

Title: **LATERALLY LOADED RIGID PILES IN COHENSIONLESS SOIL**

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ABSTRACT

In this paper, limiting force profile for laterally loaded rigid piles in sand is differentiated from on-pile force profile, from which elastic-plastic solutions are established and presented in explicit expressions. Nonlinear responses of the piles are characterised by slip depths mobilised from mudline and the pile-tip. At the states of tip-yield and rotation point yield, expressions for some critical depths are developed, which allow the on-pile force profiles to be constructed. The solutions and the expressions are developed concerning a constant subgrade modulus (k) and a linearly increasing modulus with depth (Gibson k), respectively. Capitalised on three measurable parameters, the solutions agree well with measured data and numerical predictions. Non-dimensional responses are presented for various eccentricities at the tip-yield state and in some cases at the rotation point yield state. Nonlinear responses are obtained for typical eccentricities from elastic state right up to failure. Comments are made regarding displacement based capacity. A case study is elaborated to illustrate (i) the use of the current solutions; (ii) the impact of the k distributions; (iii) the evaluation of stresses on pile surface; and (iv) the deduction of soil modulus. The current solutions are easy to be implemented and suitable for general design.

Keywords: piles, lateral loading, rigid piles, nonlinear response, closed-form solutions

1. INTRODUCTION

In-situ full-scale and laboratory model tests (Mayne et al. 1995) were widely conducted on laterally loaded rigid piles (including piers and drilled shaft etc.). They have enabled simple expressions to be established for computing bearing capacity. To assess nonlinear pile-soil interaction, centrifuge and numerical finite element (FE) modelling (Laman et al. 1999) were fulfilled previously. These tests and modelling uphold that the success of any design largely relies on input parameters used for describing in particular, the distributions of soil shear modulus (G_s) and limiting force per unit length along the pile (p_u profile, also termed as LFP). Naturally, various techniques have been proposed to gain these parameters, albeit generally for either elastic or ultimate state. The accuracy of these expressions, modelling and techniques is not the motivation for the current study. However, it is not clear about the difference between the p_u profile and the resistance per unit length along a pile (herein referred to as *on-pile force profile*). Its impact on design needs to be quantified. In addition, a stringent nonlinear model is indeed required to uniquely back-estimate the parameters upon measured nonlinear response, to capture overall pile response at any stage, as the model may then be utilised as a boundary element for simulating more complex soil-structure interaction.

Available expressions are generally deduced in light of force equilibrium on the pile (see Fig. 1), and bending moment equilibrium about the pile-head or tip. The force in turn is calculated utilising a postulated *on-pile force profile*, which differs from one author to another (Brinch Hansen 1961; Broms 1964; Petrasovits and Award 1972; Meyerhof et al. 1981; Prasad and Chari 1999). The on-pile force profile actually characterises the mobilisation of the resistance along the unique p_u profile (*independent of pile displacement*). It appears as the solid lines shown in Figs. 2a-2c-left, which is mobilised along the stipulated linearly increasing LFP (Broms 1964) indicated by the two dashed lines. It is elaborated herein for four typical states of yield between pile and soil.

- (1) *Pre-tip yield and tip-yield states*: Tip yield refers to that the force per unit length at pile tip just attains the limiting p_u [Fig. 2a-left]. Prior to and upon the *tip-yield* state, the *on-pile force profile* follows the positive LFP down to a depth of z_0 , below which, it is governed by elastic interaction. This generates an on-pile force profile akin to that adopted by Prasad and Chari (1999), as explained later in this paper.

- (2) *Post-tip yield* state: Further increase in load beyond the tip yield state enables the limiting force to be fully mobilised from the tip as well and to a depth of z_1 . This induces a new portion of the *on-pile force profile* that follows the negative LFP over the depth l to z_1 . The overall profile is more or less like that illustrated in Fig. 2b.
- (3) Continual increase in the load renders the depths z_0 and z_1 to approach each other. The depths tend to merge eventually with the depth of rotation z_r (i.e. $z_0 = z_r = z_1$), which is practically unachievable. To the depth z_r , the *on-pile force profile* now follows the positive LFP from the head; and the negative LFP from the tip. This impossible ultimate *on-pile force profile* is referred to as fully plastic (ultimate) state and is depicted in Fig. 2c. It is adopted previously by some investigators (Brinch Hansen 1961; Petrasovits and Award 1972).

Any solutions underpinned by a single stipulated *on-pile force profile* unfortunately would not guarantee compatibility with the altering profiles (see Fig. 2-left) recorded at different loading levels for a single pile.

Capacity of a laterally loaded pile is defined as the load at a specified displacement, say, 20% pile diameter upon a measured load-displacement curve (Broms 1964). It is also taken as the load inducing a certain rotation angle upon a measured load-rotation curve (Dickin and Nazir 1999). These two independent definitions would not normally yield identical capacities even for the same test. To resolve this artificial impingement, new displacement-based solutions need to be developed.

In this paper, elastic-plastic solutions are developed for laterally loaded rigid piles. They are endeavoured to be presented in explicit expressions, to facilitate the computation of:

- The distributions of force, displacement, rotation, and bending moment down the pile;
- The slip depths from mudline or pile tip, and the depth of rotation.

The solutions thus allow (1) nonlinear responses of the piles to be predicted; (2) the *on-pile force profiles* at any loading levels to be constructed; (3) the new yield (critical) states to be determined; and (4) the displacement-based pile capacity to be estimated. By stipulating a linear LFP, the study primarily aims at piles in sand, assuming a uniform modulus with depth or a linearly increasing modulus.

A spreadsheet program will be developed to facilitate numeric calculation of the solutions. Comparison with measured data and FE analysis will be presented to illustrate the accuracy and highlight the characteristics of the new solutions. An example study will

be elaborated to show all the aforementioned facets, apart from the impact of modulus profile, the back-estimation of soil modulus, and the calculation of stress distribution around a pile surface.

2. ELASTIC-PLASTIC SOLUTIONS

2.1 Features of Laterally Loaded Rigid Piles

Under a lateral load, T_t applied at an eccentricity, e above mudline, elastic-plastic solutions for infinitely long, flexible piles have been developed previously (Guo 2006). Capitalised on a generic LFP, these solutions, however, do not allow pile-soil relative slip to be initiated from the pile tip. Thus, theoretically speaking, they are not applicable to a rigid pile discussed herein. With regard to a rigid pile in sand, a linear LFP may be stipulated, upon which the *on-pile force (pressure) profile* alters as presented previously in Figs. 2a-2c-left.

2.2 Coupled Load Transfer Model

Modelling force (stress) development along the pile and in the surrounding soil in elastic state is critical to development of new solutions. Under lateral loading, stress distribution is unsymmetrical, and shows three-dimensional (3^D) features. Its solution has in general been resorted to complex numerical methods. The essence in the modelling, however, is to capture (1) the non-uniform distribution of force (pressure) on the pile surface in radial (Prasad and Chari 1999) and longitudinal dimensions; and (2) the alteration of modulus of subgrade reaction due to non-uniform soil displacement field around the pile. These two features nowadays can be well captured using a load transfer approach (Guo and Lee 2001) deduced from a simplified soil-displacement model. The approach requires much less computing effort than numerical modelling, it is thus utilised herein to model elastic response. Pertinent expressions/conclusions are recaptured herein:

(1) The stresses change in the soil around a horizontally loaded pile may be approximated by [also owing to Sun and Pires (1993)]:

$$[1] \quad \sigma_r = 2G_s u \frac{\gamma_b}{r_o} \frac{K_1(\gamma_b r/r_o)}{K_o(\gamma_b)} \cos \theta_p \quad \sigma_\theta = \sigma_z = 0$$

$$\tau_{r\theta} = -G_s u \frac{\gamma_b}{r_o} \frac{K_1(\gamma_b r/r_o)}{K_o(\gamma_b)} \sin \theta_p$$

$$\tau_{\theta z} = -G_s \omega \frac{K_o(\gamma_b r/r_o)}{K_o(\gamma_b)} \sin \theta_p \quad \tau_{zr} = G_s \omega \frac{K_o(\gamma_b r/r_o)}{K_o(\gamma_b)} \cos \theta_p$$

where G_s is soil shear modulus; σ_r is radial stress; σ_θ , σ_z are circumferential stress, and vertical stress, which are negligible; u , and ω are local lateral displacement, and rotational angle of the pile body at depth z ; θ_p is an angle between the direction of the loading and the line joining the centre of the pile cross-section to the point of interest; r is a radial distance from the pile axis; $K_i(\gamma_b)$ ($i = 0, 1$) is modified Bessel functions of the second kind, and of order i ; r_o is an outside radius of a cylindrical pile. The factor γ_b in eq. [1] may be estimated by

$$[2] \quad \gamma_b = k_1 (r_o/l)$$

where l is the pile embedded length; k_1 is 2.14, and 3.8 for pure lateral loading (free length or eccentricity, $e = 0$), and pure moment loading ($e = \infty$), respectively. k_1 may be stipulated to increase hyperbolically from 2.14 to 3.8 as the free length e increases from mudline to infinitely large.

- (2) Radial stress σ_r and shear stress $\tau_{r\theta}$ are proportional to the local displacement u (see eq. [1]). The stress σ_r can be well predicted (see Fig. 3) using eq. [1] compared with measured data, as elaborated later in the section entitled *Case Study*.
- (3) A pile is defined as rigid, should the pile-soil relative stiffness, (E_p/G_s) exceed a critical ratio $(E_p/G_s)_c$, where $(E_p/G_s)_c = 0.052(l/r_o)^4$; and E_p is Young's modulus of an equivalent solid pile. Given $l/r_o = 12$, for instance, the critical ratio $(E_p/G_s)_c$ is 1,078.

In drawing the abovementioned conclusions, the pile-soil interaction is characterised by a series of springs distributed along the pile shaft and within elastic state. In reality, each spring has a limiting force per unit length p_u at a depth z [FL^{-1}]. If less than the limiting value p_u , the on-pile force (per unit length), p at any depth is proportional to the local displacement, u and to the modulus of subgrade reaction, kd [FL^{-2}] (see Fig. 1b):

$$[3] \quad p = kdu \quad \text{(Elastic state)}$$

where d is pile outside diameter or width [L]; p is force per unit depth [FL^{-1}] for elastic zone. The gradient k [FL^{-3}] may be written as $k_0 z^m$ [k_0, FL^{-m-3}], with $m = 0$ and 1 being referred to as *Constant k* and *Gibson k* hereafter.

(1) The magnitude of a constant k may be related to the shear modulus G_s by

$$[4] \quad kd = \frac{3\pi G_s}{2} \left\{ 2\gamma_b \frac{K_1(\gamma_b)}{K_0(\gamma_b)} - \gamma_b^2 \left[\left(\frac{K_1(\gamma_b)}{K_0(\gamma_b)} \right)^2 - 1 \right] \right\} \quad (\text{Constant } k)$$

(2) The average modulus of subgrade reaction concerning a Gibson k is $k_0(l/2)d$, for which the shear modulus G_s in eq. [4] is replaced with an average \bar{G}_s over the pile embedment. This allows eq. [4] to be rewritten more generally as

$$[5] \quad k_0 \left(\frac{l}{2} \right)^m d = \frac{3\pi \bar{G}_s}{2} \left\{ 2\gamma_b \frac{K_1(\gamma_b)}{K_0(\gamma_b)} - \gamma_b^2 \left[\left(\frac{K_1(\gamma_b)}{K_0(\gamma_b)} \right)^2 - 1 \right] \right\} \quad (\text{Gibson } k)$$

In the use of eqs. [4] and [5], the following points are worthy to be mentioned.

