

Determination of Risks to Manufacturer and Buyers for Lifetime Warranty Policies by Considering Uncertainties of Lifetime

A. Rahman ¹, and G. Chattopadhyay²

¹ Griffith School of Engineering, Griffith University, Gold Coast campus, Southport, Qld 4222, Australia

² Faculty of Science, Engineering and Health, CQUniversity Australia, Bryan Jordan Drive, Gladstone, QLD4680, AUSTRALIA

ABSTRACT

The warranty period offered by the manufacturer has been progressively increasing since the beginning of the 20th Century. Due to fierce competition and customer demand, manufacturers have started selling products with lifetime warranty policies. Under such policies, both the manufacturer and the buyer are exposed to uncertainties and risks of warranty pricing and product performance since products lifetime are uncertain and are not defined well in these policies. This paper extends the work of Chattopadhyay and Rahman (COMADEM 2005) to determine the manufacturer's and buyer's risk preferences by capturing the uncertainties of warranty duration in the lifetime warranty. Using the exponential utility function, the decision models are developed to maximise the manufacturer's certainty profit equivalent. Risk preference models are developed to find the optimal warranty price through the use of the manufacturer's utility function for profit and the buyer's utility function for repair costs.

Keywords: Lifetime warranties, warranty costs, manufacturer's and customer's risk preferences

1. INTRODUCTION

By offering a lifetime warranty, both the manufacturer and the buyers are exposed to uncertainties and risks of warranty pricing (manufacturer) and product performance (buyers) due to the uncertainties of length of the product lifetime. Under the typical situation of a lifetime warranty transaction, a buyer of a product pays for the warranty at the time of product purchase which in some occasion is factored into the product price. The manufacturer provides rectification service in case of product failures due to design, manufacturing and quality assurance problems during the defined lifetime of the product.

By offering a lifetime warranty for a product, the manufacturer is risking in warranty pricing that whether its offer for such warranty will be accepted by the buyers. At the same time, buyers are unsure about the benefits of buying products sold with lifetime warranty policies. Anticipation of higher product failures encourages a buyer to pay for the higher warranty price which in turn encourages the manufacturer to charge a higher warranty price. In case of product covered with warranty, the buyer returns to the original manufacturer for rectification of product failures. But when the product is not covered by warranty, the buyer may take it to other service providers for rectifications. If the original manufacturer is the only one that can repair the product because of some technological monopoly, the manufacturer could charge a higher price for services and more buyers would be interested to have warranty cover. If buyers pay higher repair price for any product failures, they might be interested to get warranty cover and even pay a higher warranty price for free repair or replacement policies.

In line with Chun and Tang [1], Chattopadhyay and Rahman [2], proposed risk preference models that focused on these two perspectives to estimate costs and risks to manufacturers and buyers for lifetime warranty policies considering a constant lifetime. But in the case of lifetime warranty, the length of

coverage period is totally uncertain due uncertain useful life of the product since the warranty period terminates as a result of technical and commercial changes, technological obsolescence, and ownership changes [3].

This paper is extending the risk preferences models developed by Chattopadhyay and Rahman, [2] in finding the optimal lifetime warranty prices through the use of the manufacturer’s utility function for manufacturer’s profit and the buyer’s utility function for repair cost. This paper proposes the warranty price models that maximise the manufacturer/dealers certainty equivalent by applying Non-homogeneous Poisson process (NHPP) for products with time dependent failure mode.

The outline of this paper is as follows. Section 1 briefly introduces manufacturers’ and buyers’ risk preferences towards a lifetime warranty policy considering the uncertainties of warranty duration. Section 2 provides brief overview of lifetime warranty policies. In section 3, risk preference models for both manufacturer and buyer towards a lifetime warranty policy are developed using exponential utility function. Finally, in the concluding section contribution of this research work and some recommendation of future research scopes are discussed in Section 4.

