11. The axial shortening values and possibility distribution plot of column No. 10 and column No. 11 relevant to input data are automatically tabulated and produced in the output sheet (See Figure 5). Consequently, the possibility distribution plot can be developed by applying the vertex method which uses the  $\alpha$ -cut representations of the fuzzy sets. The differential of two possibility distribution plots (i.e. for column No. 10 and column No.11) can be seen as  $C10 \text{ Vs } C11 = C10 - C11$ . The calculation for this is as follows:

1. Take an  $\alpha$ -cut at 0.0,  $C10_{0.0} = [6.2, 13.9]$  and  $C11_{0.0} = [4.5, 6.4]$ , thus  $C10 \text{ vs } C11_{0.0} = [1.7, 7.5]$
2. Take an  $\alpha$ -cut at 1.0,  $C10_{1.0} = [8.7, 10.6]$  and  $C11_{1.0} = [5.4]$ , thus  $C10 \text{ vs } C11_{1.0} = [3.3, 5.2]$
3. The resulting distribution  $C10 \text{ vs } C11_{1.0} = [1.7, 3.3, 5.2, 7.5]$  is displayed in Figure 6.

This plot will provide the most likely and upper and lower least likely differential shortening between column No. 10 and column No. 11. Based on the threshold structural damage relationship (see Figure 7), the risk distribution profile can be developed accordingly. In this application, there are two possible scenarios demonstrated here. The first low risk scenario shows the threshold value of 6 mm being largely to the far right of the possibility distribution and representing a risk area of only 20 per cent (see Figure 8a). The other high risk scenario shows the threshold level (6 mm) located towards the centre of the possibility distribution and representing a risk area of 60 per cent (see Figure 8b). Where high risks are displayed, the structural engineer would need to consider mitigating possible impacts through redesign or structural health monitoring whilst the building is being constructed.

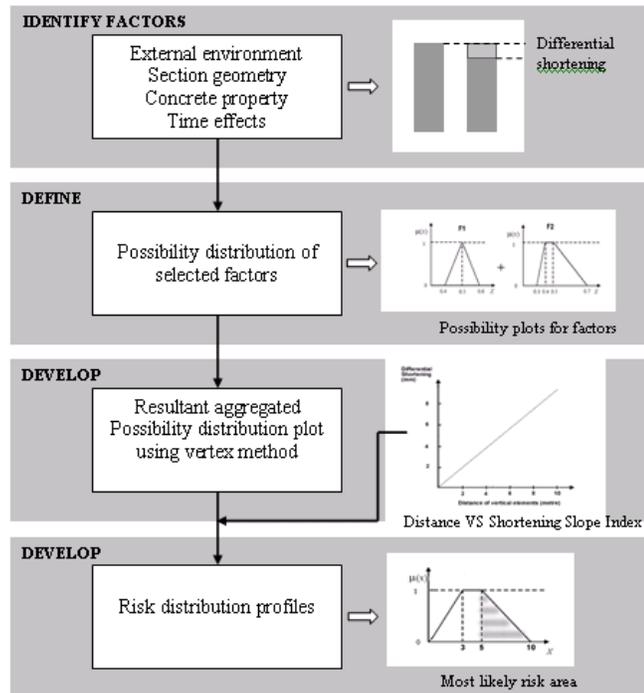


Figure 3: Risk Distribution Profile Process Flowchart

**Griffith UNIVERSITY**  
**Prediction of Axial Shortening of Vertical Elements in Reinforced Concrete structures**  
 Prepared by **Mr. You-Te (Andrew) Hwang**  
 Griffith Engineering School, Gold Coast

General Input		Input Possibility Ranges of Factors		
<b>Building property</b>		<b>Compressive strength (f<sub>c</sub>)</b>	<b>Environment coefficient (k<sub>e</sub>)</b>	
No. of Stories in the Building	40	Input range:	Input range:	
Considered Floor Level	4	(Range: 25-65)	(Range: 0.5-0.7)	
Required story level to calculate axial shortening	1			
<b>Column property (B)</b>		<b>Shrinkage strain coefficient (k<sub>1</sub>)</b>	<b>Basic creep coefficient (k<sub>2</sub>)</b>	
Column Length	3000.00 mm	Input range:	Input range:	
Gross Section Area of considered Column	0.16 m <sup>2</sup>	(Range: 0.4-1.5)	(Range: 0.4-5.0)	
Transformed Sectional Area of Column	0.167 m <sup>2</sup>			
<b>Elastic property</b>		<b>Creep coefficient (k<sub>2</sub>)</b>		
Modulus of Elasticity of Concrete	28 Gpa	Input range:		
Modulus of Elasticity of Steel	200 Gpa	(Range: 0.1-0.9)		
<b>Creep &amp; Shrinkage property</b>		<b>Maturity coefficient (k<sub>3</sub>)</b>		
Ultimate Shrinkage	4.042E-04 mm/mm	Input range:		
Ultimate Creep	9.1790E-06 mm/mm	(Range: 0.9-1.7)		
Relative Humidity	65%			
<b>Rate of Construction</b>				
Rate of Construction	7 days per floor			

