Harmonic Analysis of HVDC Transformer using Harmonic Balance - Finite Element Method (HB-FEM)

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Abstract — Harmonic balance analysis is a powerful numerical method for solving nonlinear electromagnetic (EM) problems. It is also can be used to solve the nonlinear quasi-static electromagnetic field for DC biased transformer problems used in HVDC Transmission system and power electronics. This paper introduces a relatively new numerical method, namely, the harmonic balance - finite element method (HB-FEM) and its application to time-periodic steady-state nonlinear EM field problems. The validation of DC biased problem in nonlinear magnetic field and harmonic analysis are also presented in this paper.

Keywords: Harmonic Balance, HVDC Transmission System, HB-FEM, Transformer, Nonlinear Magnetic Field.

I. Introduction

The harmonic balance technique was introduced to analyse low frequency electromagnetic field problems in the later 1980's [1], where the harmonic balance techniques were combined with FEM (HB-FEM) to accurately solve the problems arising from time-periodic steady-state nonlinear magnetic field [2-4]. The HB-FEM uses a linear combination of sinusoids to build the solution and represents waveforms using the sinusoid; coefficients are combined with finite element method. It can directly solve the steady-state response of the EM field in the multi-frequency domain and so is often considerably more efficient than the traditional FEM time-domain approach when field exhibit widely separated harmonics in frequency spectrum domain and mildly nonlinear behaviour.

The advantage of using sinusoids to approximate a quasi-static response becomes particularly important when the EM field response contains dominant sinusoids at widely separated frequencies. In this paper, the basic concept of HB-FEM in low frequency EM field analysis and its application in time-periodic steady-state nonlinear magnetic field is presented. HB-FEM is more efficient than the traditional FEM time-domain approach when field exhibit widely separated harmonics in frequency spectrum domain and mildly nonlinear behaviour.

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II. The Basic Concept of Harmonic Balance in EM Field

When linear EM field systems are excited by a sinusoid, the steady-state response is sinusoidal at the same frequency as the input. While nonlinear EM field systems are capable of a dazzling variety of significant and bizarre behavior, the systems of interest to designers generally have a periodic steady-state response to a sinusoidal input; the period of the response is usually equal to that of the input. Since the response is periodic, it is representable as a Fourier series, that is, as a linear combination of sinusoids whose periods divide the period of the response. If the excited EM field system contains two or more sinusoids that are not harmonically related, the system responds in steady-state with components having the sum and difference frequencies of the input sinusoids and their harmonics. If the response contains an infinite number of sinusoids usually all but a few are negligible.

One of the most obvious properties of a nonlinear system is the generation of harmonics. The harmonics generated in EM fields can be described in the following three ways:

a) When a linear EM object is excited by sources which contain the harmonics, it will exhibit the harmonic field.

b) When a nonlinear EM object is excited by a sinusoidal signal, it will exhibit harmonic fields.

c) When both linear and nonlinear EM objects are excited by the sources which contain the harmonics, the result is a complex harmonic field.

A. Harmonic Balance in EM field

Harmonic balance can be also applied to EM field analysis as the fields that contain the harmonics also satisfy Maxwell's equations. For example, we use the following equations to describe the quasi-static EM fields. These can be defined as follows:

$$\nabla \times \nu \nabla \times A - \sigma(\partial A / \partial t - \nabla \varphi) - J_s = 0 \qquad (1)$$

where the electric field ***E***, magnetic vector potential ***A***, scalar potential φ on the arbitrary node i in the descretised system and the current density J_s can be respectively expressed as:

$$A^i = A_0^i + \sum_{k=1}^{\infty} \{A_{ks}^i \sin(k\omega t) + A_{kc}^i \cos(k\omega t)\} \quad (2)$$

$$\varphi^i = \varphi_0^i + \sum_{k=1}^{\infty} \{\varphi_{ks}^i \sin(k\omega t) + \varphi_{kc}^i \cos(k\omega t)\} \quad (3)$$

$$E^i = E_0^i + \sum_{k=1}^{\infty} \{E_{ks}^i \sin(k\omega t) + E_{kc}^i \cos(k\omega t)\} \quad (4)$$

$$J_s = J_0 + \sum_{k=1}^{\infty} \{J_{ks} \sin(k\omega t) + J_{kc} \cos(k\omega t)\} \quad (5)$$

where the vector A_0, E_0, J_0 and scalar φ_0 are the DC components respectively. In practical applications, harmonic k is not infinite. Only a finite number is required in the real system.