- (i) The diameter d is incorporated into eq. [3], compared to the previous expression by Guo and Lee (2001). This d is seen on the left-hand side of eqs. [4] and [5]. The new introduction greatly facilitates the establishment of the current solutions presented later on.
- (ii) The G_s and \bar{G}_s are ‘proportional’ to the pile diameter (width). For instance, given $l = 0.621\text{m}$, $r_0 = 0.0501\text{ m}$, the factor γ_b was estimated as $0.173 \sim 0.307$ using eq. [2] and $k_1 = 2.14 \sim 3.8$. The γ_b was revised as 0.178 given $e = 150\text{mm}$. $K_1(\gamma_b)/K_0(\gamma_b)$ was computed to be 2.898 . As a result, the kd (Constant k) is evaluated as $3.757G_s$, whereas the $0.5k_0dl$ (Gibson k) calculated as $3.757\bar{G}_s$. Conversely, shear modulus may be deduced from k via $G_s = kd/3.757$, or $\bar{G}_s = 0.5k_0dl/3.757$, as shown later in *Case Study*.
- (iii) Equation [5] is approximately valid. The average k ($= 0.5k_0l$) and \bar{G}_s refer to those at the middle embedment of a pile; whereas the k ($= k_0l$) and the shear modulus G_L refer to those at the pile-tip.
- (iv) The modulus ratio k_m/k_T (k_m and k_T are due to pure moment loading and lateral loading, respectively) was calculated using eqs. [2] and [4], and it is provided in Table 1. The calculation shows that

- The ratio k_m/k_T reduces from 1.56 to 1.27 as the slenderness ratio l/r_0 ascends from 1 to 10, with a highest value of 3.153 at $l/r_0 = 0$.
- The ratio k/k_T reduces from k_m/k_T ($e = \infty$) to 1 ($e = 0$) as the free length e decreases.
- The modulus k may be underestimated by 30~40% for a pile having a l/r_0 of 3~8, neglecting the impact of high eccentricity.

Given a pile-head load exerted at $e > 0$, the displacement is overestimated using $k = k_T$ than otherwise. Consequently, the solutions are conservatively deduced using $k = k_T$ in this paper. Influence of the e on the k is catered for by selecting the k_1 in determining γ_b via eq. [2]. The outmost difference can be obtained by comparing with the upper bound solutions capitalised on $k = k_m$.

2.3 Limiting Force Profile (LFP)

Upon reaching plastic state, the interaction force between pile and soil interface attains a maximum. Given a cohesionless soil, *the net limiting force per unit length along pile-soil interface (i.e. LFP)*, p_u varies linearly with depth, z , which may be described by (Fleming et al. 1992):

$$[6] \quad p_u = A_r z d$$

where $A_r z$ is pressure on the pile surface [FL^{-2}] that is contributed by radial and shear stresses obtained using eq. [1], which is shown later in *Case Study*; A_r is given by:

$$[7] \quad A_r = \gamma'_s K_p^2$$

where $K_p [= \tan^2 (45^\circ + \phi'_s/2)]$ is the coefficient of passive earth pressure; ϕ'_s is an effective frictional angle of soil; γ'_s is an effective unit weight of the soil (dry weight above the water table, buoyant weight below). Equation [7] is consistent with the experimental results (Prasad and Chari 1999), and the pertinent recommendation (Zhang et al. 2002), in contrast to other expressions for the A_r provided in Table 2.

2.4 Critical States

Displacement u of a rigid pile varies linearly with depth z (see Fig. 1), and it is expressed as $u = \omega z + u_0$, in which ω and u_0 are rotation and mudline displacement of the pile, respectively. It is rewritten herein in other forms to identify critical states for pile-soil relative slip (yield) developed firstly from the mudline, and later on from the pile tip.

(1) Pile-soil slip developed from mudline to a depth z_0 : As depicted in Fig. 2a, there exists a depth z_0 (called slip depth), above which the pile-soil interface is in plastic state; otherwise in elastic state. At the slip depth, it is noted that

(i) The pile displacement reaches a local threshold u^* of

$$[8] u^* = \omega z_0 + u_0$$

(ii) The limiting force per unit length obtained using eq. [3] is equal to that derived from eq. [6], which permits the threshold, u^* concerning Gibson k to be given by

$$[9] u^* = A_r / k_o$$

(2) Pile-soil slip developed from pile-tip to a depth z_1 : As can be seen from Figs. 2a and 2b, pile-soil relative slip (yield) may also initiate from the pile-tip (depth l), and expand upwards to a depth z_1 , at which the local displacement u just touches $-u^*$. This warrants the following relationship, resembling eq. [8].

$$[10] -u^* = \omega z_1 + u_0$$

Upon the pile-tip yield (i.e. $z_1 = l$), the z_0 is rewritten as \bar{z}_o . The *on-pile resistance per unit length* is prescribed by eq. [3] regarding the portion bracketed by the depths z_0 and z_1 , and by eq. [6] for the rest portions of $0 \sim z_0$, and $z_1 \sim l$.

(3) Depth of rotation point z_r : The pile rotates about a depth $z_r (= -u_0/\omega)$, at which no displacement $u (= \omega z_r + u_0 = 0)$ is anticipated.

Equations [9] and [10] are deduced using *Gibson k* featured by $p = k_o dz_u$. Should the p be governed by *Constant k* with $p = k du$, eqs. [9g] and [10g] are deduced instead as tabulated in Table 3. Note that the results for *Constant k* are highlighted using [] brackets, and are placed adjacent to those for *Gibson k*. For instance, assuming a *Constant k*, it is noted in Fig. 2a-right that $u^* = A_r z / k$; and in Fig. 2b-right that the displacement u at the depth z_1 is given by $u = -u^* z_1 / z_0$, and the limiting displacement at the pile-tip is equal to $u^* l / z_0$ in either figure.

With respect to *Constant k*, solutions for a rigid pile were deduced previously for a uniform p_u profile with depth (Scott 1981), and for a linear p_u profile (Motta 1997). They are presented in new (explicit) form for a linear p_u (Guo 2003), characterised by the slip depths (see Table 3). The latest expressions allow nonlinear responses to be readily

estimated. They are therefore extended in this paper to cater for *Gibson k*, and to examine responses at the newly defined critical states.

2.5 Nonlinear Solutions for *Pre-tip Yield State*

The unknown rotation ω and displacement u_0 in eq. [8] are determined using equilibriums of horizontal force on the pile, and bending moment about the pile-tip, as elaborated in A1, Appendix A. They are expressed primarily as functions of the pile-head load, T_t , and the slip depth, z_o . As a result, the solutions for a rigid pile prior to tip yield (*pre-tip yield state*) are simplified to offer the following expressions.

$$[11] \quad \frac{T_t}{A_r d l^2} = \frac{1}{6} \frac{1 + 2z_o/l + 3(z_o/l)^2}{(2 + z_o/l)(2e/l + z_o/l) + 3}$$

$$[12] \quad \frac{u_0 k_o}{A_r} = \frac{3 + 2[2 + (z_o/l)^3]e/l + (z_o/l)^4}{[(2 + z_o/l)(2e/l + z_o/l) + 3](1 - z_o/l)^2}$$

$$[13] \quad \omega = \frac{A_r}{k_o l} \frac{-2(2 + 3e/l)}{[(2 + z_o/l)(2e/l + z_o/l) + 3](1 - z_o/l)^2}$$

$$[14] \quad \frac{z_r}{l} = \frac{-u_0}{\omega l}$$

where $T_t/(A_r d l^2)$ is the normalised pile-head load; $u_0 k_o/A_r$ is the normalised mudline displacement; ω is the rotation angle (in radian) of the pile; and z_r/l is the normalised depth of rotation. Regarding these solutions, the following remarks are worthy to be stressed.

- (1) Two soil-related parameters k_o and A_r are involved in the expressions. The k_o is related to G_s ; while A_r calculated using the unit weight γ_s' , and angle of soil friction, ϕ_s' . Only the three measurable soil parameters are thus required.
- (2) Nonlinear responses are characterised by the sole variable z_o/l . Assigning a value to z_o/l , for instance, a pair of pile-head load T_t and mudline displacement u_0 are calculated using eq. [11] and eq. [12], respectively. Note that e/l is a constant.
- (3) The T_t is proportional to the pile diameter (width) as per eq. [11]. The u_0 implicitly involves with the pile dimensions via the k_o that in turn is related to pile slenderness ratio (l/r_o) via the γ_b (eq. [5]).

- (4) The free length e defined as the height of the loading point (T_t) above ground level may be regarded as a fictitious eccentricity ($e = M_o/T_t$), to cater for moment loading M_o at mudline level.

These remarks are equally applicable to the solutions based on *Constant k* that are provided in Table 3, in which eq. [xg] corresponds to eq. [x]. Nevertheless, a plastic (slip) zone for *Gibson k* is not initiated (i.e. $z_o > 0$) from mudline until the T_t exceeds $A_r d l^2 / (24e/l + 18)$; whereas for *Constant k*, the slip is developed immediately upon loading. In general, elastic-plastic solutions are preferred to elastic solutions (Scott 1981; Sogge 1981).

Features of the current solutions are highlighted for two extreme cases of $e = 0$ and ∞ .

- At $e = 0$, the usage of relevant expressions for $z_o \leq \bar{z}_o$ are provided in Table 4.
- Given $e = \infty$, eqs. [11]-[14] do reduce to those obtained for pure moment loading M_o (with $T_t = 0$), for which the normalised ratios for the u_o , ω and M_o are provided in Table 5. For instance, the moment per eq. [11] degenerates to $M_o (= T_t e)$ given by:

$$[15] \quad \frac{M_o}{A_r d l^3} = \frac{1}{12} \frac{1 + 2 \frac{z_o}{l} + 3 \left(\frac{z_o}{l}\right)^2}{2 + \frac{z_o}{l}}$$

2.6 Solutions for *Post-tip Yield State (Elastic-Plastic, and plastic State)*

Equations [11] – [14] for *pre-tip* and *tip-yield* states are featured by the yield to the depth z_o (being initiated from mudline only). At a sufficiently high load level, another yield zone to depth z_1 may be developed from the pile-tip as well. As load increases further, the two yield zones expand gradually towards the practically impossible ultimate state of equal critical depths (i.e. $z_o = z_1 = z_r$, see Fig. 2c). The advance of the z_1 from depth l to z_r is herein referred to as *Post-tip yield state* (Figs. 2a-2c). Horizontal force equilibrium of the entire pile, and bending moment equilibrium about the pile-head (rather than the tip) were used to deduce the solutions (see A2 Appendix A). They are featured uniquely by a newly introduced variable $C (= A_r / (u_o k_o))$, which is the reciprocal of the normalised displacement, see eq. [12]). The variable C must not exceed its value C_y calculated for the *tip-yield* state (i.e. $C < C_y$, and u_o being estimated using eq. [12] and $z_o = \bar{z}_o$), to induce the *post-tip yield* state. The equations/expressions for estimating z_r , T_t and u_o are highlighted in the following:

(1) The rotation depth z_r is governed by the C and the e/l

$$[16] \quad \left(\frac{z_r}{l}\right)^3 + \frac{3+C^2}{2(1+C^2)} \frac{e}{l} \left(\frac{z_r}{l}\right)^2 - \frac{1}{4(1+C^2)} \left(2+3\frac{e}{l}\right) = 0$$

Solution of eq. [16] may be written as

$$[17] \quad z_r/l = \sqrt[3]{A_0} + \sqrt[3]{A_1} - D_1/6$$

$$[18] \quad A_j = (D_o/8 - D_1^3/216) + (-1)^j [(27D_o^2 - 2D_oD_1^3)/1728]^{1/2} \quad (j = 0, 1)$$

$$[19] \quad D_1 = \frac{3+C^2}{1+C^2} \frac{e}{l} \quad D_o = \frac{2+3e/l}{1+C^2}$$

(2) The normalised head-load, $T_t/(A_r d l^2)$ is derived from eqs. [A15] - [A17] as

$$[20] \quad \frac{T_t}{A_r d l^2} = \left(1 + \frac{C^2}{3}\right) \left(\frac{z_r}{l}\right)^2 - 0.5$$

(3) The mudline displacement, u_0 is obtained using the definition of the variable C .

$$[21] \quad u_0 = A_r / (k_o C)$$

(4) The slip depths from the pile head, z_0 and the tip, z_1 (see Table 6) are computed using $z_0 = z_r(1-C)$ and $z_1 = z_r(1+C)$, respectively, as deduced from eqs. [8] and [10].

Equation [17] provides the rotation depth, z_r (thus z_0 and z_1) for each value of u_0 (via the C and e). The z_r in turn allows the rotation angle, ω to be estimated using $\omega = -u_0/z_r$. As for the *Constant k*, the counterparts for eqs. [17] - [21] are provided in Table 6, and those for z_0 and z_1 in Table 7. The variable $C [= A_r z_r / (u_0 k)]$, as per eq. [21g] becomes the product of the reciprocal of the normalised displacement $u_0 k / A_r l$ and the normalised rotation depth z_r / l .

