An Efficient Fixed-Point Harmonic-Balanced Method Taking Account of Hysteresis Effect Based on the Consuming Function

Xiaojun Zhao[1], Lin Li[1], Junwei Lu[2], Zhiguang Cheng[3], Tiebing Lu[1], Hongxia Huang[1], Gang Liu[1]
[1]School of Electrical and Electronic Engineering, North China Electric Power University,
Beinong Road, Beijing, 102206, P. R. China
[2]Griffith School of Engineering, Griffith University, Brisbane, QLD4111, Australia
[3]R&D Center of Baoding Tianwei Group, Baoding, 071056, P. R. China

Abstract-**The harmonic balance finite element method (HBFEM) is combined with the fixed-point technique to calculate the steady-state magnetic field in a laminated core model (LCM). The magnetic intensity is divided into linear and nonlinear parts in the form of harmonics. The hysteresis model based on the consuming function is taken into account in the harmonic computation. The magnetic field and magnetizing currents are calculated simultaneously by considering the electromagnetic coupling. Comparison between measured and calculated magnetizing current verifies the effectiveness and validity of the proposed method.**

I. INTRODUCTION

The computation of steady-state solutions of nonlinear time-periodic magnetic field can be performed in time domain. The straightforward treatment is the so called "brute force" method of using time stepping technique starting from arbitrary initial values. However, the solutions in time domain generally require several periods to reach steady state. Variables in electromagnetic field under steady-state excitations can be approximated by the triangular series. Harmonic balance theory was introduced in the computation of magnetic field and combined with finite element method [1]. The magnetic field can be solved directly in harmonic frequency domain, without expensive computational time. The HBFEM has been used to solve the steady-state magnetic field with eddy current problems and design the switching power supplies [2-3]. In recent years the HBFEM has been developed by introducing differential reluctivity tensor [4] and transmission-line modeling technique [5] respectively. Harmonic analysis of magnetizing current and flux density has been performed by the HBFEM to investigate the DC bias phenomena in power transformers [6-7].

It is a challenging work to consider the hysteresis effects of transformer in HBFEM. The hysteresis loops are simulated by introducing an additional term in magnetic intensity, which is related to frequency and derivative of magnetic flux density [8]. However, inaccuracy of the mathematical model and difficulty of predicting the relevant parameters have limited its application in modeling hysteresis effects. Moreover, hysteresis effects give rise to the discontinuity of the magnetic reluctivity in the HBFEM. Therefore the HBFEM is not effective when hysteresis effects of iron core are involved in computation.

In this paper a properly established hysteresis model based on the consuming function is applied to model the hysteresis loops. The fixed-point technique is integrated with the HBFEM to calculate the magnetic field considering hysteresis effects. The fixed-point reluctivity replaces the magnetic reluctivity and overcomes its defect of discontinuity. The proposed method is verified with experimental results for the magnetizing current.

II. FIXED-POINT HARMONIC-BALANCED THEORY

A. Fixed-Point Technique

In HBFEM, the magnetic reluctivity ν is defined as follows,

$$\nu = H(t) / B(t) \tag{1}$$

where H and B represents magnetic intensity and flux density. When the hysteresis effects are involved in the finite element analysis of magnetic field, the discontinuity of magnetic reluctivity may affect the numerical results.

A new relationship between magnetic intensity and flux density is represented by introducing the fixed-point reluctivity ν_{FP} [9-10].

$$\boldsymbol{H}(\boldsymbol{B}) = \nu_{\mathrm{FP}}\boldsymbol{B} - \boldsymbol{M}(\boldsymbol{B}) \tag{2}$$

Unlike the time-dependent magnetic reluctivity ν in (1), ν_{FP} is a constant value. $\boldsymbol{M}$ is a magnetization-like quantity which varies nonlinearly with the flux density $\boldsymbol{B}$. Therefore the magnetic intensity $\boldsymbol{H}$ is divided into two parts, the linear part related to ν_{FP} and the nonlinear part related to $\boldsymbol{M}$.

B. Harmonic Balance Method

The nonlinear magnetic field is described by the Maxwell's equations,

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} \tag{3}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{4}$$

where $\boldsymbol{J}$ is the current density. Consequently the vector potential equation in two-dimensional case can be obtained,

$$\nabla \times \nu_{\mathrm{FP}}(\nabla \times A) + \sigma \frac{\partial A}{\partial t} = J - \nabla \times \boldsymbol{M} \tag{5}$$

where A is the magnetic vector potential and σ is the electric conductivity.

