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Response to Comments on, "An Analysis of a Staked Dipole Probe on a Lossy Earth Plane Using the Finite-Difference Time-Domain Method"

James R. Wait

There are two items that I need to respond to. The first deals with the basis of my expression for the effective complex area of a horizontal electric dipole (HED) over a layered conductive half-space. The second related item deals with a basic flaw in their estimate of the error when the HED is lifted relative to the air/earth interface. In both cases, we orient the HED to have maximum response.

A. Effective Area of Loop for Stratified Earth Model

Let E_v be the vertical electric field in air for a distant very low frequency (VLF) transmitter. Then

$$E_v \cong \hat{n}_o H_h \quad (1)$$

where H_h is the horizontal magnetic field and $\hat{n}_o = j\mu_o\omega/jk = 120\pi$. The voltage v induced in the equivalent loop of area A is

$$v = j\mu_o\omega A H_h. \quad (2)$$

But for the HED of effective length L

$$v = E_h L \quad (3)$$

where $E_h = WE_v$ in terms of the wave tilt W . Then, on equating (2) and (3), we get $A \cong WL/jk$, as given in my comments [2].

B. Tilt Corrections for Staked and Unstaked Dipoles

Thiel and Mittra (T and M) assert that the total effective area of their HED of length L , raised to a mean height t above the homogeneous half-space, is given by

$$A_{\text{total}} \cong L/\gamma + L/t = (L/\gamma)[1 + \gamma t]. \quad (4)$$

This is equivalent to writing

$$A_{\text{total}} \cong A(1 + \gamma t) \quad (5)$$

where $\gamma t = (1 + j)t/\delta$ in terms of the skin depth of the lower half-space. As T and M indicate in their example, if $|\gamma t|$ or $t/\delta \ll 1$, the correction to the raised HED is negligible. But they are really dealing with the staked HED. Only in that case, does the correction result from the voltage induced in a closed loop of area LXt .

When the elevated HED is not terminated, the correction is quite different. Now the critical parameter is the mean tilt rather than the mean height. Again, for the distant VLF transmitter, the relatively large vertical electric field E_v will induce a secondary voltage in the tilted HED, which is approximately $E_v L\Theta$, where Θ is the tilt angle relative to the air/earth interface. In this case

$$A_{\text{total}} \cong A[1 + (\Theta/W)] \quad (6)$$

where now the correction Θ/W can be very significant if the tilt angle Θ is comparable with or greater than the magnitude of the wave tilt W . It is interesting to compare the corrections for staked and unstaked cases. The relevant ratio is $|(\gamma t)/[\Theta W^{-1}]| \cong kt/\Theta \cong kL$. For $L = 10$ m, as in the example by T and M, and for a frequency of 20 kHz, this ratio is 4×10^{-3} . Thus, for this tilted dipole (with zero height at one end to $2t$ at the other end), the correction for the unstaked case is 250 times that for the staked case.

The key results given by (5) and (6), while based on a highly idealized model, are consistent with the more general analysis in [3].

Of course, I well appreciate that there are many instances in which the unstaked or unterminated horizontal dipoles are to be preferred, as T and M point out, but there are also arguments on the other side of the ledger.

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Manuscript received February 17, 1998; revised May 21, 1998.
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Publisher Item Identifier S 0196-2892(99)00472-6.

Reply to Comments by James R. Wait

David V. Thiel and Raj Mittra

We are happy with the additional analysis given by Wait in his latest comments on our paper [1].

It may be that Wait's concern for the contaminating field components for horizontal electric field measurements has resulted from the very significant difficulties that we have with making electric field wave tilt measurements in air just above ground. We certainly agree

Manuscript received November 26, 1997; revised January 7, 1998.

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Publisher Item Identifier S 0196-2892(99)00329-0.

that making such wave tilt measurements is very difficult. However, our extensive experimental work with the insulated dipole resting on the surface of the earth [2]–[4], in addition to the numerical modeling results that were presented in our original paper [1], has given us confidence that this method of approach is very satisfactory.

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