2.7 Load, Displacement, Slope and On-pile Force Profiles

2.7.1 Response at tip yield state

Upon tip yield ($z_0 = \bar{z}_o$), eq. [10] is transformed into the following form to resolve the \bar{z}_o , by replacing u_0 with that given by eq. [12], ω with that by eq. [13], and z_1 with l .

$$[22] \quad \bar{z}_o^{-3} + (2e+l)\bar{z}_o^{-2} + (2e+l)l\bar{z}_o - (e+l)l^2 = 0$$

The \bar{z}_o/l was estimated for $e/l = 0 \sim 100$ using eq. [22], and it is illustrated in Fig. 4a. Regarding the *tip-yield* state, the responses of $u_0 k_o l^m / (A_r l)$, and $\omega k_o l^m / A_r$ were calculated and are presented in Figs. 4b and 4c. Two extreme cases of $e = 0$ and ∞ are provided in the following.

- Provided that $e = 0$, the \bar{z}_o/l is computed as 0.5437. The normalised $T_t / (A_r d l^2)$ is thereby estimated as 0.113, $u_0 k_o / A_r$ as 3.383, $\omega k_o l / A_r$ as **-4.3831**, z_r/l as 0.772, and C_y as 0.296, in terms of eqs. [11], [12], [13], [14], and [21], respectively. These values are tabulated in Table 4.
- Given $e = \infty$, the \bar{z}_o/l is 0.366. The normalised values are thus obtained (see Table 5) with $T_t / (A_r d l^2) = 0$, $u_0 k_o / A_r = 2.155$, $\omega k_o l / A_r = \mathbf{-3.154}$, $z_r/l = 0.683$, and $C_y = 0.464$.

The counterparts for *Constant k* were obtained, and are provided in the square [] brackets in Tables 4 & 5, as well. For instance, eqs. [14g] and [22g] offer $\bar{z}_o = 0.618l$ and $z_r = 0.764l$ (see Table 4) for $e = 0$; whereas $\bar{z}_o = 0.5l$ and $z_r = 0.667l$ (see Table 5) for $e = \infty$.

As the e increases from mudline to infinitely large, the u_0 reduces by 36% [38%]; and the ω reduces by 28% [29.2%]. The estimated depths z_r and \bar{z}_o permit the normalised *on-pile force profiles* of $p / (A_r \bar{z}_o d)$ at the *tip-yield* state to be constructed. This is achieved by drawing lines in sequence between adjacent points (0, 0), $(A_r \bar{z}_o, \bar{z}_o)$, (0, z_r), and $(-A_r l, l)$, which are normalised by $A_r \bar{z}_o$ and \bar{z}_o , respectively for the first and the second coordinates, as exemplified in the following for $e = 0$, and ∞ .

- Figure 5a provides the profiles constructed using *Constant k*. The $\bar{z}_o/l = 0.618$ and $z_r/l = 0.764$ for $e = 0$ offer a pressure at the pile-tip level of $1.6A_r \bar{z}_o$, which raises to $2A_r \bar{z}_o$, as the e increases to ∞ (accompanied by the reduction of \bar{z}_o/l to 0.5 and z_r/l to 0.667).
- Figure 5b depicts the normalised profiles obtained using a *Gibson k*, in view of the $\bar{z}_o = 0.544l$ and $z_r = 0.772l$ (Table 4) upon $e = 0$; whereas $\bar{z}_o = 0.366l$ and $z_r = 0.683l$ (Table 5) concerning $e = \infty$. The pile-tip pressure increases from $1.84A_r \bar{z}_o$ to $2.73A_r \bar{z}_o$, as the e shifts towards infinitely large from 0.

The constructed profiles for $e = 0$ and ∞ (see Fig. 5b) well bracket the ‘*Test data*’ provided by Prasad and Chari (1999). The tri-linear feature of the ‘*Test*’ profile is also

captured using the Gibson k . Likewise, good comparisons with measured p profiles were noted for a few other cases (not shown herein). The profile is governed by the gradient A_r , the slip depths z_0 (from the head) and z_1 (from the tip), and the rotation depth z_r . Overall, Fig. 5 indicates the measured z_0 (Prasad & Chari, 1999) conforms to *pre-tip yield* state, as Fig. 4a does (and further confirmed by the reported capacity shown later in Fig. 9). It also reflects the impact of

- The k profile, as a better match is observed using a Gibson k ;
- Lack of measured points around the critical depth (z_0);
- Ignoring the increase in the modulus owing to the free length (e) (see Table 1);
- The nonlinear spring behaviour in reality vs the elastic-plastic spring adopted herein.

2.7.2 State of yield at rotation point (YRP, Completely plastic state)

The pile-head displaces infinitely large as the C approaches zero, see eq. [21]. This ultimate state featured by $z_r = z_0$ is referred to as *yield at rotation point* (YRP). Despite practically unachievable, the state provides an upper bound, for which, it is noted that

- (1) Equation [20] reduces to that proposed by Petrasovits and Award (1972), as the *on-pile force profile* does (see Fig. 2c).
- (2) The z_r/l is independent of the modulus k . It should be estimated using eq. [17] by substituting 0, $2 + 3e/l$, and $3e/l$ for the C , D_0 , and D_1 , respectively in eqs. [18] and [19] regarding $e \neq \infty$; otherwise using eq. [16] directly. This is shown for the following two typical cases.
 - Regarding $e = 0$, eq. [19] allows $D_0 = 2$, and $D_1 = 0$ to be deduced. $A_0 = 0.5$, and $A_1 = 0$ are thus obtained in terms of eq. [18]. The z_r/l is evaluated as 0.7937.
 - Substituting ∞ for the e in eq. [16], the z_r/l is directly computed as 0.7071.
- (3) *On-pile pressure profile* can be constructed by drawing lines between adjacent points $(0, 0)$, $(A_r \bar{z}_0, \bar{z}_0)$, $(-A_r \bar{z}_0, z_r)$, and $(-A_r l, l)$ (Fig. 2c-left).

2.8 Maximum Bending Moment and Its Depth

2.8.1 Pre-tip yield ($z_0 < \bar{z}_0$) and tip yield ($z_0 = \bar{z}_0$) states

The depth z_m at which maximum bending moment occurs is given by eq. [23] if $z_m \leq z_0$, otherwise by eq. [24], in light of eqs. [A5a] and [A5b] (see Appendix A).

$$[23] \quad \frac{z_m}{l} = \sqrt{\frac{2T_t}{A_r dl^2}} \quad (z_m \leq z_0)$$

$$\begin{aligned}
\frac{z_m}{l} &= \frac{3(z_o/l)^3(z_o/l + 2e/l) + 1}{8(2 + 3e/l)} + \\
[24] \quad &+ \sqrt{3} \sqrt{[(z_o/l)^3(z_o/l + 2e/l) + 16e/l + 11]} \times \frac{\sqrt{3(z_o/l)^3(z_o/l + 2e/l) + 1}}{8(2 + 3e/l)} \quad (z_m > z_o)
\end{aligned}$$

Equation [23] offers $T_t/(A_r d l^2) = 0.5(z_m/l)^2$. Following determination of \bar{z}_o using eq. [22], the z_m/l was computed for a series of e/l ratios at *tip-yield* state. It is plotted in Fig. 4d against e/l . The calculation shows that:

(1) The z_m may in general be computed by using eq. [23], satisfying $z_m < \bar{z}_o$ (regardless of e/l). Nevertheless, a low load level (pre *tip-yield* state) along with a small eccentricity e may render $z_o < z_m$ (not shown herein).

(2) The z_m converges to mudline, as the e/l approaches infinitely large (see Table 5).

Substituting the z_m/l from eq. [23] for that in eq. [A7a] allows the maximum bending moment (M_m) for $z_m \leq z_o$ to be gained.

$$[25] \quad M_m = (2z_m/3 + e)T_t \quad (z_m \leq z_o)$$

Otherwise, substituting the z_m/l from eq. [24] for that in eq. [A7b] permits the M_m for $z_m > z_o$ to be deduced as:

$$\begin{aligned}
[26] \quad M_m &= \left(\frac{3l^4 - 2z_m^3 z_o + 2z_m z_o^3 - 4l^3 z_o + z_m^4}{(l - z_o)^2 (3l^2 + 2lz_o + z_o^2)} e + z_m \right) T_t + \frac{dA_r}{6} z_o^2 (2z_o - 3z_m) \\
&- \frac{A_r d}{6} (z_o - z_m)^2 \left(z_m + 2 \frac{3l^3 z_o (l - z_o) + (z_o^5 - z_m^2 l^3)}{(l - z_o)^2 (3l^2 + 2lz_o + z_o^2)} \right) \quad (z_m > z_o)
\end{aligned}$$

Valid to a high load level, eq. [25] is independent of pile-soil relative stiffness. Therefore, it is essentially identical to that for a flexible pile (Guo 2006). Likewise, the counterpart expressions for the z_m and M_m were derived regarding a constant k , and they are provided in Table 3. In particular, eqs. [23] and [23g] are identical for either k profile, so are eqs. [25g] and [25g], although they all are still mentioned later.

With respect to the *tip-yield* state, as $z_m < z_o$ (Figs. 4a and 4d), the M_m was calculated using eq. [25]. It is presented in Figs. 6a and 6b in form of $M_m/(A_r d l^3)$. The M_m was also evaluated using eq. [25g], and it is plotted in Fig. 6b. The following points are worthy to be stressed for the *tip-yield* state.

(1) The maximum moment M_m obtained is higher using *Constant k* than *Gibson k* (see Fig. 6b). With $e = \infty$, and $z_m = 0$ [0], eqs. [25] and [25g] allow the M_m to be calculated as $T_t e$ (either k). More specifically, it is noted that:

- At $e = 0$ (see Table 4), the $M_m/(A_r dl^3)$ is estimated as 0.036[0.038], as \bar{z}_o/l was calculated as 0.5473[0.618], and the associated $T_t/(A_r dl^2)$ is 0.113[0.118].
 - At $e = \infty$ (see Table 5), the $M_m/(A_r dl^3)$ is evaluated as 0.0752 [0.0833] with the previously calculated values of $\bar{z}_o/l = 0.366[0.50]$, and $T_t = 0[0]$.
- (2) Increase in the e enables M_m to approach M_o , e.g. $M_m = 0.947M_o$ given $e/l = 2$ and Gibson k (see Figs. 6a and 6b). Note that a ratio $e/l > 3$ is normally encountered in overhead catenary systems.
- (3) The solid circle dotted line of $e = \infty$ in Figs. 6a or 6b denotes the upper limit of the *tip-yield* state for the corresponding k . They were obtained using eq. [15] or $\bar{z}_o/l/6$ (see Table 5), and overlap with those from eq. [25] [eq. [25g]] as anticipated.

The bending moments are subsequently compared with those induced at ‘*yield at rotation point*’.

2.8.2 Yield at rotation point (YRP)

At the YRP state, the relationships of T_t versus z_m , and T_t versus M_{\max} are found identical to those for pre-tip yield state ($z_m \leq z_0$), respectively. Responses at rotation point (see Fig. 2c) may be obtained by substituting $C = 0$ for that in the solutions addressing the *post-tip yield* state, e.g. eq. [20] is transformed into

$$[27] \quad \frac{T_t}{A_r dl^2} = 0.5 \left(\frac{z_m}{l} \right)^2$$

in which the depth z_m is related to the z_r given by the following relationship

$$[28] \quad \frac{z_m}{l} = \left[2 \left(\frac{z_r}{l} \right)^2 - 1 \right]^{0.5}$$

where z_r/l is still calculated from eq. [16] or [17], as discussed previously in the section entitled ‘*State of yield at rotation point*’. Equation [28] was derived utilising the *on-pile force profile* depicted in Fig. 2c, and shear force $T(z_m) = 0$ at depth z_m . The normalised maximum bending moment is derived using eqs. [A7a] and [27] as

$$[29] \quad \frac{M_m}{A_r dl^3} = \frac{T_t}{A_r dl^2} \frac{e}{l} + \frac{1}{3} \left(\frac{z_m}{l} \right)^3$$

The normalised M_m was computed using eq. [29]. It is plotted in Fig. 6a as ‘*both k at YRP*’, as eqs. [27] ~ [29] are all independent of the k profiles. The moment of $M_o (= T_t e)$ is

also provided in the figure. Equation [29] may be converted into an identical form to eq. [25] using eqs. [27] and [28]. About the YRP state, it is noted that

- The z_o/l ($= z_r/l$) of 0.7937 ($e = 0$) and 0.707 ($e = \infty$) allow the z_m/l to be evaluated as 0.5098 and 0; upon which $T_t/(A_r d l^2)$ is calculated as 0.130 ($e = 0$) and 0 ($e = \infty$).
- Equation [29] permits the $M_m/(A_r d l^3)$ to be computed as 0.0442 ($e = 0$) and 0.0976 ($e = \infty$). In particular, the $M_m/(A_r d l^3)$ for $e = \infty$ (see Fig. 2c) is deduced as $[1-2(z_r/l)^3]/3$, in light of moment equilibrium about ground line and the *on-pile force profile*.