Due to the periodic characteristics of the electromagnetic field under steady-state excitation, all variables can be approximated by the Fourier-series with a finite number of harmonics,

$$J^i(t) = \sum_{n=1,3,5,\ldots}^{N} \left(J_{ns}^i \sin n\omega t + J_{nc}^i \cos n\omega t \right) \tag{6}$$

$$A^i(t) = \sum_{n=1,3,5,\ldots}^{N} \left(A_{ns}^i \sin n\omega t + A_{nc}^i \cos n\omega t \right) \tag{7}$$

$$M_x^e(t) = \sum_{n=1,3,5,\ldots}^{N} \left(M_{xns}^e \sin n\omega t + M_{xnc}^e \cos n\omega t \right) \tag{8}$$

$$M_y^e(t) = \sum_{n=1,3,5,\ldots}^{N} \left(M_{yns}^e \sin n\omega t + M_{ync}^e \cos n\omega t \right) \tag{9}$$

where i represents node number, e represents element number and N is the truncated highest harmonic number. The magnetic intensity $\boldsymbol{H}$ and flux density $\boldsymbol{B}$ have the similar expressions with $\boldsymbol{M}$ shown in (8) and (9).

The Galerkin's method is applied in (5) to obtain the weighted residual in one element Ω_{e},

$$\iint_{\Omega_{\mathrm{e}}} \nu_{\mathrm{FP}} \left(\frac{\partial N_{\mathrm{e}i}}{\partial x}\frac{\partial A_{\mathrm{e}}}{\partial x} + \frac{\partial N_{\mathrm{e}i}}{\partial y}\frac{\partial A_{\mathrm{e}}}{\partial y} \right) \mathrm{d}x\mathrm{d}y + \iint_{\Omega_{\mathrm{e}}} \left(\sigma \frac{\partial A_{\mathrm{e}}}{\partial t} N_{\mathrm{e}i} \right) \mathrm{d}x\mathrm{d}y = \iint_{\Omega_{\mathrm{e}}} N_{\mathrm{e}i} \left(J + \frac{\partial M_y}{\partial x} - \frac{\partial M_x}{\partial y} \right) \mathrm{d}x\mathrm{d}y \tag{10}$$

where $N_{\mathrm{e}i}$ is the interpolation function for a linear triangular element.

The harmonic forms of all variables are substituted into (10) and the harmonic-balanced equation in one element can be represented by

$$\boldsymbol{S}_{\mathrm{e}}\boldsymbol{A}_{\mathrm{e}} + \boldsymbol{T}_{\mathrm{e}}\boldsymbol{A}_{\mathrm{e}} - \boldsymbol{K}_{\mathrm{e}} - \boldsymbol{P}_{\mathrm{e}} = \frac{\nu_{\mathrm{FP}}}{4\Delta_{\mathrm{e}}} \begin{bmatrix} S_{11}\boldsymbol{I} & S_{12}\boldsymbol{I} & S_{13}\boldsymbol{I} \\ S_{21}\boldsymbol{I} & S_{22}\boldsymbol{I} & S_{23}\boldsymbol{I} \\ S_{31}\boldsymbol{I} & S_{32}\boldsymbol{I} & S_{33}\boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{A}_{\mathrm{e}1} \\ \boldsymbol{A}_{\mathrm{e}2} \\ \boldsymbol{A}_{\mathrm{e}3} \end{bmatrix} + \frac{\sigma\omega\Delta_{\mathrm{e}}}{12} \begin{bmatrix} 2\boldsymbol{N} & \boldsymbol{N} & \boldsymbol{N} \\ \boldsymbol{N} & 2\boldsymbol{N} & \boldsymbol{N} \\ \boldsymbol{N} & \boldsymbol{N} & 2\boldsymbol{N} \end{bmatrix} \begin{bmatrix} \boldsymbol{A}_{\mathrm{e}1} \\ \boldsymbol{A}_{\mathrm{e}2} \\ \boldsymbol{A}_{\mathrm{e}3} \end{bmatrix} - \begin{bmatrix} \boldsymbol{K}_{\mathrm{e}1} \\ \boldsymbol{K}_{\mathrm{e}2} \\ \boldsymbol{K}_{\mathrm{e}3} \end{bmatrix} - \begin{bmatrix} \boldsymbol{P}_{\mathrm{e}1} \\ \boldsymbol{P}_{\mathrm{e}2} \\ \boldsymbol{P}_{\mathrm{e}3} \end{bmatrix} = \boldsymbol{0} \tag{11}$$

where $\boldsymbol{S}_{\mathrm{e}}$ and $\boldsymbol{T}_{\mathrm{e}}$ represents the coefficient matrix related to magnetic nonlinearity and eddy current respectively. $\boldsymbol{I}$ is the unit matrix and N is the harmonic matrix. $\boldsymbol{K}_{\mathrm{e}}$ is the harmonic vector related to the excitation and $\boldsymbol{A}_{\mathrm{e}}$ is the harmonic vector potential.

