

# Demonstrations of Adaptive Quantum Measurements in Optics

Brendon L. Higgins<sup>1</sup>, Dominic W. Berry<sup>2,3</sup>, Byron M. Booth<sup>1</sup>, Morgan W. Mitchell<sup>4</sup>, Andrew C. Doherty<sup>5</sup>, Stephen D. Bartlett<sup>6</sup>, Howard M. Wiseman<sup>1</sup> and Geoff J. Pryde<sup>1</sup>

<sup>1</sup>Centre for Quantum Dynamics, Griffith University, Brisbane, 4111, Australia

<sup>2</sup>Department of Physics, Macquarie University, Sydney, 2109, Australia

<sup>3</sup>Institute for Quantum Computing, University of Waterloo, Waterloo, ON N2L 3G1, Canada

<sup>4</sup>ICFO—Institut de Ciències Fotoniques, Mediterranean Technology Park, 08860 Castelldefels (Barcelona), Spain

<sup>5</sup>Physics Department, The University of Queensland, Brisbane 4072, Australia

<sup>6</sup>School of Physics, The University of Sydney, Sydney 2006, Australia

## Abstract

Quantum measurement protocols make use of the quantum nature of the universe to gain more information about a system using fewer resources than would otherwise be possible. Achieving precision at the fundamental limits imposed by quantum mechanics nevertheless remains a challenge. Adaptive quantum measurement protocols may allow us to step closer to these ultimate limits. We present an overview of some interesting recent results of quantum measurement schemes as demonstrated in single-photon optical implementations, in the contexts of unambiguous phase estimation and multiple-copy non-orthogonal state discrimination.

*Keywords:* phase; nonorthogonal; discrimination

## Introduction

The quantum mechanical nature of the universe enforces fundamental limits on the precision one can obtain from any act of measurement. However, optimal measurements often require difficult to create non-local quantum states. Thus, determining practical measurement schemes which use quantum properties of local states to achieve precision better than that possible with the same number of resources classically is therefore an important scientific goal, not only for their applicability to real-world situations, but also as they provide useful insight into the fundamental operations of quantum measurement.

An adaptive measurement is one in which some information about a system is gathered, processed, and used to decide how subsequent information should be collected. In the context of classical measurement, it has long been known that adaptivity often has the potential to increase measurement precision at a rate beyond that achievable using non-adaptive techniques. Similarly, it is well established that applying adaptivity to the quantum measurement regime can allow one to improve precision beyond that possible without adaptive measurements.

Here we consider several different adaptive measurement schemes demonstrated in quantum optics, and show that while adaptive measurements can improve performance as compared to non-adaptive schemes, in some situations the scaling of precision nevertheless remains identical to that offered by non-adaptive measurements. We show this in two contexts. Firstly, we consider the estimation of an optical phase. Secondly, we consider the discrimination of two non-orthogonal quantum states when multiple copies are available. In both cases results show that while the demonstrated adaptive schemes are more precise in absolute terms, in the asymptotic limit of resources the scaling of precision is the same.

## Phase Estimation

The estimation of a completely unknown phase is an important and fundamentally interesting quantum primitive. By employing techniques from quantum information science, quantum measurement, and quantum computation, we constructed an adaptive phase estimation scheme using only single-photon states with precision (quantified by standard deviation) scaling at the fundamental limit imposed by quantum mechanics (Higgins et al., 2007). The scheme is a generalisation of the quantum phase estimation algorithm (QPEA) of Cleve et al. (1998) in which  $2^k$ -fold oscillating phase fringe measurements were used to construct each  $k$ th digit of a binary expansion of the phase, in order from least to most significant. The key insight of the generalised QPEA is to employ  $M$  times each  $2^k$ -fold fringe measurement. To accommodate this, the adaptivity already inherent in the QPEA is extended, such that at each step the measurement performed is the one that minimises the expected uncertainty.

We have subsequently shown (Higgins et al., 2009) that adaptation is not necessary to achieve precision scaling at the fundamental limit. The key insight to this result is that by replacing the constant  $M$  with a function  $M(k)$ , the most significant digits in the binary expansion of the phase can be measured using more resources, and thus with greater precision, than the least significant digits. Thus, the non-adaptive requirement of disallowing the use of information gathered in the estimation of the least-significant digits is mitigated. Both schemes were demonstrated (Fig. 1) using single photons, generated through spontaneous parametric downconversion, passed a controlled multiple number of times through a half-wave plate which implemented the phase shift.

### State Discrimination

The quantum mechanical property that two non-orthogonal quantum states cannot be reliably distinguished is vital to the operation of quantum encryption schemes such as the famous BB84 (Bennett and Brassard, 1984) protocol. Acín et al. (2005) have shown theoretically that when multiple copies of an unknown pure state are available a protocol that adaptively measures each copy individually achieves exactly the minimum error possible, even for collective measurements over all copies at once (which is very difficult to achieve practically). We have experimentally demonstrated the schemes presented by Acín et al. using single-photons with quantum states encoded in the polarisation basis.

Less is known for mixed states. We have found that the adaptive strategy of Acín et al. is not optimal compared to the optimal collective measurement, nor even a simple fixed measurement scheme known as the majority vote. Subsequently, we have devised a mechanism to construct the optimal adaptive measurement scheme for mixed states. The scheme reduces to the adaptive scheme of Acín et al. for highly-pure states, and the majority vote for highly-mixed states. We have also demonstrated these schemes (Fig. 2).

A recent theoretical result by Hayashi (2009) has shown that for any quantum state the error as a function of the number of copies scales, as the number of copies goes to infinity, at the same rate for the optimal fixed measurement scheme as for the optimal adaptive measurement scheme. Although this situation is unlikely in practice, the result demonstrates a remarkable parallel to the result of non-adaptive phase estimation.

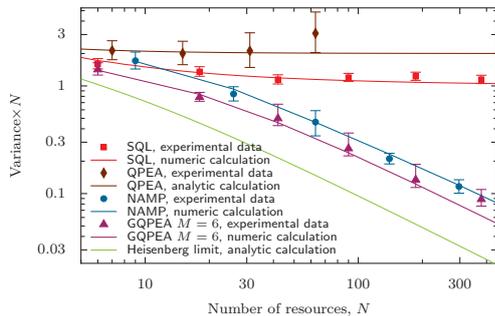


Figure 1: Uncertainty of phase estimation schemes with increasing resources  $N$ .

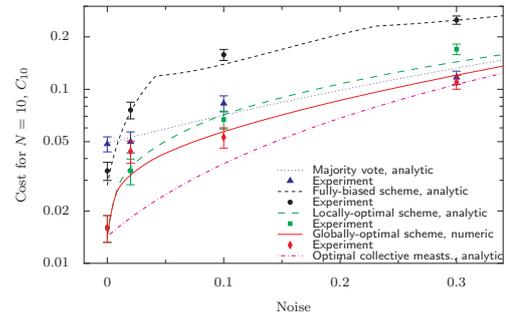


Figure 2: Probability of error (cost) of state discrimination schemes for 10 copies with increasing mixture, implemented as depolarising noise.

### Conclusions

The use of adaptivity in quantum measurement protocols provides interesting insights into the fundamentals of quantum mechanics. We have constructed, demonstrated and compared a number of adaptive and non-adaptive quantum measurement schemes in two contexts, and have found that while non-adaptive schemes can provide the same scaling of precision in the asymptotic regime, adaptivity is required to obtain the optimal performance in non-collective measurements, both in terms of non-asymptotic scaling and achieving the lowest constant overhead.

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