Finally, eqs. [23] and [25] are supposedly valid from the tip-yield state through to the ultimate YRP state (with $z_m < z_o$). This is not pursued herein, but will be seen later in *Case Study*.

2.8.3 Tip-yield to YRP

Figure 6a demonstrates that as the yield (slip) extends from the pile tip (see Fig. 2a) to the rotation point (see Fig. 2c), $M_m/(A_r d l^3)$ (Gibson k) increases by 22.8% (from 0.036 to 0.0442) at $e = 0$ (see Table 4); or by 29.9% (from 0.0752 to 0.0976) at $e = \infty$ (see Table 5). (Slightly less increase in percentage is observed using Constant k). The increase in M_{max} consequently would not exceed $\sim 30\%$ from the *tip-yield* to the YRP states. In contrast, as the e increases from 0 to ∞ , the M_m increases by 109% at *tip-yield* state (from 0.036 to 0.0752), or by 120% at rotation point yield state (from 0.0442 to 0.0976). The eccentricity has more profound impact on the M_{max} than the states of yield. These conclusions about the M_m are valid to other rigid piles of identical non-dimensional parameters.

2.9 Calculation of Nonlinear Response

Response of the pile is characterised by two sets of expressions concerning pre- ($z_o < \bar{z}_o$), and post- ($z_o > \bar{z}_o$) tip yield states. It thus may be obtained pragmatically via two steps:

- (1) Regarding the *pre-tip yield* state, a series of slip depth z_o ($< \bar{z}_o$) may be specified. Each z_o allows a set of load (T_t), displacement (u_o), and rotation (ω) to be evaluated using eqs. [11], [12], and [13], respectively; and furthermore, the moment (M_m) to be estimated using eq. [25] or eq.[26].

(2) As for the *post-tip yield* state, a series of C ($0 \leq C \leq C_y$) may be assigned. Each C permits a rotation depth z_r to be calculated using eq. [17] ($e \neq \infty$) otherwise eq. [16] ($e = \infty$). The depth z_r in turn allows a load and a displacement to be calculated using eqs. [20] and [21] respectively; and a rotation angle to be assessed as $-u_0/z_r$. The force T_t estimated is then used to determine M_m using eq. [25] or [29].

The two steps allow entire responses of the pile-head load, displacement, rotation, and maximum bending moment to be ascertained. For instance, non-dimensional responses of $u_0 k_o/A_r$ [$u_0 k/A_r l$], $\omega k_o/A_r$ [$\omega k/A_r$], and $M_m/(A_r d l^3)$ were predicted along with $T_t/(A_r d l^2)$, and those at *tip-yield* state ($z_o = \bar{z}_o$), using Gibson k [also Constant k] for a pile having $l/r_o = 12$, and at six typical ratios of e/l . The ultimate moment at YRP state and $e/l = 0$ was also predicted using eq. [29]. All these predictions are shown in Fig. 7. The effect of k profile on the normalised u_0 and ω is evident, but on the normalised M_{max} is noticeable only at low load levels (as anticipated). Conversely, two measured responses [e.g. $T_t \sim u_0$ and $T_t \sim M_{max}$ (or ω) curves] may be fitted by using the current solutions, to allow values of A_r and k to be uniquely back-figured in a principle discussed previously for a flexible pile (Guo 2006), and further illustrated later in *Case Study*.

3. COMPARISON WITH EXISTING SOLUTIONS

The current solutions have been implemented into a spreadsheet program called GASLSPICS operating in Windows EXCELTM. The results presented so far and subsequently were all obtained using this program.

3.1 Comparison with Existing Experiments and Numerical Solutions

Model tests were conducted by Nazir (Laman et al. 1999) at a centrifugal acceleration of 33g ($g = \text{gravity}$) on a pier with a diameter $d = 30\text{mm}$ and an embedment length $l = 60\text{mm}$ (Test 1); at 50g on a pier with $d = 20\text{mm}$ and $l = 40\text{mm}$ (Test 2); and at 40g on a pier with $d = 25\text{mm}$ and $l = 50\text{mm}$ (Test 3), respectively. They are designed to mimic the behaviour of a prototype pier with $d = 1\text{m}$ and $l = 2\text{m}$. Embedded in dense sand, the prototype pier was gradually loaded to a maximum lateral load of 66.7 kN, generating a moment M_o of 400 kNm about mudline ($e = 6\text{m}$). The sand bulk density γ_s' was 16.4 kN/m³, and frictional angle ϕ_s' was 46.1°. Young's modulus of the pier was 207 GPa, and Poisson's ratio was 0.25. In the 40g test (Test 3), lateral loads were applied at a free-length (e) of 120 mm above mudline (Laman et al. 1999) on the model pier. The pier

rotation angle (ω) was measured under various moments (M_0) during the test, and it is plotted in Fig. 8a in prototype scale. Tests 1 and 2 were conducted to examine the effect of modelling scale. The test results are plotted in Fig. 8c.

Three-dimensional finite element analysis (FEA^{3D}) was undertaken (Laman et al. 1999) to simulate the tests as well, adopting a hyperbolic stress-strain model, for which the initial and unloading-reloading Young's moduli alter with confining stress. The predicted moment (M) is plotted against rotation (ω) in Figs. 8a and 8c. It compares well with the median value of the three centrifugal tests, except for the initial stage.

To undertake the current predictions, the two parameters A_r and kd were calculated.

- (1) The A_r was estimated as 621.7 kN/m^3 using eq. [7], $\gamma_s' = 16.4 \text{ kN/m}^3$, and $\phi_s' = 46.1^\circ$
- (2) The kd was calculated as $3.02G_s$ (or $1.2E_s$) in light of eq. [4]. Initial Young's modulus, E_s was computed as 25.96 MPa , and unloading-reloading E_s as 58.63 MPa , in view of the published data (Laman et al. 1999), and an average confining pressure of 16.4 kPa over the pile embedment. This offers an initial modulus of subgrade reaction kd of 31.36 MPa , and a unloading-reloading kd of 70.83 MPa . The kd was thus chosen (see Table 8) as 34.42 MPa ($d = 1 \text{ m}$) to simulate Test 3; and as 51.63 MPa ($d = 1 \text{ m}$) to mimic Test 2.

The T_t and the ω were estimated via eqs. [11g] and [13g], with the A_r and the selected kd , and assuming a *Constant k*. The M_0 was obtained as $T_t * e$, and is plotted against ω in Fig. 8a and 8c as 'Current CF'. The figures show the measured responses for Tests 2 and 3 were well modelled. The *tip-yield* occurred at a rotation angle of 3.8° (see Table 8). Furthermore, a Gibson k was stipulated with $k_0 = 17.5 \text{ kN/m}^3/\text{m}$. With the A_r , the M_0 and ω were calculated using eqs. [11] and [13], and are plotted in Fig. 8a. The prediction is also reasonable well. Next, the displacement u_0 is calculated using eq. [12] and eq. [12g], respectively, and is plotted in Fig. 8b against the respective T_t . A softer response is noted given Gibson k than a uniform k . The real k profile is not known until the measured $T_t \sim u_0$ becomes available.

The A_r for Test 2 (see Fig. 8c) may be raised slightly from the current value of 621.7 kN/m^3 , to achieve a better agreement with the measured ($M_0 \sim \omega$) curve. A high value of A_r was used to achieve a good prediction (not shown herein) for the same pier tested in a 'loose' sand. The accuracy of any predictions is essentially dominated by the p_u (Guo 2006) mobilised along flexible piles, and perhaps also by the k (or G_s) for rigid piles. The

current solutions are sufficiently accurate, in terms of capturing nonlinear response manifested in the tests and the FE analysis (FEA^{3D}).

To validate the current solutions, a continuum-based analysis on a rigid pile ($l = 5$ m, $d = 1$ m) was also conducted using the finite-difference program FLAC^{3D} (Itasca 2002). The primary parameters adopted are as follows: $\gamma_s' = 16.4$ kN/m³, $\phi_s' = 46.1^\circ$, $G_s = 5.0$ MPa, and $E_p = 200$ GPa. The study reveals the p_u is more or less proportional to $z^{1.7}$ (i.e. $p_u \propto z^{1.7}$), as it was noted for majority of flexible piles in sand (Guo 2006). The current solutions (using $p_u \propto z$, eq. [16]) thus predict a lower stiffness (ratio of pile-head load over displacement) at a large displacement than that obtained from FLAC^{3D}. The good comparisons noted among the current solutions, the measured data and the FE analysis vindicate the acknowledged fact that (1) FLAC^{3D} offers 10~20% higher stiffness than FE analysis; (2) Equation [6] is sufficiently accurate for rigid piles.

3.2 Comments on Existing Uncoupled Expressions for T_o

Capacity T_o (i.e. T_t at a defined state) of a laterally loaded rigid pile is currently evaluated using such simple expressions as those outlined in Table 2. They were broadly presented in terms of either the normalised rotational depth z_r/l (Methods 1, 4 and 8), or the normalised eccentricity e/l (Methods 2, 3, 5, 6 and 7). Nevertheless, the capacity (alike to T_t) relies on the two facets of e/l and z_r/l , apart from the critical value A_r and pile dimensions. This may be inferred from eq. [11] or eq. [20]. In eq. [11], for instance, the impact of z_r/l on T_t is represented via z_o/l .

Some features of the expressions in Table 2 are highlighted in the following:

- (1) The relative weight of free-length compared to the constant in the denominator in general varies from e/l (Broms 1964) to $1.5e/l$ (McCorkle 1969; Balfour-Beatty-Construction 1986).
- (2) Each expression may offer a capacity T_o close to measured data via adjusting the gradient A_r . For example, the A_r for McCorkle's method may be selected as 20~30% that suggested for Balfour Betty method, as the T_o offered by the former is approximately 3.76 times the latter, given an identical A_r .
- (3) The expressions are not explicitly related to magnitude of displacement or rotation angle. The A_r reported may correspond to different stress states.

The aforementioned ambiguity regarding T_o and A_r prompts us to redefine the capacity T_o as the T_t at (a) *Tip-yield* state, or (b) Yield at rotation point state (YRP).

- (i) The T_o for tip-yield state is obtained by substituting \bar{z}_o for the z_o in eq. [11]; whereas
- (ii) The T_o for the YRP state is evaluated using eq. [27], in which the z_m is calculated from z_r using eq. [28].

The ratio $T_o/(A_r d l^2)$ was estimated for a series of e/l ratios, concerning the two states; and is plotted in Fig. 9. The mudline displacement may be accordingly estimated using eq. [12] (or eq. [12g]) concerning the *tip-yield* state, given k_o [or k]; whereas it is infinitely large upon the YRP state. Fig. 9 indicates that

- (1) The T_o at the YRP state exceeds all other predictions, and **overlies** on the upper bound given by Fleming et al (1992). On the other hand,
- (2) The T_o dictated by the test results of Prasad and Chari (1999) offers the lower bound, which occurs at a *pre-tip yield* state as mentioned previously.

Especially, \bar{z}_o/l at the tip-yield state is obtained as 0.618 if $e = 0$ is stipulated. Regardless of the e , substituting 0.618 for the z_o/l in eq. [11g], a new expression for $T_o/(A_r d l^2)$ is developed, and provided in Table 2 as Mtd 9. Interestingly, it offers a T_o (not shown herein) close to the YRP state.