2. A BRIEF OVERVIEW

Because of high popularity and huge competition, manufacturers are becoming more and more interested in offering lifetime warranty for their products. Lifetime warranty means the manufacturer commitment to provide free or cost sharing rectification of the sold product in case of failure due to design, manufacturing or quality problems throughout the useful life of the product or the buyer’s ownership period of the product. One important difference between normal warranty and lifetime warranty is the coverage period. In case of normal warranty, warranty coverage period is fixed whereas it is uncertain for lifetime warranty and is often difficult to tell about life measures for the period of coverage [4]. Details can be found in [3]. Figure 1 shows the framework for long term warranty measures. The lifetime of the product is assumed to be terminated in some finite, random time horizon. Termination of such warranty may arise from any one of the following reasons [5].

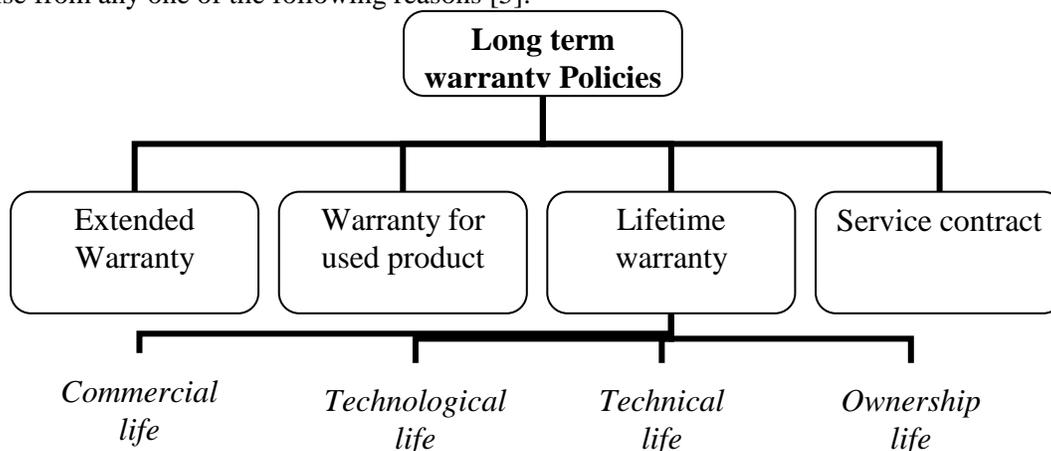


Figure. 1: A framework for the lifetime warranty measure (Chattopadhyay and Rahman, 2008)

Technical life/ Physical life – the period over which the product might be expected to last physically, or when replacement or major rehabilitation is physically required.

Technological life – the period until technological obsolescence dictating replacement due to the development of a technologically superior alternative

Commercial life/ Economic life/ Functional life – the period, over which the need for the product exists, the period until economic obsolescence dictates replacement with a lower cost alternative (economic or operational convenience).

Ownership life/ Social and legal life – the period until human desire or legal requirement dictates replacement or change of ownership occurs.

Since both manufacturer and buyer are exposed to uncertainties and risks of such warranty pricing and product performance during the lifetime of the product it is essential to develop risk preference models for both manufacture and buyer taking into account the uncertain nature of the warranty coverage period. Limited work so far takes into account the manufacturer and buyer's risk preferences toward how much a manufacturer should charge and how much a buyer is willing to pay for such warranty service. Ritchken and Tapiero [6] proposed a framework in which warranty policies for non-repairable items can be evaluated according to risk preferences of both the manufacturer/dealer and the buyers where they emphasised the design and pricing of warranties to which the manufacturers are indifferent in an expected utility sense. Chun and Tang [1] proposed a warranty model for the free-replacement, fixed-period warranty policy that determines the optimal warranty price for a fixed warranty period. In line with Chun and Tang, Chattopadhyay and Rahman, [2] proposed risk preference models for lifetime warranty where they considered a fixed lifetime of the product which is similar to a normal warranty. They assumed an increased failure rate for the product over time, constant repair costs throughout the warranty period, and a producer's and customers' risk aversion for future repair costs. Using the exponential utility function and the gamma failure rate distribution, they derived the decision model that maximizes the producer's certainty profit equivalent. To overcome this shortcoming, this paper captures the uncertain lifetime measures in developing the risk preference models for lifetime warranty in finding the optimal warranty price through the use of the manufacturer's utility function for manufacturer's profit and the buyer's utility function for repair cost.