B. Nonlinear Medium Description

Nonlinear phenomena in EM fields are caused by nonlinear materials. The nonlinear materials are normally field strength dependent. Therefore when the time-periodic quasi-static EM field is applied to the nonlinear material, the electromagnetic properties of material will be functions of the EM field and time dependent as well. Simple nonlinear materials and their harmonic expressions are defined as follows:

$$\nu(t) = \nu(B(t)) = \nu_0 + \sum_{k=2n-2}^{\infty} \{\nu_{ks} \sin(k\omega t) + \nu_{kc} \cos(k\omega t)\} \quad (6)$$

where ν (=1/μ) is the nonlinear magnetic reluctivity and the Fourier coefficients obtained from

$$\nu_0 = \frac{1}{T}\int_0^T \nu(t)dt \quad (7)$$

$$\nu_{ns} = \frac{2}{T}\int_0^T \nu(t)\cdot\sin(n\omega t)dt \quad (8)$$

$$\nu_{nc} = \frac{2}{T}\int_0^T \nu(t)\cdot\cos(n\omega t)dt \quad (9)$$

C. Boundary Conditions in the simulation domain:

Since the trigonometric functions are orthogonal functions, the harmonic potential P_k (degrees of freedom) on the boundary satisfy Dirichlet and Neumann boundary condition. The frequency-domain representation, or spectrum on each boundary node, can then be expressed as follows:

a) Dirichlet boundary condition:

$$P_k = \{P_0,\ P_{1s},\ P_{1c},\ P_{2s},\ P_{2c},\ \cdots\}^T \quad (10)$$

b) Neumann boundary condition:

$$\frac{\partial P_k}{\partial n} = \{\frac{\partial P_0}{\partial n},\ \frac{\partial P_{1s}}{\partial n},\ \frac{\partial P_{1c}}{\partial n},\ \frac{\partial P_{2s}}{\partial n},\ \frac{\partial P_{2c}}{\partial n},\ \cdots\}^T \quad (11)$$

where the potential P_k is the sum of harmonics on each boundary node *i*.

III. HVDC Transformer

Figure 1 illustrated a typical dc transmission system which consists basically of a dc transmission line connecting two ac systems. A converter at one end of the line converts ac power into dc power while a similar converter at the other end reconverts the dc power into ac power. One converter acts therefore as a rectifier, the other as an inverter.

A. Converter Transformer:

The basic purpose of the converter transformer on the rectifier side is to transform the ac network voltage to yield the ac voltage required by the converter. Three-phase transformers, connected in either wye-wye or wye-delta, are used.

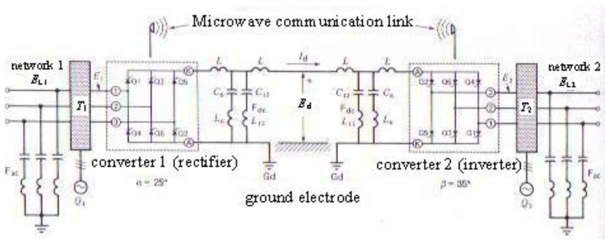


Fig. 1. HVDC transmission system [5]

The magnetostrictive strain is not truly sinusoidal in character, which leads to the introduction of the harmonics. With DC biased transformer, the saturation of magnetizing will also cause some harmonics. The harmonics in a transformer noise may have a substantial effect on an observer even though their level is 10dB or more lower than that of the 100Hz fundamental. Infect, the most striking point is the strength of the component at 100Hz or twice the normal operating frequency of the transformer.