4. CASE STUDY

4.1 Model tests by Prasad and Chari (1999)

A total of 15 steel pipe piles were tested in dry sand. Each model pile was 1,135 mm in length, 102 mm in outside diameter (d), and 5.6 mm in wall thickness. All piles were embedded to a depth (l) of 612 mm. The sand had three typical relative densities (D_r) of 0.25, 0.5, and 0.75; bulk densities (γ_s') of 16.5 kN/m³, 17.3 kN/m³, and 18.3 kN/m³; and frictional angles (ϕ_s') of 35°, 41°, and 45.5°, respectively. Lateral loads were imposed at an eccentricity of 150 mm on the piles until failure, which offers the results (Prasad and Chari 1999) of

- (1) Distribution of σ_r across the pile diameter at a depth of about 0.276 m, shown previously in Fig. 3, with a maximum $\sigma_r = 66.85$ kPa;
- (2) Pressure profile (p) along the pile that was plotted previously in Fig. 5 as ‘Test data’;
- (3) Normalised capacity $T_o/(A_r d l^2)$ versus normalised eccentricity (e/l) relationship, signified as ‘Prasad & Chari (1999)’ in Fig. 9;
- (4) Lateral pile-head load (T_t) \sim mudline displacement (u_0) curves, as plotted in Fig. 10a;
- (5) Shear moduli at the pile-tip level (G_L) being 3.78, 6.19 and 9.22 MPa (taking the Poisson’s ratio of the soil ν_s as 0.3) for $D_r = 0.25, 0.5$ and 0.75 , respectively.

Incorporating the effect of diameter, the measured moduli should be revised, for instance for Gibson k , as 0.385 MPa ($= 3.78d$), 0.630 MPa ($= 6.19d$) and 0.94 MPa ($= 9.22d$). These values are then quite consistent with 0.22~0.3 MPa deduced from a model pile of similar size (having $l = 700$ mm, $d = 32$ mm or 50 mm) embedded in a dense sand (Guo and Ghee 2005).

Responses (2) and (3) were addressed in previous sections. Other responses are studied herein, along with the M_m and z_m , in order to illustrate the use of the current solutions, and the impact of the k profiles.

4.1.1 Analysis using Gibson k

The measured $T_t \sim u_0$ relationships (see Fig. 10a) were fitted using the current solutions for Gibson k , following the section entitled ‘*Calculation of Nonlinear Response*’. This allows the parameters k_o and A_r to be deduced (see Table 9) for each test in the specified D_r . (Note: ideally two measured curves are required for the two parameters. The initial elastic gradient and subsequent nonlinear portion of the $T_t \sim u_0$ curve may serve this purpose). The $k_o d$ in turn permits shear modulus G_L to be determined. The A_r allows maximum bending moment M_m to be evaluated, along with its depth of occurrence z_m . The M_m is plotted in Fig. 10b against T_t , and in 10c against z_m , respectively. The study indicates that:

- The A_r was 244.9 kN/m³, 340.0 kN/m³, and 739. kN/m³ for $D_r = 0.25, 0.5$, and 0.75 respectively, which is $\pm \sim 15$ % different from 224.68 kN/m³, 401.1 kN/m³, and 653.3 kN/m³ obtained using eq. [7].
- The G_L was 0.31 MPa, 0.801 MPa, and 1.353 MPa, which differ by -19.48%, 27.7% and 43.9% from the revised moduli for $D_r = 0.25, 0.5$ and 0.75 , respectively.

- The *tip-yield* for $D_r = 0.5$ and 0.75 occurs in close proximity to the displacement u_0 of $0.2d$ ($= 20.4$ mm, see Fig. 10a), whereas the *tip-yield* for $D_r = 0.25$ occurs at a much higher displacement u_0 of 39.5 mm than $0.2d$. The latter yields a higher load T_0 of 0.784 kN than 0.529 kN observed at $0.2d$ upon the measured $T_t \sim u_0$ curve. The *tip-yield*, nevertheless, is associated with a rotation angle of $2.1 \sim 4.0$ degrees (see Table 9), conforming to $2 \sim 6$ degrees (see Dickin and Nazir, 1999) deduced from model piles tested in centrifuge.

The calculation is elaborated here, for instance, regarding the test with $D_r = 0.25$, for which, $A_r = 244.9$ kN/m³, and $k_o = 18.642$ MPa/m². It is presented for four typical yielding states (see Tables 10 and 11), shear modulus, and distribution of stress σ_r across the pile diameter.

(1) *Tip-yield* state: The ratio \bar{z}_o/l at the *tip-yield* state is obtained as 0.5007 from eq. [22]. This ratio allows relevant responses to be calculated, as tabulated in Table 10.

(i) The $T_t/(A_r d l^2)$ is determined, in terms of eq. [11], as:

$$T_t/(A_r d l^2) = \frac{1}{6} \frac{1 + 2 \times 0.50 + 3 \times 0.5^2}{(2 + 0.5)(2 \times 0.245 + 0.5) + 3} = 0.0837$$

The T_t is thus estimated as 0.784 kN ($= 0.0837 \times 244.9 \times 0.102 \text{ m} \times 0.612^2$ kN/m).

(ii) The $u_0 k_o / A_r$ is computed using eq. [12] as 3.005 , and u_0 is evaluated as 39.5 mm ($= 3.005 \times 244.9 / 18642$ m).

(iii) The z_m/l is estimated as 0.4093 using eq. [23], as z_m ($= 0.251$ m) $< z_o$ ($= 0.306$ m).

(iv) In view of $z_m < z_o$, $M_m/(A_r d l^3)$ is estimated using eq. [25] as 0.0434 [$= (2/3 \times 0.4093 + 0.245) \times 0.0837$], and M_m as 0.248 kNm.

In brief, upon the *tip-yield* state, it is noted that $T_t = 0.784$ kN, $u_0 = 39.5$ mm, $z_m = 0.251$ m, and $M_m = 0.248$ kNm, as shown in Figs. 10a-10c. The moment is of similar order to that recorded in similar piles tested under soil movement (Guo and Ghee 2005).

(2) *Pre-tip yield* state ($z_o/l = 0.3$, and 0.5): Given, for instance, $z_o/l = 0.3$ ($< \bar{z}_o/l$), the z_m/l is computed as 0.364 using eq. [24]. Thereby, it follows that $T_t = 0.605$ kN, $u_0 = 22.3$ mm, and $z_m = 0.223$ m. M_m is estimated as 0.18 kNm using eq. [26] ($z_m > z_o$).

As z_o/l increases to 0.5, T_t increases to 0.783 kN, M_m to 0.248 kNm, and z_m to 0.251 m. The two points, each given by a pair of M_m and z_m , are plotted in Fig. 11a.

(3) *Post-tip yield state* (e.g. $C < C_y$): At the *tip-yield* state, the value of $u_0 = 39.5$ mm allows the C_y to be computed as 0.3326 [= 244.9/(0.0395×18,642)]. Assigning a typical 0.2919 ($< C_y$) to the C , for instance, the responses were estimated as follows:

(i) The z_r/l is evaluated in steps (see Table 12) that follow:

- $D_1 = 0.6968$, and $D_0 = 2.52049$ are gained using eq. [19], which allow $A_1 = 3.9127 \times 10^{-6}$, and $A_0 = 0.627$ to be obtained using eq. [18].
- z_r/l is computed as 0.756 (= $0.627^{1/3} + 3.9127^{1/3} \times 0.01 - 0.6968/6$) using eq. [17].

(ii) The T_t is calculated as 0.814 kN, as $T_t/(A_r d l^2) = 0.087$ using eq. [20].

(iii) The u_0 is 45 mm [= 244.9/(18642×0.29193)], as per eq. [21].

(iv) The z_o/l is calculated as 0.535 [= (1-0.29193)×0.7555].

(v) The z_m/l is calculated as 0.4171 [= (2×0.087)^{0.5}] as per eq. [23], and $z_m = 0.255$ m.

(vi) $M_m/(A_r d l^3)$ is 0.0453 using eq. [25], and $M_m = 0.261$ kNm.

(4) YRP state: The condition of $C = 0$ at the YRP state allows D_1 , D_0 , A_0 , A_1 to be estimated (see Table 12), which in turn enable z_r/l to be computed as 0.774 (= $0.68^{1/3} + 0.017 - 0.735/6$). Values of z_m/l , $T_t/(A_r d l^2)$, and $M_m/(A_r d l^3)$ are thus calculated as 0.445 ($< z_o/l$), 0.099, and 0.0536, respectively (see Table 10) using eqs. [28], [27], and [29]. Upon YRP state, it follows $T_t = 0.926$ kN, $z_m = 0.272$ m, and $M_m = 0.307$ kNm.

(5) Distributions of the bending moment $M(z)$ and the shear force $T(z)$ with depth were determined in light of eqs. [A7a] and [A7b] and eqs. [A6a] and [A6b] regarding the two ratios of $z_o/l = 0.3$ and 0.5. They are plotted in Figs. 11a and 11b, and agree with the M_{\max} and z_{\max} predicted before (Figs. 10a-10c). For instance, $T(z)$ at a slip depth of $z_o = 0.5l$ was estimated using $\omega = -0.087$, $u_0 = 39.5$ mm (see Table 11), and eqs. [30] and [31].

$$[30] \quad T(z) = 0.5A_r d (z_o^2 - z^2) + k_o d \omega (l^3 - z_o^3) / 3 + k_o d u_0 (l^2 - z_o^2) / 2 \quad (z \leq z_o)$$

$$[31] \quad T(z) = k_o d \omega (l^3 - z^3) / 3 + k_o d u_0 (l^2 - z^2) / 2 \quad (z > z_o)$$

Local shear force $T(z) \sim$ displacement $u(z)$ relationships were predicted using eqs. [A5a] and [A5b], for the normalised depths z/l of 0, 0.3, 0.5, 0.62 and 0.9. They are plotted in Fig. 12

- (6) The \bar{G}_s is estimated as 0.1549 MPa ($= 0.5 \times 18.642 \text{ MPa/m}^2 \times 0.612 \text{ m} \times 0.102 \text{ m} / 3.757$) as per the discussion for eq. [5], in terms of $k_0 = 18.642 \text{ MPa/m}^2$ in Table 9. The shear modulus G_L (at the pile-tip level) is thus inferred as 0.310 MPa ($= 2\bar{G}_s$).
- (7) The measured σ_r on the pile surface was mobilised by a local displacement u of 21.3 mm, at a head-displacement u_0 of 57 mm (after tip yield, and with $z_0 = 0.361 \text{ m}$, and $z_r = 0.470 \text{ m}$).

The discussion about eq. [5] indicates $\gamma_b = 0.178$ and $K_1(\gamma_b)/K_0(\gamma_b) = 2.898$. These values along with $\bar{G}_s = 0.1549 \text{ MPa}$, and $u = 21.3 \text{ mm}$ allow the maximum σ_r (with $r = r_o = 0.051 \text{ m}$ and $\theta_p = 0$) to be obtained using eq. [1], with $\sigma_r = 2 \times 154.9 \times 0.0213 \times 0.178 / 0.051 \times 2.898$. The rationale of using the elastic eq. [1] is explained later on. The stress σ_r across the diameter is predicted as $\sigma_r = 66.85 \cos \theta_p$ (compared to $\tau_{r\theta} = -33.425 \sin \theta_p$) in light of eq. [1]. It compares well with the measured data (Prasad and Chari 1999), as shown previously in Fig. 3.

The maximum σ_r may be cross examined using eq. [6]. In the loading direction, the net force on the pile surface per unit projected area (net pressure) due to the components σ_r and $\tau_{r\theta}$ are determined, respectively, as:

$$\int_0^\pi \sigma_r \cos \theta_p d\theta_p = 105.0 \text{ (kPa)} \quad \int_0^\pi \tau_{r\theta} \sin \theta_p d\theta_p = -52.5 \text{ (kPa)}$$

Thereby, the total net pressure is 52.5 kPa ($= 105.0 - 52.5$). On the other hand, the 'net pressure' at the depth of 0.276 m is estimated as 67.6 kPa ($= 244.9 \times 0.276$) using eq. [6]. The former is less than the latter, as is the measured force compared to the predicted one (see Fig. 10a), showing the effect of k profile (discussed later) and neglecting the τ_{zr} and σ_θ .

Other features noted herein are: (i) The moment M_m occurs below the slip depth ($z_m > z_0 = 0.3l = 0.1836 \text{ m}$) under $T_t = 0.605 \text{ kN}$; or within the depth ($z_m < z_0 = 0.5l = 0.306 \text{ m}$) at $T_t = 0.783 \text{ kN}$ (see Fig. 11). (ii) The rotation depth of z_r is largely around $0.62l$, indicating by a negligible displacement of $u(z_r = 0.62l) \approx 0$ (see Fig. 12). The force below the depth has an opposite direction, as is observed in some field tests. Finally (iii) The non-dimensional responses, e.g. $T_t/(A_r dl^2)$, are independent of the parameters A_r and k_o , and are thus identical concerning the three piles tested in different D_r .

