

Unidirectionality of time induced by T violation

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Abstract. The physical nature of time has been an enigma for centuries. Despite the progress made in linking various arrows of time together, the underlying reason for the observed direction of time has remained a deep mystery. However, an increasing number of experiments are confirming a violation of time reversal invariance (T) in meson decay processes. T violation represents a fundamental break in the symmetry between the past and future. I show here that such processes induce destructive interference between different paths that the universe can take through time. The interference eliminates all paths except for two that represent continuously forwards and continuously backwards time evolution. These results illustrate that there are previously-unknown large-scale physical effects arising from the T violation processes in meson decay. Moreover these effects offer a solution to the age-old problem of the cause of the unidirectionality of time.

1. Introduction

Nature has been shown to violate the time reversal (T) and charge-parity conjugation (CP) invariances [1]. For T violating processes

$$H \neq THT^{-1}$$

where H is the Hamiltonian for one specific direction of time, THT^{-1} is the version of the Hamiltonian in the reversed direction of time and T is Wigner's time inversion operation [2]. In principle, one can write down the Schrödinger equation for T violating processes provided the direction of time evolution and the corresponding version of the Hamiltonian are known. Then by applying the time reversal operation we obtain the Schrödinger equation involving the THT^{-1} version of the Hamiltonian for evolution in the opposite direction of time. But we do not have a dynamical equation of motion for the situation where the *direction of time evolution cannot be specified* and for which there is no argument for favouring one version of the Hamiltonian over the other. This problem becomes critical when we attempt to describe the universe as a closed system because being closed precludes any external clock-like device for use as a reference for the direction of time. There is no argument for favouring one direction of time over the other, and so there is no reason to favour H over THT^{-1} . This means that if H is included in the dynamical equation of motion of the universe then so must THT^{-1} . Current quantum theory is unable to provide a description in this case because of its inability to deal with two different Hamiltonians for a single system in one dynamical equation. There is, therefore, no satisfactory quantum formalism for describing a universe that exhibits T violation processes. The resolution of this problem calls for *a major shift in the way we think about time and dynamical equations of motion.*

This hurdle has thwarted previous attempts to understand the implications of T violation for the nature of time [3, 4, 5]. I will show here, however, that it can be overcome by using Feynman’s sum over histories method [6] to construct the set of all possible paths that the universe can take through time. This set includes paths that zigzag forwards and backwards through time, for which the universe evolves according to H in one direction of time and THT^{-1} for the reverse direction. T violation induces destructive interference that restricts the possible paths through time the universe can take. The interference eliminates paths which meander forwards and backwards in time and leaves only two main paths corresponding to evolution continuously forwards and evolution continuously backwards. This analysis resolves the problem of modelling the dynamics of T violation processes in the absence of a preferred direction of time by incorporating two Hamiltonians, one for forwards and one for backwards evolution in a single dynamical equation. Remarkably it demonstrates that there are *large-scale* physical effects of T violations and it offers a physical explanation of the unidirectionality of time. It stands to overhaul conventional notions of time.

In what follows I will describe how the *paths through time* are constructed, how T violation processes induce destructive *interference between paths* and how this gives rise to the *unidirectional nature of time*. Fuller details can be found in Ref. [7]

2. Paths through time

Let the universe be modelled as a closed system. This immediately excludes any external clock devices that could be used as references for the direction of time. As a consequence, the evolution of the universe needs to be unbiased with respect to the direction of time. For convenience, let the two directions of time be labelled as “forward” and “backward” although these are arbitrary labels. We begin with the universe in a state $|\psi_0\rangle$ which we shall call the “origin state” without reference to the direction of time. Let the evolution of the universe in the forwards direction over the time interval τ be

$$|\psi_F(\tau)\rangle = U_F(\tau)|\psi_0\rangle$$

where $U_F(\tau) = \exp(-i\tau H_F)$ is the forwards time evolution operator, and H_F is the Hamiltonian for forwards time evolution. (Throughout this paper we shall use units in which $\hbar = 1$.) Then the backwards time evolution operator is given by

$$U_B(\tau) = TU_F(\tau)T^{-1} = \exp(i\tau H_B)$$

where $H_B = TH_F T^{-1}$ is the Hamiltonian for backwards time evolution. This gives the state of the universe after backwards evolution over a time interval of the same magnitude as