Deviation from a "square-law" magnetostrictive characteristic would result in even harmonics (at 200, 400, 600Hz, etc.), while the different values of magnetostrictive strain for increasing and decreasing flux densities – a pseudo-hysteresis effect – lead to the introduction of odd harmonics (at 300, 500, 700Hz, etc.). If any part of the structure has a natural frequency at or near 100, 200, 300, 400Hz, etc., the result will be an amplification of noise at that particular frequency.

B. HBFEM Model of HVDC Transformer

The source of model is voltage driven source to the magnetic system which is always coupled to the external circuits [2]. The current in the input circuits will be unknown but saturation of the current waveform occurs because of the nonlinear characteristic of the magnetic core [3]. The input voltage can be defined as:

$$\{V_{ink}\} = \{V_k\} + S_{ck}[Z_k]\{J_k\} \tag{12}$$

where matrix $[Z_k]$ is the circuit impedance including the resistance of windings corresponding to the harmonics. The input voltage $\{V_{ink}\}$ including all the harmonic components which have the known values are expressed as follows:

$$\{V_{ink}\} = \{V_{0k}, V_{1sk}, V_{1ck}, V_{2sk}, V_{2ck}, \cdots\}^T \tag{13}$$

S_{ck} and V_0 are the area of windings and DC component respectively.

Considering a three-phase transformer, connected in wye-wye, a computer simulation model with a neutral *NN* and external circuits for both primary and secondary windings is obtained using HBFEM theory. According to Galerkin procedure, system matrix equations of HBFEM for HVDC transformer can be obtained by means of Faraday's and Kirchhoff's laws, that is:

$$\left.\begin{aligned}
&[H]\{A\}-[G_U]\{J_{inU}\}-[G_V]\{J_{inV}\}-[G_W]\{J_{inW}\}\\
&-[G_{outU}]\{J_{outU}\}-[G_{outV}]\{J_{outV}\}-[G_{outU}]\{J_{outV}\}=\{0\}\\
&[C_{inU}]\{A\}+[I]\{V_{NN}\}+S_{cu}[Z_{inU}]\{J_{inU}\}=[V_{inU}]\\
&[C_{inV}]\{A\}+[I]\{V_{NN}\}+S_{cu}[Z_{inU}]\{J_{inU}\}=[V_{inU}]\\
&[C_{inW}]\{A\}+[I]\{V_{NN}\}+S_{cu}[Z_{inU}]\{J_{inU}\}=[V_{inU}]\\
&[C_{outU}]\{A\}+[I]\{V_{NN}\}-S_{cu}[Z_{outU}]\{J_{outU}\}=[V_{outU}]\\
&[C_{outV}]\{A\}+[I]\{V_{NN}\}-S_{cu}[Z_{outU}]\{J_{outV}\}=[V_{outV}]\\
&[C_{outW}]\{A\}+[I]\{V_{NN}\}-S_{cu}[Z_{outU}]\{J_{outW}\}=[V_{outW}]\\
&S_{in,cu}[I]\{J_{inU}\}+S_{in,cu}[I]\{J_{inV}\}+S_{in,cu}[I]\{J_{inW}\}=0\\
&S_{out,cu}[I]\{J_{outU}\}+S_{out,cu}[I]\{J_{outV}\}+S_{out,cu}[I]\{J_{outW}\}=0
\end{aligned}\right\} \tag{14}$$

where $[G_k]$ is obtained from a single element, that is $[G^e] = \Delta^e / 3$, and $[Z_{ink}]$, $[Z_{outk}]$ and $S_{in,cu}$, $S_{out,cu}$ are external circuit impedances and cross sectional areas of windings respectively, $[\boldsymbol{I}]$ is unit matrix, $[C_k] = \frac{\omega d_0 \Delta}{3S_{ck}}[N \quad N \quad N]$ is geometric co-efficient related to transformer windings, and current density $\boldsymbol{J}$ can be presented as:

$$\{J_k^e\} = \{J_0 \quad J_{1s} \quad J_{1c} \quad J_{2s} \quad J_{2c} \quad J_{3s} \quad J_{3c} \quad \cdots\}^T \text{ (15).}$$