4.1.2 Analysis using Constant k

The solutions for a *Constant k* (see Tables 3 and 7) were utilised to match each measured pile-head and mudline displacement $T_t \sim u_0$ curve, the k was thus deduced (using the same A_r as that for Gibson k). This resulted in the *dashed lines* in Fig. 10, the shear modulus $G_s (= G_L)$ and the angle at *tip-yield* furnished in Table 9. The predicted curves of $M_{\max} \sim T_t$, and $M_{\max} \sim z_m$ are also plotted in Figs. 10b and 10c, respectively. This analysis indicates:

- The shear moduli deduced for $D_r = 0.25, 0.5$ and 0.75 are 0.105 MPa, 0.327 MPa, and 0.461 MPa, respectively, showing -45.6%, 3.5% and -1.9% difference from the revised measured values of 0.193 MPa, 0.316 MPa, and 0.470 MPa, respectively.
- The *tip-yield* (thus pile-head force T_0), as illustrated in Fig. 10a, occurs at a displacement far greater than $0.2d (= 20.4 \text{ mm})$, and at a rotation angle of 10~15 degrees (see Table 9) that are ~5 times those inferred using a Gibson k .

In parallel to the *Gibson k* , the calculation for the pile in $D_r = 0.25$ is again presented herein (with $A_r = 244.9 \text{ kN/m}^3$, and $k = 3.88 \text{ MN/m}^3$), and is focused on the difference from the Gibson k analysis.

- (1) The ratio \bar{z}_o/l at the *tip-yield* state was obtained as 0.5885 using eq. [22g]. It allows the following to be gained: $T_t/(A_r d^2) = 0.0885$, and $u_0 k/(A_r l) = 2.86$ in terms of eqs. [11g] and [12g]; $z_m/l = 0.4208$ via eq. [23g] ($z_o/l > z_m/l$); and $M_m/(A_r d^3) = 0.0465$ using eq. [25g]. Accordingly, it follows $T_t = 0.8285 \text{ kN}$, $u_0 = 110.4 \text{ mm}$, $z_m = 0.257 \text{ m}$, and $M_m = 0.266 \text{ kNm}$.
- (2) Responses for the *pre-tip yield* state are tabulated in Table 10 for $z_o/l = 0.3$ and 0.5 . Given $z_o/l = 0.3$, T_t , u_0 , and z_m were estimated as 0.462 kN, 21.3 mm, and 0.193 m, based on eqs. [11g], [12g], and [24g], respectively. M_m was calculated as 0.129 kNm via eq. [26g] (with $z_m > z_o$). As the z_o/l increases to 0.5, T_t increases to 0.723 kN; u_0 to 65.3 mm; z_m raises to 0.241 m (as per eq. [23g] with $z_m < z_o$); and M_m to 0.224 kNm (according to eq. [25g]). The two points given by the pairs of M_m and z_m agree well with the respective $M(z)$ profiles.
- (3)-(4) Calculation for the *post-tip yield* state is not presented here. Upon the rotation point yield, an identical response to that for a Gibson k (see Table 13) is obtained.

- (5) Profiles of bending moment, $M(z)$ and shear force, $T(z)$ at the slip depths of $z_0 = 0.3l$ and $0.5l$ were determined using expressions given previously (Guo 2003). They are plotted in Figs. 11a and 11b. Local shear force-displacement relationships at five different depths were evaluated and are plotted in Fig. 12.
- (6) The G_s was estimated as 0.105 MPa ($= 3.88 \times 0.102 / 3.757$ MPa) using eq. [4].
- (7) A head-displacement u_0 of 92 mm (prior to tip yield) was needed to mobilise the radial pressure σ_r of 66.85 MPa at the depth, associating with a local displacement u of 31.3 mm. The displacement occurs at $z_0 = 0.342$ m, and $z_r = 0.447$ m (note $G_s = 0.105$ MPa). Distribution of the σ_r is predicted identical to that for Gibson k .

The results shown in Figs. 11 and 12 for *Constant k* largely support the comments on Gibson k about the M_m , z_m , z_r and the non-dimensional responses. The measured force (thus σ_r) at a u_0 of 92 mm far exceeds the predicted one (see Fig. 10a), which is opposite to that from Gibson k . The actual k should be bracketed by the uniform k and Gibson k .

4.1.3 Effect of k profiles

The impact of the k profiles is evident on the predicted $T_t \sim u_0$ curve; whereas it is noticeable on the predicted M_m only at initial stage (see Fig. 10). The latter is owing to the fact that beyond the initial low load levels, the M_m is given by the same value of A_r and the same eqs. [23] and [25]. The deduced A_r is $\pm \sim 15\%$ different from eq. [7]. The deduced (constant) k is $\pm \sim 3.5\%$ different from the revised measured k , except for the pile in $D_r = 0.25$, and those deduced from Gibson k , which are explained herein.

Given Gibson k , the local displacement u of 21.3 mm ($> u^* = 13$ mm = $244.9/18640$ m) for inducing the measured σ_r was associated with plastic response, as the $u = 31.3$ mm for a Constant k was ($u > u^* = 18.6$ mm = $244.9 \times 0.276 / 3880$ m). In contrast, the stress hardening exhibited (see Fig. 10) beyond a mudline displacement u_0 of 57 mm implies a higher value of A_r (thus p_u) than 244.9 kPa/m adopted herein, and a higher local limit u^* than the currently adopted 13.0~18.6 mm. The derived k from eq. [1] needs modifications in view of the following:

- Any plastic component of displacement, $u - u^*$, may render the modulus k to be underestimated. The displacement u exceed the elastic limit u^* by 64 % [= $(21.3 - 13) / 13$] using Gibson k ; or by 68% [= $(31.3 - 18.6) / 18.6$] using Constant k . With stress $\sigma_r \propto ku$ (i.e., eq. [1]), the k may be supposedly underestimated by $\sim 68\%$, even if the σ_r has been well mimicked using the k profiles. The hardening effect may render less

degree of underestimation, which then seems to be consistent with the 19.5~45.6% underestimation of the measured modulus (Table 9, $D_r = 0.25$), with a predicted $G_L = 0.105\sim 0.31$ MPa.

In contrast, overestimation of the (Gibson) k is noted for $D_r = 0.5\sim 0.75$, although the displacement of $0.2d$ and the angle (slope) for the capacity T_o are close to those used in practice. Real k profile should be bracketed by the constant and Gibson profiles.

4.2 Comments on Current Predictions

- (1) The current solutions were developed to cater for net lateral resistance along the shaft only. Longitudinal resistance along the shaft, and transverse shear resistance on the tip are neglected. The shear resistance may become apparent regarding very short, stub piers such as pole foundations (Vallabhan and Alikhanlou 1982). In these circumstances, use of the current solutions will be conservative.
- (2) The modulus k was stipulated as a constant or linearly increase with depth (Gibson type), along with a linear p_u profile. The two k profiles should bracket well possible k profiles encountered in practice. Along rigid piles, the linear p_u profile is normally expected, whereas along flexible piles, the p_u may be proportional to $z^{1.7}$ (Guo 2006). Given a pile in a multi-layered sand, the p_u may even become uniform, for which pertinent solutions published previously (Scott 1981) may be utilised.
- (3) Equations [1] and [5] are rigorous for the constant k , but approximate for the Gibson k . The modulus deduced incorporates the effect of pile diameter.
- (4) The elastic-plastic load-displacement curve cannot capture the impact of stress hardening as demonstrated in the pile in $D_r = 0.25$.

5. CONCLUSIONS

Elastic-plastic solutions were developed for laterally loaded rigid piles using the load transfer approach. They are presented in explicit form regarding *pre-tip* and *post-tip yield* states respectively. Simple expressions are developed for determining the depths z_o , z_1 , and z_r used for constructing *on-pile force profiles*, and for calculating moment M_m and its depth z_m . They are generally plotted against e/l , and are elaborated for $e = 0$ and ∞ . The solutions and expressions are shown to be consistent with available FE analysis and

relevant measured data. They are implemented into a spreadsheet program called GASLSPICS, which was used to conduct a detailed investigation into a well documented case. Comments are made regarding estimation of capacity. In particular, the following features about the current solutions are noted:

- (1) *The p_u profile is differentiated from on-pile force profile. The former is unique, whereas the latter is mobilised along a specified LFP and may be constructed for any states (e.g. pre-tip yield, tip-yield, post-tip yield, and rotation point yield states).*
- (2) *Characterized by the slip depths z_0 and z_1 , the solutions allow nonlinear response (e.g. load, displacement, rotation and maximum bending moment) to be readily estimated, using the parameters k and A_r . Conversely, the two parameters can be deduced using two measured nonlinear responses. The back-estimation is legitimate, as stress distributions along depth and around pile diameter are integrated into the solutions.*
- (3) *For the investigated piles, the deduced A_r is with $\pm\sim 15\%$ error from eq. [7]; whereas shear moduli have $\sim\pm 3.5\%$ discrepancy from the measured data (except for the $\sim 46\%$ underestimation noted for stress hardening case).*
- (4) *Maximum bending moment raises $\sim 30\%$ as the tip-yield state moves to the YRP state. It increases 2.1~2.2 times as the e increases from 0 to $3l$ at either state (N. B. $M_{\max} \approx M_0$ given $e/l > 3$).*
- (5) *The impact of k profile is bracketed by the solutions concerning a uniform k and a Gibson k . Without catering for the influence of the eccentricity, the k is underestimated by $\sim 40\%$ for a rigid pile ($l/r_0 = 3\sim 8$).*

The current solutions can accommodate the increase in resistance owing to dilation by modifying A_r , while not able to capture the effect of stress-hardening.

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List of Symbols

The following symbols are used in the paper:

- A_r = coefficient for the LFP [FL⁻³];
 C = $A_r/(u_0k_o)$ (Gibson k), or $A_rz_o/(u_0k_o)$ (Constant k), used for *post-tip yield state*;
 C_y = value of the C at the tip yield state;
 $d(r_o)$ = outside diameter (radius) of a cylindrical pile [L];
 D_r = sand relative density;
 E_p = Young's modulus of an equivalent solid cylinder pile [FL⁻²];
 E_s = Young's modulus of soil [FL⁻²];
 e = eccentricity (free- length) [L] i.e. the height from the loading location to the mudline; or $e = M_o/T_i$;
 G_s, \bar{G}_s = shear modulus of the soil, and average of the G_s [FL⁻²];
 G_L = shear modulus of the soil at pile tip level [FL⁻²];
 k, k_o = modulus of subgrade reaction [FL⁻³], $k = k_o z^m$, $m = 0$ and 1 for Constant and Gibson k , respectively; and k_o , a parameter [FL^{-3-m}];
 K_a = $\tan^2(45^\circ - \phi_s'/2)$, the coefficient of active earth pressure;
 k_m, k_T = modulus k due to pure bending moment, and pure lateral loading [FL⁻³];
 K_p = $\tan^2(45^\circ + \phi_s'/2)$, the coefficient of passive earth pressure;
 k_1 = parameter for estimating the load transfer factor, γ_b ;
 $K_i(\gamma_b)$ = modified Bessel function of second kind of i^{th} order;
 l = embedded pile length [L];
LFP = net limiting force profile per unit length [FL⁻¹];
 M_m = maximum bending moment within a pile [FL];
 M_o = bending moment at the mudline level [FL];
 p, p_u = force per unit length, and limiting value of the p [FL⁻¹];
 s = integration variable;
 $T(z)$ = lateral force induced in a pile at a depth of z [F];
 T_t, T_o = lateral load applied at an eccentricity of 'e' above mudline, and T_t at a defined (tip yield or YRP) state [F];
 u, u_o = lateral displacement, and u at mudline level [L];
 u^* = local threshold u^* above which pile-soil relative slip is initiated [L];
YRP = yield at rotation point;
 z = depth measured from the mudline [L];
 z_m = depth of maximum bending moment [L];
 $z_o(z_1)$ = slip depth initiated from mudline (pile-base)[L];
 z_r = depth of rotation point [L];
 \bar{z}_o = slip depth z_o at the moment of the tip yield [L];
 γ_b = load transfer factor;
 γ_s' = effective density of the overburden soil [FL⁻³];
 θ_p = angle between the interesting point and the loading direction;
 ν_s = Poisson's ratio of soil;
 σ_r = radial stress in the soil surrounding a lateral pile [FL⁻²];
 σ_θ = circumferential stress in the soil surrounding a lateral pile [FL⁻²];

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- σ_z = vertical stress in the soil surrounding a lateral pile [FL^{-2}];
 $\tau_{r\theta}$ = shear stress in the r - θ plane [FL^{-2}];
 $\tau_{\theta z}$ = shear stress in the θ - z plane [FL^{-2}];
 τ_{rz} = shear stress in the r - z plane [FL^{-2}];
 ϕ_s' = effective frictional angle of soil;
 ω = rotation angle (in radian) of the pile

Appendix A: Solutions for Gibson k profile

A.1 Pile Response at Pre-tip Yield State

The force per unit length p gained from eqs. [3] and [6] allows the horizontal force equilibrium of the rigid pile subjected to a lateral load, T_t at the pile-head (see Fig. 1) to be written as

$$[A1] \quad T_t - \int_0^{z_0} A_r s dds - \int_{z_0}^l k_o s d(\omega s + u_0) ds = 0$$

The integration is made with respect to 's'. The moment equilibrium about the pile-tip offers

$$[A2] \quad T_t(e+l) - \int_0^{z_0} A_r s(l-s) dds - \int_{z_0}^l k_o s d(\omega s + u_0)(l-s) ds = 0$$

Equations [A1] and [A2] along with eq. [8] allow the ω and u_0 to be determined as:

$$[A3a] \quad \omega = \frac{12T_t(l+e)}{k_o(l+z_0)(l-z_0)^3} - \frac{2l^3 u^*}{(l+z_0)(l-z_0)^3}$$

$$[A3b] \quad u_0 = \frac{-12z_0 T_t(l+e)}{k_o(l+z_0)(l-z_0)^3} + \frac{-z_0^4 + 2z_0^3 l + l^4}{(l+z_0)(l-z_0)^3} u^*$$

The solutions prior to tip yield are obtained from eq. [A1] to eqs. [A3a] and [A3b]. They are recast in the normalised form of eq. [11] for the T_t , eq. [12] for the ω , and eq. [13] for the u_0 .