$$|\psi_B(\tau)\rangle = U_B(\tau)|\psi_0\rangle .$$

The matrix elements $\langle\phi|U_F(\tau)|\psi_0\rangle$ and $\langle\phi|U_B(\tau)|\psi_0\rangle$ represent the probability amplitudes for the universe in state $|\psi_0\rangle$ to evolve over the time interval τ to $|\phi\rangle$ via two paths in time corresponding to the forwards and backwards directions, respectively. Given that we have no basis for favouring one path over the other, we follow Feynman [6] and attribute an equal statistical weighting to each. Thus, *the total probability amplitude for the universe to evolve from one given state to another is proportional to the sum of the probability amplitudes for all possible paths through time between the two states*. In the current situation we have only two possible paths and so the total amplitude is proportional to

$$\langle\phi|U_F(\tau)|\psi_0\rangle + \langle\phi|U_B(\tau)|\psi_0\rangle = \langle\phi|[U_F(\tau) + U_B(\tau)]|\psi_0\rangle . \quad (1)$$

This result holds for all states $|\phi\rangle$ of the universe and so the evolution of $|\psi_0\rangle$ via both paths can be written as

$$|\Psi(\tau)\rangle = [U_F(\tau) + U_B(\tau)]|\psi_0\rangle \quad (2)$$

which is called the *symmetric time evolution* of the universe. To avoid unnecessary detail un-normalised vectors will be used to represent states of the universe. It follows that the symmetric time evolution of the universe in state $|\Psi(\tau)\rangle$ over an additional time interval of τ is given by

$$|\Psi(2\tau)\rangle = [U_F(\tau) + U_B(\tau)] |\Psi(\tau)\rangle = [U_F(\tau) + U_B(\tau)]^2 |\psi_0\rangle .$$

Repeating this for N such time intervals yields the general expression for symmetric time evolution

$$|\Psi(N\tau)\rangle = [U_F(\tau) + U_B(\tau)]^N |\psi_0\rangle . \quad (3)$$

Figure 1 gives a graphical interpretation of this result in terms of a binary tree. It is useful to write the expansion of the product on the right side of Eq. (3) as

$$|\Psi(N\tau)\rangle = \sum_{n=0}^N S_{N-n,n} |\psi_0\rangle \quad (4)$$

where $S_{m,n}$ represents a sum containing $\binom{n+m}{n}$ different terms each comprising n factors of $U_F(\tau)$ and m factors of $U_B(\tau)$, and where

$$\binom{k}{j} = \frac{k!}{(k-j)!j!}$$

is the binomial coefficient. A more manageable expression for $S_{m,n}$, with all the U_B factors to the left of the U_F factors, is [7]

$$S_{m,n} = U_B(m\tau)U_F(n\tau) \sum_{v=0}^m \cdots \sum_{\ell=0}^s \sum_{k=0}^{\ell} \exp \left[(v + \cdots + \ell + k)\tau^2 [H_F, H_B] \right] + \exp[O(\tau^3)] \quad (5)$$

where $[A, B]$ is the commutator of A and B , and there are n summations on the right side.

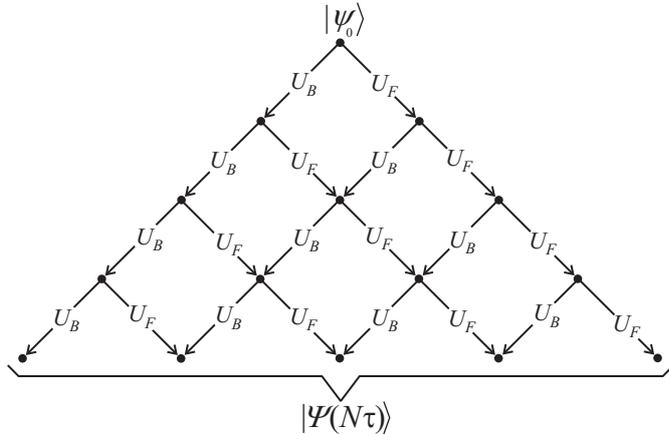


Figure 1. Binary tree representation of the generation of the state $|\Psi(N\tau)\rangle$ from the origin state $|\psi_0\rangle$ according to Eq. (3) with $N = 4$. States are represented as nodes (solid discs) and unitary evolution by links (arrows) between them. The root node (at the top) represents the origin state $|\psi_0\rangle$ and the leaf nodes (on the bottom row) represent components of the state $|\Psi(4\tau)\rangle$.

The expression $\langle\phi|S_{N-n,n}|\psi_0\rangle$ represents the evolution of the universe from $|\psi_0\rangle$ to $|\phi\rangle$ over a set of $\binom{N}{n}$ paths through time, where each path comprises a total of n steps in the forwards direction and $N - n$ steps in the backwards direction. The set of paths includes all possible orderings of the forwards and backwards steps. This set of paths is the focus of the remaining analysis.