[H] is system matrix and the detailed definitions can be obtained from [2-3].

$$[H^e] = \frac{1}{4\Delta^e}\begin{bmatrix}(b_1b_1+c_1c_1)D & (b_1b_2+c_1c_2)D & (b_1b_3+c_1c_3)D\\(b_2b_1+c_2c_1)D & (b_2b_2+c_2c_2)D & (b_2b_3+c_2c_3)D\\(b_3b_1+c_3c_1)D & (b_3b_2+c_3c_2)D & (b_3b_3+c_3c_3)D\end{bmatrix}\begin{Bmatrix}A_1^e\\A_2^e\\A_3^e\end{Bmatrix} + \frac{\sigma\omega\Delta^e}{12}\begin{bmatrix}2N & N & N\\N & 2N & N\\N & N & 2N\end{bmatrix}\begin{Bmatrix}A_1^e\\A_2^e\\A_3^e\end{Bmatrix} \tag{16}$$

where matrix D and N are:

$$D = \frac{1}{2}\begin{bmatrix}
2\nu_0 & \nu_{1s} & \nu_{1c} & \nu_{2s} & \nu_{2c} & \nu_{3s} & \nu_{3c} & \cdots\\
2\nu_{1s} & 2\nu_0-\nu_{2c} & \nu_{2s} & \nu_{1c}-\nu_{3c} & -\nu_{1s}+\nu_{3s} & \nu_{2c}-\nu_{4c} & -\nu_{2s}+\nu_{4s} & \cdots\\
2\nu_{1c} & & 2\nu_0+\nu_{2c} & \nu_{1s}+\nu_{3s} & \nu_{1c}+\nu_{3c} & \nu_{2s}+\nu_{4s} & \nu_{2c}+\nu_{4c} & \cdots\\
2\nu_{2s} & & & 2\nu_0+\nu_{4c} & \nu_{4s} & \nu_{1c}-\nu_{5c} & -\nu_{1s}+\nu_{5s} & \cdots\\
2\nu_{2c} & & & & 2\nu_0+\nu_{4c} & \nu_{2s}+\nu_{4s} & \nu_{1c}+\nu_{5c} & \cdots\\
2\nu_{3s} & & \text{Symmetry} & & & 2\nu_0-\nu_{6c} & \nu_{6s} & \cdots\\
2\nu_{3c} & & & & & & 2\nu_0+\nu_{6c} & \cdots\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{bmatrix} \tag{17}$$

and

$$N^e = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots\\
0 & 0 & -1 & 0 & 0 & 0 & 0 & \cdots\\
0 & 1 & 0 & 0 & 0 & 0 & 0 & \cdots\\
0 & 0 & 0 & 0 & -2 & 0 & 0 & \cdots\\
0 & 0 & 0 & 2 & 0 & 0 & 0 & \cdots\\
0 & 0 & 0 & 0 & 0 & 0 & -3 & \cdots\\
0 & 0 & 0 & 0 & 0 & 3 & 0 & \cdots\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{bmatrix} \tag{18}$$

The full matrix equation of HBFEM model in a compact form can be obtained as:

$$\begin{bmatrix}
[H] & -[G_1] & -[G_2] & 0 & 0\\
[C_1] & S_1[Z_1] & 0 & [I] & 0\\
[C_2] & 0 & S_2[Z_2] & 0 & [I]\\
0 & S_1[I] & 0 & 0 & 0\\
0 & & S_2[I] & 0 & 0
\end{bmatrix}\begin{Bmatrix}\{A_n\}\\\{J_{1k}\}\\\{J_{2k}\}\\V_{1NN}\\V_{2NN}\end{Bmatrix} = \begin{Bmatrix}0\\\{V_{1k}\}\\\{V_{2k}\}\\0\\0\end{Bmatrix} \tag{19}$$

IV. Validation of HBFEM

To verify HBFEM based solution, a single phase DC biased switching transformer with voltage source driven model as shown in Fig. 2 was employed as a case study. In this simulation model, both input and output circuits are included as part of simulation model, where the output side is considered as an open circuited case.