Compressive strength (f <sub>c</sub> )	
Input range:	
40	
(Range: 25-65)	
Shrinkage strain coefficient (k <sub>1</sub> )	
Input range:	
0.5	0.7
(Range: 0.4-1.5)	
Creep coefficient (k <sub>2</sub> )	
Input range:	
0.4	0.7
1.4	
(Range: 0.4-1.4)	
Maturity coefficient (k <sub>3</sub> )	
Input range:	
0.8	0.9
1.1	1.8
(Range: 0.9-1.7)	

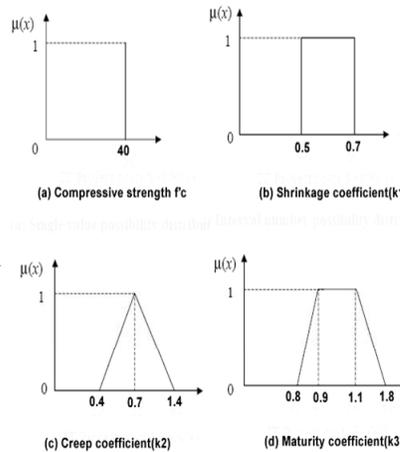


Figure 4: Example of Input Data Sheet

Output sheet				
	Possible shortening range (mm)			
	least likely	most likely	most likely	least likely
Column No. 1	6.2	8.7	10.6	13.9
Column No. 2	5.3	7.2	9.8	11.6
Column No. 3	5.8	9.1	11.1	15.4
Column No. 4	5.3	7.2	9.8	11.6
Column No. 5	4.1	4.9		5.8
Column No. 6	6.2	8.7	10.6	13.9
Column No. 7	4.6	7.2	9.8	11.6
Column No. 8	4.5	5.4		6.4
Column No. 9	5.3	7.2	9.8	11.6
Column No. 10	6.2	8.7	10.6	13.9
Column No. 11	4.5	5.4		6.4
Column No. 12	6.2	8.7	10.6	13.9
Shear wall No. 1	3.9	6.6	7.8	10.5
Shear wall No. 2	5.8	9.1	11.1	15.4

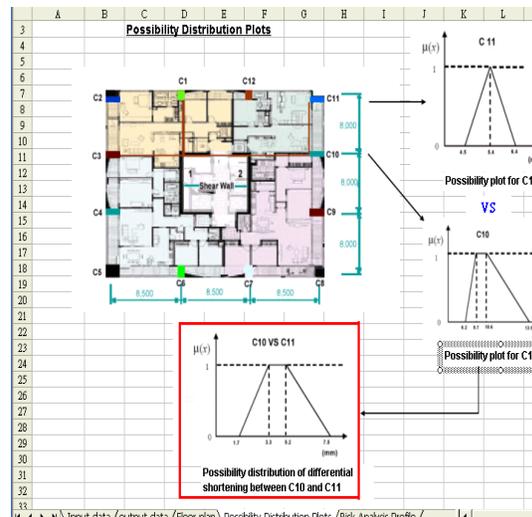
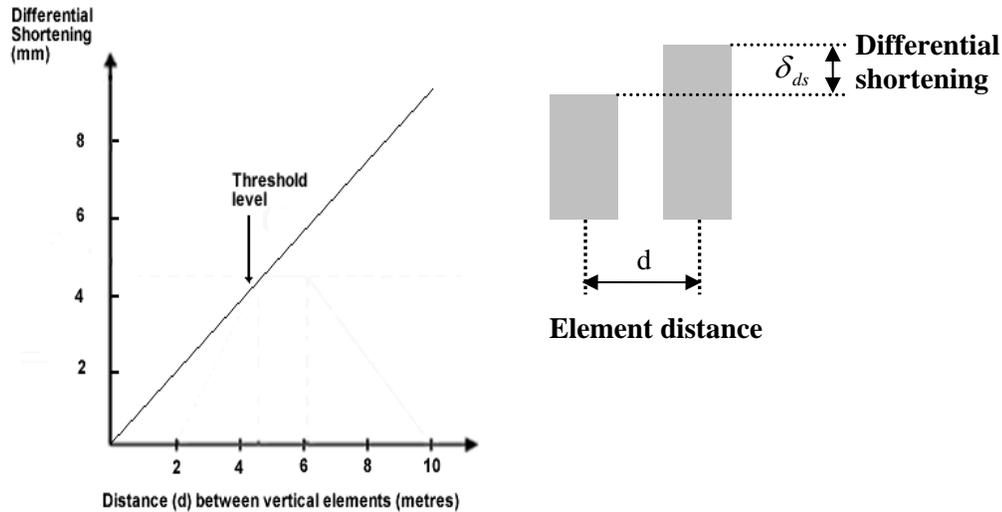
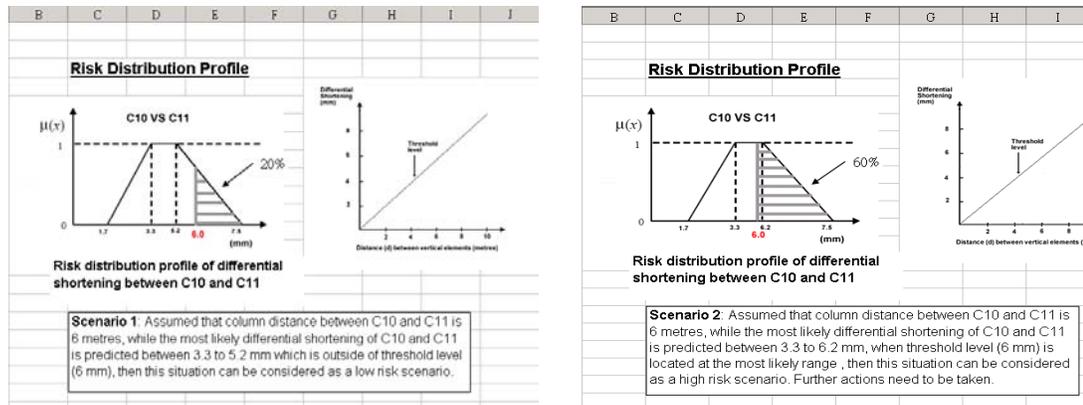