There is an important point to be made about the size of the time steps τ . To explore the consequences of the limit $\tau \rightarrow 0$ of infinitely-small time steps for a fixed total time t_{tot} , we set $\tau = t_{\text{tot}}/N$ and consider Eq. (3) for increasing values of N . We can write

$$2^{-N}[U_F(\tau) + U_B(\tau)]^N = \left\{ \exp[-i\frac{1}{2}(H_F - H_B)\tau] + \mathcal{O}(\tau^2) \right\}^N$$

which becomes $\exp[-i\frac{1}{2}(H_F - H_B)t_{\text{tot}}]$ as $N \rightarrow \infty$. In this limit, the universe evolves according to the Hamiltonian $\frac{1}{2}(H_F - H_B)$. This means that isolated subsystems of the universe that obey T invariance, such as a conventional model of a clock, would not exhibit any evolution. As clocks are taken to measure time intervals, the universe as a whole would not exhibit dynamics in the conventional sense in this $\tau \rightarrow 0$ limit. The lack of dynamics is an unphysical result if our system is to model the visible universe. This suggests that the time interval τ should be a small non-zero number for a universe-like system. Accordingly, in the following we set the value of τ to be a small time interval, for which we pick the Planck time for definiteness, i.e. $\tau \approx 5 \times 10^{-44} s$. The need for a finite time interval should not be alarming. Indeed the discreteness of space at the Planck scale is regarded as one of the triumphs of Loop Quantum Gravity [8, 9, 10].

3. Interference between paths

Being anti-Hermitian, the commutator $[H_F, H_B]$ has imaginary eigenvalues. Its presence in the nested summations in Eq. (5) gives rise to interference terms in the basis of eigenstates of $[H_F, H_B]$ of the form [7]

$$\sum_{v=0}^m \cdots \sum_{\ell=0}^s \sum_{k=0}^{\ell} \exp \left[-(v + \cdots + \ell + k)\tau^2 i\lambda \right]$$

where λ is an eigenvalue of $i[H_F, H_B]$. This leads to interference between the set of paths represented by $\langle \phi | S_{N-n,n} | \psi_0 \rangle$.

An estimate of typical values for λ can be made using neutral K meson (kaon) evolution as a prototypical T-violating process. The phenomenological model of Lee and Wolfenstein [11] and the empirical values of Yao *et al.* [12] gives a spectrum of values of λ which has a mean of zero and standard deviation of $\lambda_{\text{SD}} \approx \sqrt{f} 10^{57} s^{-2}$. Here f is a fraction given by dividing the number of particles associated with T-violating processes by the total number of particles in the universe (which is assumed to be 10^{80}).

It is shown in Ref. [7] that destructive interference leads to Eq. (4) being replaced with $|\Psi(N\tau)\rangle = [\sum_{n \approx 0} S_{N-n,n} + \sum_{n \approx N} S_{N-n,n}] |\psi_0\rangle$ for total times $N\tau > 10^{-17} s$. Ignoring terms of order τ gives

$$\begin{aligned} |\Psi(N\tau)\rangle &= \left\{ [U_B(\tau)]^N + [U_F(\tau)]^N \right\} |\psi_0\rangle \\ &= [U_B(N\tau) + U_F(N\tau)] |\psi_0\rangle, \end{aligned} \tag{6}$$

and then on writing the time as $t = N\tau$ we arrive at a key result, the *bievolution equation of motion*:

$$|\Psi(t)\rangle = [U_B(t) + U_F(t)] |\psi_0\rangle, \tag{7}$$

which is illustrated in Fig. 2. The term ‘‘bievolution’’ here refers to the dual evolution generated by the two different Hamiltonians. The approximations made in deriving this equation become exact in the limit $t \gg f^{-1/2} 10^{-13} s$.

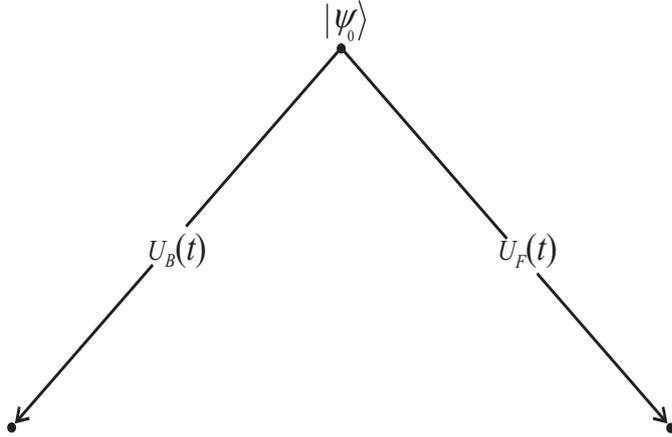


Figure 2. Binary tree representation in the presence of sufficient T violation processes. Destructive interference has eliminated all paths except for two corresponding to either continuous evolution in the forwards or continuous evolution in the backwards directions of time.