3. MODEL FORMULATION

3.1. Notations and Statistical Preliminaries

Let the number of items sold by the manufacturer, be S and p be the proportion of product sold with lifetime warranty, and $(1 - p)$ be the proportion of S products sold without such warranty. Two conditions are considered here: products are sold with warranty or without warranty. Subscripts m and b stands for manufacturer and buyer respectively.

Let k be the proportion of buyers without warranty coming back to manufacturer for repairing of the faults/defects and $1-k$ portion of buyers prefer to go to third parties for repairing the failed products.

$N_b(L)$ represents the number of valid claims made by the buyer per item and $N(L)$ is the total number of possible claims for S products over the lifetime L .

$E[N_b(L)]$ is the expected number of failure per item experienced by the buyers over the lifetime.

$U_m(Y)$ and $U_b(X)$ denote the manufacturers and buyers continuous utility functions for a manufacturer's profit Y and buyer's repair cost X respectively.

U_m is an individual manufacturer's utility function

U_b is the aggregate utility function representing the entire buyer's risk preference as a whole.

$\Lambda_m(t)$ is the manufacturer's failure intensity function.

$\Lambda_b(t)$ is the per item failure intensity function for an individual buyer during the lifetime

Let r_b represents the cost of rectification (repair cost) for a buyer in each occasion of failure if the item is not warranted.

r_m represents the manufacture's per occasion cost of rectification (repair cost) which is the actual rectification cost when the item is covered under warranty.

W is the warranty price offered by the manufacturer during the time of purchase.

Product cumulative failure distribution $F(t)$ with density function $f(t)$ in general (assumption), where,

$$f(t) = dF(t)/dt, \text{ and the failure intensity function can be expressed by } \Lambda(t) = \frac{f(t)}{(1-F(t))}.$$

3.2. Assumptions

- Item failures are statistically independent and failures in a probabilistic sense, is only a function of its age.
- The time to carry out a rectification action is negligible, and an item failure results in an immediate claim and all claims are valid.
- Failures over the warranty period modelled at the system (or item) level.
- Manufacturer's cost of rectification r_m and the buyer's cost of each rectification r_b are constant over the warranty period.
- The Manufacturer's utility function $U_m(Y)$ for profit Y is concave (risk averter) and strictly increasing as most manufacturers prefer a higher profit to a lower profit. Thus, it follows from Jensen's inequality [1], [6], that if Y has finite mean then $E[U_m(Y)] < U_m[E(Y)]$ and $[y_1 < y_2] \leftrightarrow [U_m(y_1) < U_m(y_2)]$
- The buyer's utility function $U_b(X)$ for repair cost X is concave (also risk averter) and strictly decreasing as buyer prefer a lower cost to a higher cost. Thus, it follows from Jensen's inequality that if X has finite mean then $E[U_b(X)] < U_b[E(X)]$ and $[x_1 < x_2] \leftrightarrow [U_b(x_1) > U_b(x_2)]$.

3.3. Buyer's Acceptances of Warranty

In determining the worthy of buying such warranty, a buyer may first estimate the total repair cost $r_b N_b(L)$ of his purchased product during the defined lifetime and then compare it with the given warranty price W in terms of the expected utility. Since r_b is constant, buyer's total repair cost is given by $r_b N_b(L)$ which is estimated by his perceived product failure intensity $\Lambda_b(t)$. The higher the buyer estimate of the product failure rate, the more likely he would be willing to buy the warranty [1].