The shear force at depth z , $T(z)$ is given by:

$$[A5a] \quad T_t - \int_0^z A_r s dds = T(z) \quad (z \leq z_0)$$

$$[A5b] \quad T_t - \int_0^{z_0} A_r s dds - \int_{z_0}^z k_o s d(\omega s + u_0) ds = T(z) \quad (z > z_0)$$

At $z = l$, $T(z) = 0$, eq. [A5b] is identical to eq. [A1]. Furthermore, $T(z)$ is rewritten as:

$$[A6a] \quad \frac{T(z)}{A_r d l^2} = 0.5 \left(\left(\frac{z_o}{l} \right)^2 - \left(\frac{z}{l} \right)^2 \right) + \frac{\omega l}{3u^*} \left(1 - \left(\frac{z_o}{l} \right)^3 \right) + \frac{u_0}{2u^*} \left(1 - \left(\frac{z_o}{l} \right)^2 \right) \quad (z \leq z_o)$$

$$[A6b] \quad \frac{T(z)}{A_r d l^2} = \frac{\omega l}{3u^*} \left(1 - \left(\frac{z}{l} \right)^3 \right) + \frac{u_0}{2u^*} \left(1 - \left(\frac{z}{l} \right)^2 \right) \quad (z > z_o)$$

Maximum bending moment, M_m occurs at a depth of z_m at which the shear force, $T(z_m)$ is zero. The moment, M_m is determined from the following expressions:

$$[A7a] \quad M_m = T_t(e + z_m) - \int_0^{z_m} dA_r s(z_m - s) ds \quad (z_m \leq z_o)$$

$$[A7b] \quad M_m = T_t(e + z_m) - \int_0^{z_o} dA_r s(z_m - s) ds - \int_{z_o}^{z_m} k_o s d(\omega s + u_0)(z_m - s) ds \quad (z_m > z_o)$$

Equations [A7a] and [A7b] are similar to those given previously (e.g. Scott 1981). They were used to derive the depth z_m , and the moment M_m .

A.2 Pile Response Posterior to Tip Yield

As yield expands from the pile-tip, the horizontal force equilibrium of the pile, and the moment equilibrium about the pile-head require:

$$[A8] \quad T_t - \int_0^{z_o} A_r s dds - \int_{z_o}^{z_1} k_o s d(\omega s + u_0) ds + \int_{z_1}^l A_r s dds = 0$$

$$[A9] \quad \int_0^{z_o} A_r s^2 dds + \int_{z_o}^{z_1} k_o s d(\omega s + u_0) s ds - \int_{z_1}^l A_r s^2 dds + T_t e = 0$$

These expressions may be integrated to give:

$$[A10] \quad T_t/d = 0.5 A_r z_o^2 + 0.5 k_o u_0 (z_1^2 - z_o^2) + \omega k_o (z_1^3 - z_o^3)/3 - 0.5 A_r (l^2 - z_1^2)$$

$$[A11] \quad \frac{1}{3} A_r z_o^3 + \frac{1}{4} k_o \omega (z_1^4 - z_o^4) + \frac{1}{3} k_o u_0 (z_1^3 - z_o^3) - \frac{1}{3} A_r (l^3 - z_1^3) + T_t e/d = 0$$

Equation [A10], together with eqs. [8] and [10], gives:

$$[A12] \quad \omega = -2u^*/(z_1 - z_o)$$

$$[A13] \quad u_0 = u^* (z_1 + z_o)/(z_1 - z_o)$$

$$[A14] \quad z_r = -u_0/\omega$$

$$[A15] \quad T_i = \frac{1}{6} k_o du^* (2z_1^2 + 2z_1 z_o - 3l^2 + 2z_o^2)$$

Given $C = A_r/(k_o u_0)$, it follows that:

$$[A16] \quad z_o = z_r(1 - C)$$

$$[A17] \quad z_1 = z_r(1 + C)$$

$$[A18] \quad u_0 = A_r/Ck_o$$

Equation [A15] can be simplified to the form of eq. [20], in terms of eqs. [A16]~[A18].

Table 1. k_m/k_T at various slenderness ratios of l/r_o

| l/r_o | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|-------------|------|------|------|------|------|------|------|------|------|------|
| k_m/k_T^a | 1.56 | 1.47 | 1.41 | 1.37 | 1.35 | 1.32 | 1.31 | 1.29 | 1.28 | 1.27 |

Note ^a $k_m/k_T = 3.153$ ($l/r_o = 0$); 1.27~1.22 (10~20); 1.22~1.19(20~30); 1.14~1.12 (100 ~ 200).

Table 2. Capacity of lateral piles based on limit states

| Mtds | $T_o/(A_r d l^2)$ | A_r (kN/m ³) | References |
|------|--|---|--|
| 1 | Using equilibrium against rotating point | $K_{qz} \gamma'_s$ ^a | Brinch Hansen (1961) |
| 2 | $[6(1 + e/l)]^{-1}$ | $3 K_p \gamma'_s$ | Broms (1964) |
| 3 | $[2.13(1 + 1.5e/l)l]^{-1}$ ^b | (28 ~ 228) kPa ^b | McCorkle (1969) |
| 4 | $0.5[2(z_r/l)^2 - 1]$ | $(3.7K_p - K_a) \gamma'_s$ | Petrasovits and Award (1972) |
| 5 | $(1 + 1.4e/l)^{-1}$ | $F_b S_{bu} (K_p - K_a) \gamma'_s$ ^c | Meyerhof, et al (1981) |
| 6 | $[8(1 + 1.5e/l)l]^{-1}$ ^b | (80 ~ 160) kPa ^{b, d} | Balfour Beatty Construction (1986) |
| 7 | $\frac{1 - d/l}{[1 - 0.333 \ln(l/d)]e/d}$ | $4.167 \gamma'_s$ | Dickin and Wei (1991) |
| 8 | $0.51 \frac{z_r}{l} \left(1.59 \frac{z_r}{l} - 1 \right)$ | $0.8 \gamma'_s 10^{1.3 \tan \phi'_s + 0.3}$ | Prasad & Chari (1999) |
| 9 | $\frac{0.1181}{1 + 1.146e/l}$ | $K_p^2 \gamma'_s$ | Derived using $z_o/l = 0.618$ in eq. [11g] |

Note ^a K_{qa} = also passive pressure coefficient, but it depends on l/d ratio, etc.
^b Dimensional expressions, as a uniform p_u is adopted, and A_r unit is kPa.
^c F_b = lateral resistance factor, 0.12 for uniform soil; S_{bu} = a shape factor which depends on the depth l and the angle of ϕ'_s . ^d Smaller values in presence of water.

Table 3. Responses of piles in 'k = constant' soil (*Pre-tip yield state*)

| Expressions | References | $e/l = \infty$ |
|---|------------|---|
| $u^* = A_r z_o / k$ | eq. [9g] | |
| $-u^* z_1 / z_o = \omega z_1 + u_0$ | eq. [10g] | |
| $\frac{T_t}{A_r d l^2} = \frac{z_o / l}{2[2 + z_o / l + 3e / l]}$ | eq. [11g] | 0 |
| $\frac{u_0 k}{A_r l} = \frac{(2 + 3e / l) z_o / l}{[2 + z_o / l + 3e / l](1 - z_o / l)^2}$ | eq. [12g] | $\frac{z_o / l}{(1 - z_o / l)^2}$ |
| $\omega = \frac{A_r z_o}{k l} \frac{(z_o / l)^2 + 3(z_o / l - 2)e / l - 3}{[2 + z_o / l + 3e / l](1 - z_o / l)^2}$ | eq. [13g] | $\frac{A_r z_o}{k l} \frac{z_o / l - 2}{(1 - z_o / l)^2}$ |
| $z_r / l = -u_t / \omega l$ | eq. [14g] | |
| $\frac{\bar{z}_o}{l} = -\left(1.5 \frac{e}{l} + 0.5\right) + 0.5 \sqrt{5 + 12 \frac{e}{l} + 9 \left(\frac{e}{l}\right)^2}$ | eq. [22g] | 0.5 |
| $\frac{z_m}{l} = \sqrt{\frac{2T_t}{A_r d l^2}}$ | eq. [23g] | 0 |
| $\frac{z_m}{l} = \frac{1 + (z_o / l + 3e / l) z_o / l}{3(1 + 2e / l) - (z_o / l + 3e / l) z_o / l}$ | eq. [24g] | $\frac{z_m}{l} = \frac{z_o / l}{2 - z_o / l}$ |
| $M_m = (2z_m / 3 + e) T_t$ | eq. [25g] | $T_t e$ |
| $M_m = \frac{(z_m - z_o)^3}{(l - z_o)^2 (2l + z_o)} \left(T_t e + \frac{dA_r}{3} z_o^3 \right) + T_t (e + z_m) + \frac{dA_r}{6} z_o^2 (2z_o - 3z_m)$ | eq. [26g] | N/A |
| $+ \frac{dA_r z_o}{2} \left[\frac{(z_m z_o - 2l^2 + z_m l)(z_o - z_m)^2}{(l - z_o)(2l + z_o)} \right]$ | | |

Note: Equations [3] and [6] are adopted for the derivation.