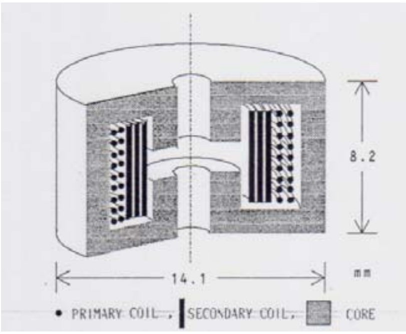
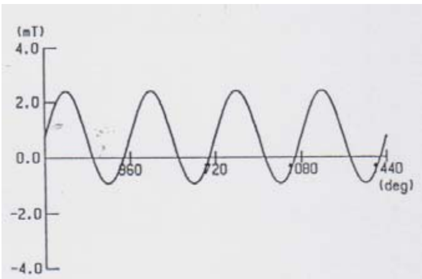
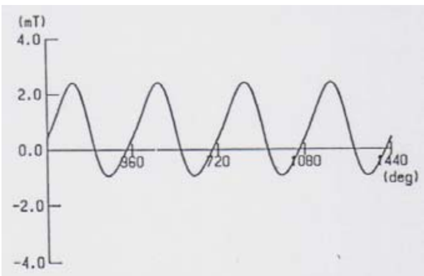


Fig. 2 Simulation model of switching transformer



(a) Experiment result



(b) Simulation result

Fig. 3. Flux density of transformer with DC biased excitation.

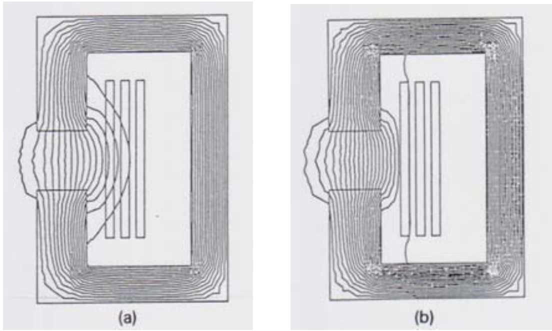


Fig. 4. Magnetic flux distribution with, (a) DC flux, (b) fundamental flux.

Figure 3 and 4 show the flux density with DC biased excitation and flux distribution for each harmonic component in the magnetic core respectively.

V. Conclusion

The HB-FEM has been effectively used to solve the harmonic problems in EM field problems. To design and analyse nonlinear and harmonic related EM field problems in HVDC transformer properly, HB-FEM based numerical techniques can be employed. The application problem revealed how the harmonic balance technique combined with the finite element method to solve the quasi-static nonlinear magnetic field problem with DC biased magnetic field. The magnetic field coupled with the circuits exited by voltage source was discussed. The comparison between numerical and experimental results has shown a good agreement.

References

[1] S. Yamada and K. Bessho. Harmonic field calculation by the combination of finite element analysis and harmonic balance method, IEEE Transactions on Magnetics, Vol.24, no 6, pp.2588-2590, Nov. 1988.

[2] J. Lu, S. Yamada and K. Bessho. Harmonic Balance Finite Element Method Taking Account of External Circuits and Motion, IEEE Transactions on Magnetics, Vol. 27, No. 5, pp 4204-4207, 1991

[3] J. Lu, S. Yamada and H. B. Harrison. Application of HB-FEM in the Design of Switching Power Supplies, IEEE Trans. on Power Electronics, Vol. 11, No 2, pp 347-355, March 1996.

[4] J. Lu. Harmonic Balance - Finite Element Method (HB-FEM) for Low Frequency Electromagnetic Field Analysis, ACES2006, pp237-244, March 2006.

[5] T. Wildi. Electriacl machines, Drives, and Power Systems, Fourth Edition, Prentice Hall, 2000.