Let n_b^* be the number of product failure when buyers are indifferent between the warranty price W and the total repair cost $r_b N_b(L)$ in terms of the expected utility. Certainty equivalent of similar concept can be found in [7]. By definition, a certainty equivalent of W is an amount of $r_b N_b(L)$ such that the decision maker is indifferent between W and $r_b N_b(L)$, therefore, it is given by

$$U_b(W) = E[U_b(r_b N_b(L))] \quad (1)$$

Therefore, we can write

$$U_b(W) = E[U_b(r_b N_b(L))] = \sum_{n_b=0}^{\infty} U_b(r_b n_b) P(N_b(L) = n_b^*) \quad (2)$$

But when failures of product follows Non-homogeneous Poisson process that is failures are time dependent then we have the following equation

$$\text{Prob}\{N_b(L) = n_b^*\} = \frac{\int_0^L \Lambda_b(t) dt \}^{n_b^*} e^{-\int_0^L \Lambda_b(t) dt}}{n_b^*!} \quad (3)$$

One form for $\Lambda_b(t)$ using Non-homogeneous Poisson process [$F(t) = 1 - \exp(-(\lambda_b t)^{\beta_b})$] is as follows:

$$\Lambda_b(t) = \lambda_b \beta_b (t)^{\beta_b - 1} \quad (4)$$

with the shape parameters $\beta_b > 1$ and inverse characteristic life $\lambda_b > 0$. This is an increasing function of time or age t . Let the buyers' risk aversions be represented by exponential utility functions,

$$U_b(Y) = -e^{cY}, \quad c > 0, \quad (5)$$

Where c is the risk parameters representing the buyers' risk preferences. A buyer's attitude toward risk can be determined by the nature of c . A buyer is more risk-averse if the risk parameter increases. A risk parameter $c = 0$, indicates a risk neutral buyer. In this study, the buyers' risk parameter c represents a wide spectrum of buyers' risk preferences.

In the exponential utility functions, the *absolute risk aversion measure* is constant for all X , implying that the exponential function represents only the *constant* risk-averse case [1].

By combining Equations (2), (3) and (4), Equation (6) can be developed

$$e^{cW} = \sum_{n_b^*=0}^{\infty} e^{cn_b^*} \frac{\int_0^L \lambda_b \beta_b t^{\beta_b - 1} dt \}^{n_b^*} e^{-\int_0^L \lambda_b \beta_b t^{\beta_b - 1} dt}}{n_b^*!} \quad (6)$$

Finally this can be expressed as Equation (7) (for details see [2]).

$$W = \ln \sum_{n_b^*=0}^{\infty} \exp(cn_b^*) \times \frac{[(\lambda_b L)^{\beta_b}]^{n_b^*} \exp[-(\lambda_b L)^{\beta_b}]}{n_b^*!} \quad (7)$$

According to the Equation 7, L is constant but in case of lifetime warranty, the upper limit of the warranty coverage is uncertain since the termination of life is uncertain and randomly variable. Product failure also occurs randomly over this uncertain coverage period (see Figure 2). Conditioned on the upper limit of coverage $L = a$, one can capture this uncertain coverage period by binding a with a lower limit l and upper limit u at statutory base since the termination of lifetime is assumed to be occurred in some finite, random time horizon. One can model this as a random variable with a distribution function $H(a)$ with $H(l) = P(a \leq l) = 0$ and $H(u) = P(a \leq u) = 1$

$h(a)$ is the probability density function of coverage period a associated with $H(a)$ and $h(a) = \frac{dH(a)}{da}$ (8)

One form of $H(a)$, which is analytically tractable, is the following truncated exponential distribution [8]:

$$H(a) \text{ is } \frac{e^{-\rho l} - e^{-\rho a}}{e^{-\rho l} - e^{-\rho u}} \text{ which gives a } h(a) = \frac{\rho e^{-\rho a}}{e^{-\rho l} - e^{-\rho u}}$$