Table 4. Response at various states ($e = 0$, Gibson k / [Constant k])

| Items | $T_t/(A_r d l^2)$ | $\frac{u_0 k_o / A_r}{[u_0 k / A_r l]}$ | $\frac{\omega k_o l / A_r}{[\omega k / A_r]}$ | $M_m/(A_r d l^3)$ |
|--|---|---|---|---|
| $\frac{z_o/l \leq \bar{z}_o/l}{\bar{z}_o/l}$ | $\frac{Eq.(11)}{[Eq.(11g)]}$ | $\frac{Eq.(12)}{[Eq.(12g)]}$ | $\frac{Eq.(13)}{[Eq.(13g)]}$ | $\frac{Eqs.(25) \text{ or } (26)}{[Eqs.(25g) \text{ or } (26g)]}$ |
| Tip yield ^a | $\frac{0.113}{[0.118]}$ | $\frac{3.383}{[3.236]}$ | $\frac{-4.3831}{[-4.2352]}$ | $\frac{0.036}{[0.038]}$ |
| YRP ^b | $\frac{0.130}{[0.130]}$ | ∞ | $\frac{0.5\pi k_o l / A_r}{[0.5\pi k / A_r]}$ | $\frac{0.0442}{[0.0442]}$ |
| Note | ^a $\bar{z}_o/l = 0.5437 / [0.618]$, $z_r/l = 0.772 / [0.764]$, $z_m/l = 0.4756 / [0.4859]$, and $C_y = 0.296 / [0.236]$. ^b At the YRP state, all critical values are independent of k distribution. Thus, $z_o/l = z_r/l = 0.7937 / [0.7937]$, and $z_m/l = 0.5098 / [0.5098]$. Also $M_o = 0$. | | | |

Table 5. Response at various yield states ($e = \infty$, Gibson k [Constant k])

| Items | $u_0 k_o / A_r / [u_0 k / A_r l]$ | $\omega k_o l / A_r / [\omega k / A_r]$ | $M_o / (A_r d l^3)$ |
|--------------------------|---|---|---|
| $z_o/l \leq \bar{z}_o/l$ | $\frac{2 + (z_o/l)^3}{(1 - z_o/l)^2 (2 + z_o/l)}$ [Table 2] | $\frac{-3}{(1 - z_o/l)^2 (2 + z_o/l)}$ [Table 2] | $\frac{1 + 2 z_o/l + 3(z_o/l)^2}{12(2 + z_o/l)}$ $[\frac{1}{6} z_o/l]$ |
| Tip yield ^a | $2.155 / [2.0]$ | $-3.1545 / [-3.0]$ | $0.0752 / [0.0833]$ |
| YRP ^b | ∞ | $0.5\pi k_o l / A_r / [0.5\pi k / A_r]$ | $0.0976 / [0.0976]$ |
| Note | ^a $\bar{z}_o/l = 0.366 / [0.50]$, $z_r/l = 0.683 / [0.667]$, $z_m/l = 0 / [0]$, and $C_y = 0.464 / [0.333]$. ^b $z_o/l = z_r/l = 0.7071 / [0.7071]$, and $z_m/l = 0 / [0]$. Also $T_t = 0$, and $M_o = M_m$. | | |

Table 6. Responses of piles in ‘ $k = \text{constant}$ ’ soil (*Post-tip yield state*)

| Expressions | References |
|---|------------|
| $\frac{T_t}{A_r dl^2} = 0.5 \left[\frac{2}{1-C^2} \left(\frac{z_r}{l} \right)^2 - 1 \right]$ where $C = A_r z_r / (ku_0)$ | eq. [20g] |
| $u_0 = A_r z_r / (kC)$ | eq. [21g] |

The ratio z_r/l is governed by the following expression:

$$\left(z_r/l \right)^3 + 1.5(1-C^2) \frac{e}{l} \left(z_r/l \right)^2 - \left(0.5 + 0.75 \frac{e}{l} \right) (1-C^2)^2 = 0$$

Thus, the z_r/l should be obtained using

$$\begin{aligned} & \left[-1.5 \frac{e}{l} C_g^2 - \left(0.5 + 0.75 \frac{e}{l} \right) C_g^4 \right] \left(z_r/l \right)^4 + \left(z_r/l \right)^3 \\ & + \left[1.5 \frac{e}{l} + \left(1 + 1.5 \frac{e}{l} \right) C_g^2 \right] \left(z_r/l \right)^2 - \left(0.5 + 0.75 \frac{e}{l} \right) = 0 \end{aligned} \quad \text{where } C_g = Cl/(kz_r).$$

z_r/l may be approximated by the following solution (Guo 2003)

$$z_r/l = 0.5(1-C^2) \left(\sqrt[3]{A_0} + \sqrt[3]{A_1} + D_o \right) \quad (\text{Iteration required}) \quad \text{eq. [17g]}$$

$$A_j = (D_o^3 + D_1) + (-1)^j [D_1(2D_o^3 + D_1)]^{1/2} \quad (j = 0, 1) \quad \text{eq. [18g]}$$

$$D_1 = \frac{2+3e/l}{1-C^2} \quad D_o = -\frac{e}{l} \quad \text{eq. [19g]}$$

It is generally ~5% less than the exact value of z_r/l . Note that eqs. [27g], [28g] and [29g] are identical to eqs. [27], [28] and [29], respectively.

Table 7. Expressions for depth of rotation

| u | Depth of rotation | Slip depths | Figure |
|----------------------|---|---|--------|
| $u = \omega z + u_0$ | $z_r = -\frac{u_0}{\omega}$ | z_o/l deduced using eqs. [11]~[14] | 2a |
| | | $\frac{z_1/l = (1+C)z_r/l}{[z_1/l = (z_r/l)/(1-C)]}$ $\frac{z_o/l = (1-C)z_r/l}{[z_o/l = (z_r/l)/(1+C)]}$ | 2b |
| | | $z_1/l = z_o/l = z_r/l$ | 2c |
| Note | $u_0, \omega, z_r, z_o, z_1$ refer to list of symbols. $C = A_r/(u_0 k_0)$ (Gibson k), $C = A_r z_r/(u_0 k)$ (Constant k) | | |

Table 8. Pile in dense sand reported by Laman, et al. (1999)

| Input parameters ($l = 2$ m, $d = 1$ m) | | | Output for tip yield state (Test 3) | | |
|--|--|----------------------------------|-------------------------------------|---------|-----------|
| A_r (kN/m ³) | k (MN/m ³) | γ_s' (kN/m ³) | Angle (deg.) | z_r/l | M (kNm) |
| 621.7 | 34.42 ^a /51.63 ^b | 16.4 | 3.83 | 0.523 | 338.4 |

^a Test 3; ^b Test 2.

Table 9. Parameters for the model piles (Gibson k / [Constant k])

| D_r | 0.25 | 0.50 | 0.75 | Notes |
|---|-----------------------------|-------------------------|-------------------------|---|
| A_r (kN/m ³) | 244.9 | 340. | 739. | Numerator: <i>Gibson k</i> Denominator: <i>Constant k</i> |
| $\frac{k_o}{[k]}$ (MPa/m ²) | $\frac{18.64}{[3.88]}$ | $\frac{48.2}{[12.05]}$ | $\frac{81.43}{[16.96]}$ | ^a Via multiplying the values of 3.78, 6.19 and 9.22 with the diameter d (0.102m), or |
| Predicted G_L (MPa) | $\frac{0.31}{[0.105]}$ | $\frac{0.801}{[0.327]}$ | $\frac{1.353}{[0.461]}$ | |
| Measured G_L (MPa) ^{a,b} | $\frac{0.385^a}{[0.193]^b}$ | $\frac{0.631}{[0.316]}$ | $\frac{0.94}{[0.47]}$ | ^b Via multiplying by $0.5d$. |
| Angle at \bar{z}_o/l (deg.) | $\frac{3.94}{[14.0]}$ | $\frac{2.11}{[10.6]}$ | $\frac{2.72}{[14.4]}$ | |

Table 10. States of yield for model piles (valid for any D_r , Gibson k / [Constant k])

| Items | z_o/l | $\frac{u_o k_o / A_r}{[u_o k / A_r l]}$ | $T_t / (A_r d l^2)$ | $\frac{\omega k_o l / A_r}{[\omega k / A_r]}$ | z_m/l | $M_m / (A_r d l^3)$ |
|----------------------|------------------------------|---|---------------------------|---|---------------------------|---------------------------|
| <i>Pre-tip yield</i> | 0.30 | $\frac{1.695^a}{[0.5517]^b}$ | $\frac{0.0647}{[0.0494]}$ | $\frac{-2.318}{[-0.8391]}$ | $\frac{0.364}{[0.3151]}$ | $\frac{0.031}{[0.0225]}$ |
| | 0.50 | $\frac{3.005}{[1.691]}$ | $\frac{0.0838}{[0.0773]}$ | $\frac{-4.005}{[-2.382]}$ | $\frac{0.409}{[0.3931]}$ | $\frac{0.0434}{[0.0392]}$ |
| | $\frac{0.535^c}{[0.5885]^d}$ | $\frac{3.426}{[2.86]}$ | $\frac{0.087}{[0.0885]}$ | $\frac{-4.530}{[-3.86]}$ | $\frac{0.4171}{[0.4208]}$ | $\frac{0.0453}{[0.0465]}$ |

YRP $z_r/l = 0.774$, $z_m/l = 0.445$, $T_t / (A_r d l^2) = 0.099$, and $M_m / (A_r d l^3) = 0.0536$.

Note: ^aNumerator: *Gibson k*; ^bDenominator: *Constant k*; ^c Post-tip yield; ^d Tip yield state.

Table 11. Effect of k profile on predicted responses (Gibson k / [Constant k])

| z_0/l | u_0 (mm) | T_t (kN) | ω (degree) | z_m (m) | M_m (kNm) |
|------------------------------|---------------------------|--------------------------|--|-------------------------|-------------------------|
| 0.30 | $\frac{22.3^a}{[21.3]^b}$ | $\frac{0.605}{[0.462]}$ | $\frac{-0.050(2.85^\circ)}{[-0.053(3.03^\circ)]}$ | $\frac{0.223}{[0.193]}$ | $\frac{0.181}{[0.129]}$ |
| 0.50 | $\frac{39.5}{[65.3]}$ | $\frac{0.783}{[0.723]}$ | $\frac{-0.087(4.93^\circ)}{[-0.15(8.61^\circ)]}$ | $\frac{0.251}{[0.241]}$ | $\frac{0.248}{[0.224]}$ |
| $\frac{0.535^c}{[0.5885^d]}$ | $\frac{45.0}{[110.4]}$ | $\frac{0.814}{[0.8285]}$ | $\frac{-0.097(5.57^\circ)}{[-0.244(13.95^\circ)]}$ | $\frac{0.255}{[0.257]}$ | $\frac{0.261}{[0.266]}$ |
| YRP | ∞ | 0.926 | $\pi/2$ (90°) | 0.272 | 0.307 |

Note ^aNumerator: *Gibson k*; ^bDenominator: *Constant k*; ^cPost-tip yield; ^dTip yield state.

Table 12. Calculation of z_r/l for post- tip and YRP states (Gibson k)

| Eqs. for Gibson k (valid for any D_r) | Eq. [19] | | | Eq. [18] | | Eq. [17] |
|---|----------|--------|--------|----------|------------------------|----------|
| | C | D_0 | D_1 | A_0 | A_1 | z_r/l |
| <i>Post-tip yield</i> | 0.2919 | 2.5205 | 0.6968 | 0.627 | 3.912×10^{-6} | 0.756 |
| YRP | 0 | 2.735 | 0.735 | 0.680 | 4.969×10^{-6} | 0.774 |

Table 13. Calculation of z_r/l for YRP state (Constant k)

| Eqs. for Constant k (valid for any D_r) | Eq. [19g] | | | Eq. [18g] | | Eq. [17g] |
|---|-----------|--------|-------|-----------|------------------------|-----------|
| | C | D_0 | D_1 | A_0 | A_1 | z_r/l |
| YRP | 0 | -0.245 | 2.735 | 5.4405 | 3.975×10^{-5} | 0.774 |

Figure Captions

Fig. 1 Schematic analysis for a rigid pile. (a) Pile - soil system. (b) Load transfer model

Fig. 2 Schematic limiting force profile, on-pile force profile, and pile deformation. (a) Tip yield state. (b) Post-tip yield state. (c) Impossible yield at rotation point (YRP)

Fig. 3 Comparison between the predicted and the measured (Prasad and Chari 1999) radial pressure, σ_r on a rigid pile surface

Fig. 4 Effect of free-length on responses of piles at tip-yield state. (a) Normalised slip depth, z_o/l . (b) Normalised $u_0 k_o l^m / (A_r l)$. (c) Normalised $\omega k_o l^m / A_r$. (d) Normalised depth z_m/l

Fig. 5 Predicted vs measured (Prasad and Chari 1999) normalised on-pile force profiles upon the tip yield. (a) Constant k . (b) Gibson k .

Fig. 6 Normalised moment $M_o (= T_o e)$ and M_m

Fig. 7 Normalised responses under various ratios of e/l . (a) Pile-head load T_t and mudline displacement u_0 . (b) T_t and rotation ω . (c) T_t and maximum bending moment M_{max}

Fig. 8 Comparison among the current predictions, the measured data, and FEA results (Laman, et al. 1999). (a) M_o versus rotation angle ω (Test 3). (b) T_t versus mudline displacement u_0 . (c) M_o versus rotation angle ω (Effect of k profiles, Tests 2 and 1).

Fig. 9 Comparison of the normalised pile capacity at various critical states

Fig. 10 Comparison between the current predictions and the measured (Prasad and Chari 1999) data. (a) Pile-head load T_t and mudline displacement u_0 . (b) T_t and maximum bending moment M_{max} . (c) Maximum bending moment M_{max} and its depth z_m

Fig. 11 Effect of k distributions on the profiles of (a) bending moment, and (b) shear force

Fig. 12 Local shear force ~ displacement relationships at five typical depths