Figure 5: Example of Output Sheet

Figure 6: Possibility Distribution Sheet



**Figure 7: Relationship between Element Distance and Differential Shortening Threshold**



**(a) Scenario 1 (low risk scenario)**

**(b) Scenario 2 (high risk scenario)**

**Figure 8: Risk Distribution Plot**

## 5. Features and Benefits of the Model

The development of the prediction model which can provide the risk distribution profile for differential column shortening is an essential assessment tool for structural engineers designing the various vertical supporting elements in super high-rise buildings; particularly when designing the highly loaded elements in the basement and lower levels of the building. Applying possibility distribution theory (fuzzy sets) to this problem offers a user friendly approach for accounting for the variability of the various parameters which sum to creep and shrinkage predictions. Traditional prediction methods for column shortening offered by the various national (i.e. AS) and international codes do not sufficiently address this issue of prediction variability and risk profiles of differential column shortening. Thus, the potential benefits of implementing this prediction model includes a reduction in computation time, consideration of variation in parameters associated with column shortening, risk profiles of column shortening outcomes, etc. All such outcomes will undoubtedly assist structural engineers with their decision making.

## 6. Summary

Despite a number of studies proposing prediction models based on standard codes and/or other modified models for axial shortening in high rise reinforced concrete buildings, researchers and practitioners still require a more comprehensive understanding of how differential shortening can lead to structural failures and undesirable serviceability. The affect of differential shortening depends on a variety of factors which need to be taken into account, such as concrete material properties, geometry, time factors and external environment. Moreover, the absence of a methodical way of reasonably assessing the risk of differential shortening between adjacent vertical elements limits the wide application of those well-proposed prediction models. Therefore, the development of this risk distribution profile framework and companion application software has been discussed as well as the methodology for refining and validating the proposed model. Ultimately, this proposed prediction framework will be tested for both validity and application through industrial trials, several case studies on field measurements (e.g. Q1 Tower, Gold Coast, Australia) and consultation with engineering experts. Finally, the findings of this study will help in the formulation of a series of guidelines for structural engineers to control and accommodate differential shortening as early as possible in the design process.

## 7. References

- Fintel, M., and Khan, F. R. (1969). "Effects of column creep and shrinkage in tall structures-Prediction of inelastic column shortening", *ACI Journal*, Vol. 66, No. 12, pp. 957-967.
- Guyonnet, D., Bourguine, B., Dubois, D., Fargier, H., Come, B., and Chiles, J. P. (2003). "Hybrid approach for addressing uncertainty in risk assessments", *Journal of Environmental Engineering*, Vol. 129, No. 1, pp. 68-78.
- Jayasinghe, M. T. R., and Jayasena, W. M. V. P. K. (2004). "Effects of axial shortening of columns on design and construction of tall reinforced concrete buildings", *Practice Periodical on Structural Design and Construction*, Vol. 9, No.2, pp. 70-78.
- Pan, L. B., Liu, P. C., and Bakoss, S. L. (1993). "Long term shortening of concrete columns in tall buildings", *Journal of Structural Engineering*, Vol. 119, No. 7, July, pp. 2258-2262.
- Park, H. S. (2003). "Optimal compensation of differential column shortening in high-rise buildings", *The Structural Design of Tall and Special Buildings*, Vol. 12, No.1, pp. 49-66.
- Standards Association of Australia (2002). "Concrete structures", *Standards Australia AS 3600*, Sydney.
- Standards Association of Australia (2005). "Concrete structures (Revision of AS 3600-2001)", *Standards Australia AS 3600 (draft for public comment)*, Sydney.
- Song, H. C., and Cho, Y. S. (2002). "Probabilistic prediction of column shortening of the Plaza Rakyat Tower", *Proceedings of the second International Conference on Advances in Structural Engineering and Mechanics*, Busan.