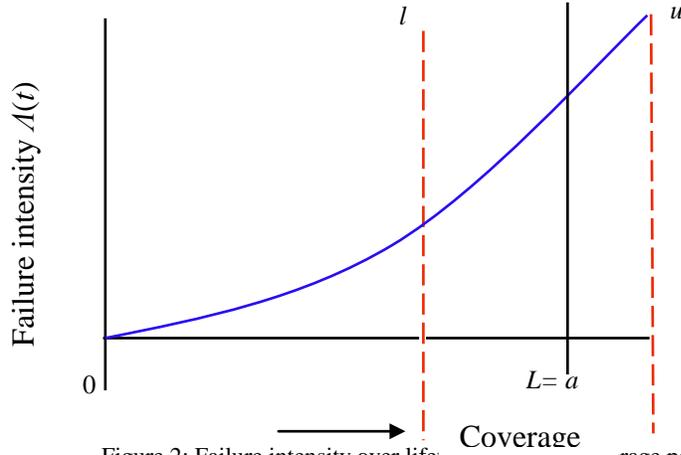


Figure 2: Failure intensity over lifetime warranty coverage period

The mean value of useful life of the sold product can be expressed by

$$\mu_L = E(a) = \frac{(le^{-\rho l} - ue^{-\rho u}) + (e^{-\rho l} - e^{-\rho u})/\rho}{e^{-\rho l} - e^{-\rho u}} \quad (9)$$

ρ is Parameter for the truncated exponential distribution used in the life distribution of products. In real life distribution of lifetime coverage might not be possible to model using a particular distribution and can be modelled using a probability mass function.

Therefore, the equation (7) can be expressed as

$$W = \ln \sum_{n_b^*=0}^{\infty} \exp(cr_b n_b^*) \times \frac{[(\lambda_b \mu_L)^{\beta_b}]^{n_b^*} \exp[-(\lambda_b \mu_L)^{\beta_b}]}{n_b^*} \quad (10)$$

By substituting μ_L in Equation 7 from Equation 6, buyers' expected optimal warranty price can be estimated (Equation 10). From the above equation it will be worthy for the buyer to accept the warranty offer if the buyer's estimated number of failure is more than and equal to indifferent failure n_b^* . Accordingly, a buyer whose expected failure is higher than n_b^* would buy the warranty if the total estimated repair cost is higher than the warranty price W . Notice that probability to buy this warranty is determined by n_b^* , which, in turn, is determined by W .

Based on the information about the buyer's willingness to pay for the warranty price W , a manufacturer may determine the warranty price such that the expected total profits during the warranty period is maximised.

3.4. Manufacturer's profit

The manufacturer's expected total profit $E[\pi(L)]$ for warranty and rectification related services during the lifetime under such warranty policy can be expressed as

$$E[\pi(L)] = p(SW - E[N(L)]r_m) + (1-p)k[E[N(L)]r_b - E[N(L)]r_m] = p(SW - E[N(L)]r_m) + (1-p)kE[N(L)](r_b - r_m) \quad (11)$$

For a risk-averse manufacturer with an increasing utility function $U_m(Y)$, the certainty equivalent for profit can be obtained from the following relationship

$$U_m(E[\pi(L)]) = U_m(p(SW - E[N(L)]r_m) + (1-p)kE[N(L)](r_b - r_m)) \quad (12)$$

As the manufacturer estimated failure intensity is time dependent, it follows the Non-homogeneous Poisson process (NHPP) and here the intensity function can be expressed as

$$\Lambda_m(t) = \lambda_m \beta_m t^{(\beta_m - 1)} \quad (13)$$

with the shape parameters $\beta_m > 1$ and inverse characteristic life parameter $\lambda_m > 0$. This is an increasing function of t .