$$\boldsymbol{A}_{ei} = \left[A_{1s}^i, A_{1c}^i, A_{3s}^i, A_{3c}^i, A_{5s}^i, A_{5c}^i, \ldots \right]^{\mathrm{T}} \tag{12}$$

$$\boldsymbol{K}_{ei} = \frac{\Delta}{3} \left[J_{1s}^i, J_{1c}^i, J_{3s}^i, J_{3c}^i, J_{5s}^i, J_{5c}^i, \ldots \right]^{\mathrm{T}} \tag{13}$$

where Δ represents the area of triangular element.

The harmonic vector $\boldsymbol{P}_{\mathrm{e}i}$ which is relevant to the magnetization-like quantity $\boldsymbol{M}$ can be defined by

$$\begin{gathered} \boldsymbol{P}_{\mathrm{e}i} = \left[P_{\mathrm{e}i,1\mathrm{s}}, P_{\mathrm{e}i,1\mathrm{c}}, P_{\mathrm{e}i,3\mathrm{s}}, P_{\mathrm{e}i,3\mathrm{c}}, P_{\mathrm{e}i,5\mathrm{s}}, P_{\mathrm{e}i,5\mathrm{c}}, \cdots \right] \\ \begin{cases} P_{\mathrm{e}i,n\mathrm{s}} = \iint_{\Omega_{\mathrm{e}}} \left(M_{yn\mathrm{s}} \dfrac{\partial N_{\mathrm{e}i}}{\partial x} - M_{xn\mathrm{s}} \dfrac{\partial N_{\mathrm{e}i}}{\partial y} \right) dxdy \\ P_{\mathrm{e}i,n\mathrm{c}} = \iint_{\Omega_{\mathrm{e}}} \left(M_{yn\mathrm{c}} \dfrac{\partial N_{\mathrm{e}i}}{\partial x} - M_{xn\mathrm{c}} \dfrac{\partial N_{\mathrm{e}i}}{\partial y} \right) dxdy \end{cases} \\ (i = 1,2,3);\ (n = 1,3,5,\ldots,N) \end{gathered} \tag{14}$$

C. *Electromagnetic Coupling*

When electromagnetic devices are excited by voltage source, the coupling between external circuits and magnetic field should be taken into account in the finite-element equations so that the magnetizing current and magnetic field can be solved simultaneously [6-7].

$$\boldsymbol{U}_{\mathrm{in}k} = \boldsymbol{C}_k \boldsymbol{A} + S_{\mathrm{c}k} \boldsymbol{Z}_k \boldsymbol{J}_k \tag{15}$$

where $\boldsymbol{U}_{\mathrm{in}k}$ is the harmonic voltage in the k-th winding, $S_{\mathrm{c}k}$ and $\boldsymbol{Z}_k$ are the cross-sectional area of the k-th winding and the corresponding impedance matrix respectively. $\boldsymbol{C}_k$ represents the coupling matrix.

Consequently the harmonic-balanced matrix equation based on the fixed-point technique is obtained by assembling (11) and (15),

$$\begin{bmatrix} \boldsymbol{F} & -\boldsymbol{G}_1 & \cdots & -\boldsymbol{G}_k & \cdots \\ \boldsymbol{C}_1 & S_{\mathrm{c}1}\boldsymbol{Z}_1 & \boldsymbol{0} & \boldsymbol{0} & \cdots \\ \vdots & \vdots & \ddots & \boldsymbol{0} & \cdots \\ \boldsymbol{C}_k & \boldsymbol{0} & \boldsymbol{0} & S_{\mathrm{c}k}\boldsymbol{Z}_k & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{J}_1 \\ \vdots \\ \boldsymbol{J}_k \\ \vdots \end{bmatrix} = \begin{bmatrix} \boldsymbol{P} \\ \boldsymbol{U}_{\mathrm{in}1} \\ \vdots \\ \boldsymbol{U}_{\mathrm{in}k} \\ \vdots \end{bmatrix} \tag{16}$$

where matrix $\boldsymbol{F}$ is the sum of coefficient matrices $\boldsymbol{S}$ and $\boldsymbol{T}$, matrix $\boldsymbol{G}$ is related to spatial distribution of current density. The harmonic vector potential $\boldsymbol{A}$ and harmonic current density $\boldsymbol{J}$ can be solved together.