4. Unidirectional nature of time

Accelerator experiments are able to discern which Hamiltonian H_F or H_B is responsible in neutral K and B meson decay experiments. Such experiments yield evidence of just *one* of the Hamiltonians H_F or H_B . Our experience, therefore, is consistent with just one of the terms on the right side of the bievolution equation Eq. (7). The opposite signs in the exponents of the definitions of U_F and U_B imply that they represent time evolution in opposing directions of time. We conclude that in Eq. (7) the expressions $U_F(t)|\psi\rangle$ and $U_B(t)|\psi\rangle$ represent the universe evolving in opposite directions of time and in each case the evolution leaves evidence of the associated Hamiltonian in the state of the universe.

To expand on this point a little, the bievolution equation Eq. (7) describes our observation of the universe as evolving according to either

$$|\Psi(t)\rangle = U_F(t)|\psi_0\rangle$$

or

$$|\Psi(t)\rangle = U_B(t)|\psi_0\rangle$$

depending on whether we observe the version H_F or H_B , respectively, of the corresponding Hamiltonian in meson decay experiments. Each of these equations is the solution of the conventional Schrödinger equation for the corresponding version of the Hamiltonian and its associated direction of time. This demonstrates how the *unidirectional nature of time emerges phenomenologically from our physical observations*. It also demonstrates the *consistency between the bievolution equation and the conventional quantum theory*.

5. New standing of the matter-antimatter arrow of time

These results significantly elevate the importance of the matter-antimatter arrow in the study of the nature of time. The underlying T violation is based on the empirical evidence that different versions of the Hamiltonian operate for different directions of time evolution. Moreover, as discussed in the previous section, each direction of time can be uniquely identified by physical evidence left by the Hamiltonian. This is quite different to other arrows of time. In particular, in a universe in which all physical laws are T invariant, the big bang expansion occurs equally in both directions of time and, by symmetry, the entropy of the universe also increases equally in both directions of time. The thermodynamic arrow, therefore, does not distinguish between the different directions of time evolution because entropy increases in the direction of time evolution *regardless of the direction*. This suggests that T violation and the matter-antimatter arrow play a far more significant role in determining the nature of time than previously imagined [3, 4, 5].

6. Absence of T violation processes

In many cosmological models, the early universe would not have had significant numbers of T violation processes [13]. In such models, T violation processes become significant only towards the end or even after the inflation period. It is instructive, therefore, to compare the above analysis with a universe that obeys T invariance. As $H_F = H_B = H$ in this case we can write $U_F + U_B = 2 \cos(H\tau)$ and so

$$|\Psi(N\tau)\rangle = 2^N \cos^N(H\tau)|\psi_0\rangle . \quad (8)$$

According to the power method of determining the largest eigenvalue of a matrix, as N increases the right side approaches the eigenstate of $\cos^N(H\tau)$ that has the largest absolute eigenvalue, which in this case is the eigenstate of H with zero eigenvalue. Hence in the limit as $N \rightarrow \infty$

$$H|\Psi(N\tau)\rangle = 0 . \quad (9)$$

According to Eq. (3), $|\Psi(N\tau)\rangle$ is a superposition of states each of which represents a net time ranging from $-N\tau$ to $+N\tau$. It therefore represents a *history* of the universe. The zero energy eigenstate implies that the positive energy of matter fields is balanced by the negative energy of the gravitational potential energy [14, 15, 16].

Eq. (9) corresponds to the Hamiltonian constraint of the Wheeler-DeWitt equation [17, 18]. The solution of the Wheeler-DeWitt equation represents the entire history of the universe with no bias towards either direction of time. Time symmetric evolution is unbiased in the same way in the sense that it represents evolution without a fixed direction of time and the state in Eq. (9) represent the history of the universe before T violation processes were significant. This suggests that there is a close connection between the Wheeler-DeWitt equation and time symmetric evolution.

7. Conclusion

Our general understanding of T violating processes has previously been hindered by the lack of a formalism for accommodating their time asymmetry in a single dynamical law. We have removed this obstacle by deriving a general method for incorporating both versions of the Hamiltonian for a T violating process, one for forwards and the other for backwards evolution, in a single dynamical equation of motion. Moreover we have shown that T violation in meson decay provides, at least in principle, a physical mechanism for the phenomenological unidirectionality of time. This analysis, therefore, offers a new picture of the physical nature of time itself.

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