Since the items are statistically similar then with more information on product failure manufacture can have probability of n_m failures over life L for any item as given by:

$$\Pr ob\{N_m(L) = n_m\} = \frac{\left\{ \int_0^L \Lambda_m(t) dt \right\}^{n_m} e^{-\int_0^L \Lambda_m(t) dt}}{n_m!} \quad (14)$$

$$E[N_m(L)] = \lambda_m^{\beta_m} (L^{\beta_m}) \quad (15)$$

$$E[N(L)] = S \lambda_m^{\beta_m} (L^{\beta_m}) \quad (16)$$

Manufacturer estimates expected number of failures from $N(L)$ over the lifetime of S items based on the failure intensity $\Lambda_m(t)$ and number of products sold S . The number of failures over the warranty period, is a function of product useful life (lifetime), and is a random variable. Similar to the buyers' risk aversions, the manufacturer's risk aversions is assumed to be exponential utility functions, which is given by

$$U_m(Y) = e^{-aY}, \quad a > 0, \quad (17)$$

Where, a is the risk parameters representing the manufacturer's risk preferences. $a > 0$ indicates risk averse manufacturer, whereas $a < 0$ indicates risk seeker and $a = 0$ means a risk neutral manufacturer. According to [2], utility of expected profit for warranty and no-warranty strategy for indifferent manufacturer's decision point is as follows:

$$e^{-a[pS(W - (\lambda_m L)^{\beta_m} r_m) + (1-p)Sk(\lambda_m L)^{\beta_m} (r_b - r_m)]} = \sum_{n_m=0}^{\infty} e^{-a(r_b - r_m)Skn_m} \cdot \frac{\left[\int_0^L \lambda_m \beta_m t^{\beta_m - 1} \right]^{n_m} e^{-\int_0^L \lambda_m \beta_m t^{\beta_m - 1}}}{n_m!} \quad (18)$$

Or,

$$W = -\frac{1}{apS} \times \ln \sum_{n_m=0}^{\infty} e^{-adSkn_m} \cdot \frac{[(\lambda_m L)^{\beta_m}] e^{-(\lambda_m L)^{\beta_m}}}{n_m!} + \frac{q}{p} k(\lambda_m L)^{\beta_m} (r_b - r_m) - (\lambda_m L)^{\beta_m} r_m \quad (19)$$

According to the Equation (19), L is constant which contradicts with the uncertain lifetime. To capture the uncertainty of lifetime (useful life) the Equation (19) can be rewritten as

$$W = -\frac{1}{apS} \times \ln \sum_{n_m=0}^{\infty} e^{-adSk n_m} \cdot \frac{[(\lambda_m \mu_L)^{\beta_m}] e^{-(\lambda_m \mu_L)^{\beta_m}}}{n_m!} + \frac{q}{p} k (\lambda_m \mu_L)^{\beta_m} (r_b - r_m) - (\lambda_m \mu_L)^{\beta_m} r_m \quad (20)$$

The developed Buyer's and manufacturer's risk preference model for lifetime warranty can be solved numerically by using mathematical software such as MATLAB, MAPLE etc.

4. CONCLUSIONS

This paper extended the buyer's and manufacturers risk preference models developed by Chattopadhyay and Rahman [2] for product lifetime warranty by capturing uncertainty and randomness of useful life (lifetime warranty coverage). These models have been proposed with Non homogeneous Poisson's process for failure intensity function, a constant repair cost, and concave utility function. Using the exponential utility function, the decision models have been developed to maximise the manufacturer/dealer's certainty profit equivalent. Risk preference models are developed to find the optimal warranty price through the use of the manufacturer's utility function for profit and the buyer's utility function for repair costs. These models can be useful for manufacturers when making decisions about warranty pricing. Due to page limitations, numerical examples together with the sensitivity analysis of the models could not be presented. Numerical examples and the sensitivity of the models will be illustrated in future publication.

There is a scope for future research with other forms of failure distributions, impact of preventive maintenance and scope for replacements during lifetime with possible trade-ins. Another possible extension is to include discounting of the future repair costs and corrections for inflation which are important for products sold with long service life.

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