D. *Fixed-Point Iterative procedure*

The convergence of harmonic solutions of magnetizing current and magnetic vector potential depends on the fixed-point reluctivity ν_{FP}. In time-stepping finite element analysis the value of ν_{FP} is recommended as follows [9],

$$\nu_{\mathrm{FP}} = \frac{\nu_{\mathrm{dmax}} + \nu_{\mathrm{dmin}}}{2} \approx \frac{\nu_0}{2} \tag{17}$$

where ν_{dmax} and ν_{dmin} are the minimum and maximum and maximum differential reluctivities of the magnetizing curve

$H=F(B)$. v_0 is the reluctivity of the vacuum.

However the recommended fixed-point reluctivity is not applicable in harmonic computation. Strong nonlinearity may lead to unstable oscillation and even non-convergence of the harmonic solutions. In fact a large reluctivity may result in a stable oscillation of harmonic solution in iterative process, whereas a small value gives rise to fast convergence of harmonic solution.

An alternative iterative scheme is presented for the sake of stable iteration and fast convergence. v_{FP} is initially set to be $3v_0 \sim v_0$, and reset in the range $v_0/10 \sim v_0/40$ after a few initial iterations. When $\Delta \boldsymbol{M}=\boldsymbol{M}_p - \boldsymbol{M}_{p-1}$ (p represents the number of iterative step) is small enough, the iteration is stopped.

III. Hysteresis Model Based on Consuming Function

A. Consuming Function

A simplified hysteresis model based on consuming function was presented to approximate the hysteresis loss of power transformers [11-12].

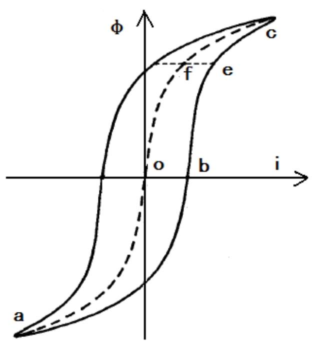

Fig.1 hysteresis curve

In Fig. 1 the dotted line "*aoc*" represents the middle magnetizing curve, which is the locus of the mid points of the hysteresis loop and not related to hysteresis loss. The distance between the periphery of hysteresis loop and middle magnetizing curve, such as "*ef*", represents hysteresis loss. The loss part is defined as consuming function. The consuming function includes the reverse characteristic of the magnetizing curve [11],

$$\begin{cases} \varphi = \varphi_m \cdot \cos \omega t \\ f(\varphi) = D(d\varphi / dt) = -i_{ob} \cdot \sin \omega t \end{cases} \tag{18}$$

where φ_m is the magnitude of flux φ, i_{ob} is the coefficient of consuming function and can be defined by the maximum distance "*ob*" between the mid point locus and the periphery of the hysteresis loop.

B. Hysteresis Effects of LCM

The LCM made by the TianWei Group, Baoding, China, has been tested to investigate the hysteresis effects. Two coils, search coil and exciting coil, are wound on the ferromagnetic core. The exciting coil is fed by different alternating voltages of 50 Hz to obtain a series of hysteresis loops.

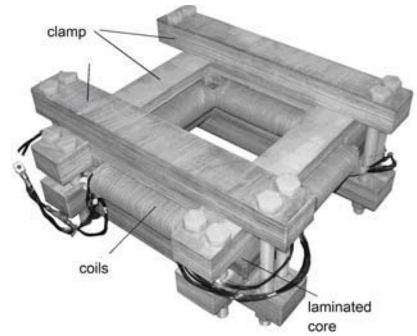

Fig.2 Laminated core model

In order to obtain the hysteresis loop of the LCM, the experiment data u and i are manipulated as follows:

$$\begin{cases} \varphi = (1/N_c)\int u \cdot dt \\ B = \varphi / S \\ H \cdot l = i \cdot N \end{cases} \tag{19}$$

where N_c is the number of turns of the search coil, u is the induced voltage obtained by the search coil, S is the cross-sectional area of the LCM, l is the equivalent length of the magnetic circuit, and i is the magnetizing current.

From (19) it is concluded that a similar function expressed in (18) can be assumed in the *B*-*H* hysteresis loops,

$$\begin{cases} B = B_m \cdot \cos \omega t \\ H_c(B) = -H_{ob} \cdot \sin \omega t \end{cases} \tag{20}$$

where consuming function H_c corresponds to the loss part in the magnetic intensity. H_{ob}, termed as consuming coefficient, is related to the magnitude of flux density B_m and can be collected from the experimental hysteresis data. The measured data relevant to H_{ob} are partly listed in Table I when the exciting coil is fed by different alternating voltages.

TABLE I
Consuming Coefficients Related to Magnitude of Flux Density in the LCM

B_m/T	0.2024	0.4068	0.6112	0.9159	1.1196
H_{ob}/(A/m)	4.2337	8.2774	11.6918	17.7434	21.5094
B_m/T	1.3243	1.5269	1.6818	1.7829	1.9214
H_{ob}/(A/m)	25.8237	29.3191	26.4671	28.0403	29.8274

The total magnetic intensity H can be expressed as follows,

$$H = H_m + H_c \tag{21}$$

where H_m corresponds to the middle magnetizing curve. The middle magnetizing curves cross through the vertexes of the hysteresis loops. Hence the normal magnetizing curve can be approximately treated as a substitution of the middle magnetizing curve in the numerical computation.

IV. Computational Results

The harmonic vectors of magnetizing current and magnetic field are calculated in (16). The truncated harmonic number should be properly selected to decrease the computational error ($N \geq 11$ in this paper). The calculated results are analyzed through the harmonic solutions.

A. Magnetizing Current

The magnetizing currents under different excitations are compared with the experimental results in Fig. 3-4.

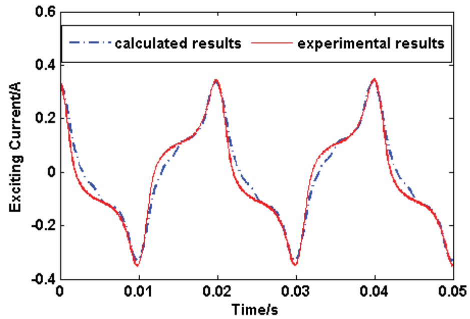

Fig.3 Magnetizing current (U=260V)

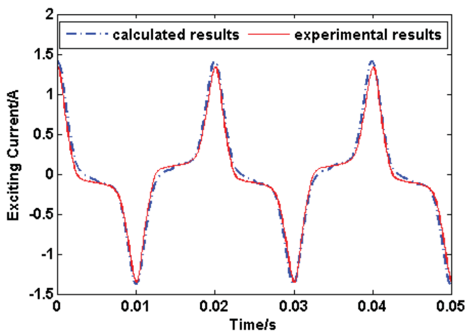

Fig.4 Magnetizing current (U=320V)

B. Magnetic Field

The flux density $\boldsymbol{B}$ is obtained from the calculated harmonic vector potential $\boldsymbol{A}$, and the magnetic intensity $\boldsymbol{H}$ can be determined by the proposed hysteresis model. Fig. 5 shows the computational region of the LCM. A point in the LCM, such as D, can be selected to observe the calculated results of magnetic field. The flux density in the Y-direction (B_y) is so small that B_x and H_x are mainly focused. The computational hysteresis loop is presented in Fig. 6.

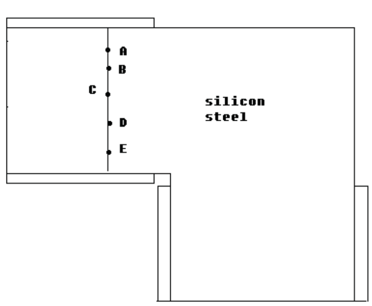

Fig.5 Computational region of the LCM

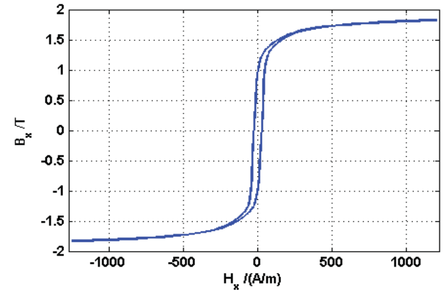

Fig.5 Calculated hysteresis loop

V. CONCLUSIONS

The fixed-point technique is combined with the HBFEM to calculate the time-periodic magnetic field. The hysteresis loops are simulated by the hysteresis model based on the consuming function. The computational results are compared with the measured ones to verify the validity of the proposed method. The fixed-point harmonic-balanced finite element method is an efficient method to compute the magnetic field in harmonic domain.

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