

# Effect of Sizes and Positions of Web Openings on Strut-and-Tie Models of Deep Beams

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## Abstract

The trend towards high-rise concrete structures has placed an increased importance on the design of reinforced concrete deep beams. Deep beams also have useful applications in foundations and offshore gravity type structures. Numerous design models for deep beams have been suggested, however even the latest design manuals still offer little guidance for the design of deep beams in particular when complexities exist in the beams like web openings. A method commonly suggested for the design of deep beams with openings is the strut-and-tie model which in the present study is developed through topology optimisation of continuum structures. During the optimisation process, both the stress and displacement constraints are satisfied and the performance of progressive topologies is maximised. In all, eight concrete deep beams with web openings are optimised and compared in an attempt to examine the effect of sizes and positions of web openings on the strut-and-tie models. Through comparison with published experimental crack patterns, the study confirms the effectiveness of the optimisation technique in developing the strut-and-tie model representing the load transfer

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mechanism in concrete deep beams under ultimate load, which in turn would assist detailed design of such structural elements.

**Key words:** Concrete deep beams, strut-and-tie model, web openings, topology optimisation, stress and displacement constraints

## 1. Introduction

Reinforced concrete deep beams are of considerable interest in structural engineering practice. Deep beams have useful applications in tall buildings, foundations and offshore gravity type structures acting as transferring and supporting elements. A deep beam is a type of non-flexural member having a depth comparable to its span length. It is generally defined as a member that has a span ( $L$ ) to depth ( $D$ ) ratio of less than 5. Deep beams cannot be designed as conventional beams using well accepted design methods, as the underlying assumptions in beam theory (i.e. plane sections remain plane after bending) are not valid when  $L/D < 5$ . Over the past two decades, as the need for deep beams continued to grow throughout the construction industry, numerous design models for deep beams were suggested. However even the latest design manuals, such as the ACI code (2002), Eurocode (1992), the British (1997) and Australian Standards (2002) still offer little guidance for the design of deep beams in particular when complexities exist in the beams like web openings.

In various forms of construction, openings in the web area of deep beams are frequently provided for accessibility and to allow essential services to pass through the beam. The presence of openings induces geometric discontinuity into the deep beam which only enhances the complexity of the nonlinear stress distribution along the depth of the beam. Numerous investigations on various attributes of deep beams have been conducted by Rogowsky *et al.* (1986), Kong (1990), Tan *et al.* (1995, 1997a,b,c), Ashour (1997), Foster and Gilbert (1998), Tan *et al.* (2003), to name a few. However only limited research has been conducted dealing with deep beams with openings (Kong and Sharp 1977; Kong *et al.* 1978; Mansur and Alwis 1984; Almeida and Pinto 1999; Maxwell and Breen 2000; Ashour and Rishi 2000; Tan *et al.* 2003).

A method commonly suggested for the design of deep beams is the strut-and-tie model generalised from the truss analogy. The rationale of such model is that the considerably higher shear strength in deep beams than shallow beams is a result of the internal arching which is the process that transfers the load directly to a support through the development of concrete compression struts while steel reinforcement acts as tension ties (Foster and Malik 2002). The strut-and-tie model can be used to represent the actual load transfer mechanism in a structural concrete member under ultimate load. For deep beams with web openings, it is well recognised that the actual flow of the force around the openings can be modelled and designed for using the strut-and-tie model (Tan *et al.* 2003), which radically differs from the simple model for deep beams without openings. However many existing methods used in deriving the strut-and-tie model can be laborious and complex, because the compression strut from the loaded area generally separates and tracks around the opening before joining together again at the supports. This is especially true when predicting the correct strut-and-tie model for members with complex loading, support and geometric conditions (Guan *et al.* 2003; Guan and Parsons 2003). Hence it would be advantageous if a simple and effective method of generating the strut-and-tie model can be derived. In the present study, the strut-and-tie models of deep beams are developed through topology optimisation of continuum structures, where inefficient material is gradually removed from an over-designed area. During the optimisation process, both the stress and displacement constraints are satisfied and the performance of progressive topologies is maximised.

In all, eight concrete deep beams with web openings are optimised and compared in an attempt to examine the effect of sizes and positions of web openings on the strut-and-tie models. The optimal strut-and-tie models achieved compare favourably with published

experimental crack patterns, confirming the effectiveness of the optimisation technique in developing the load transfer mechanism in concrete deep beams under ultimate load.

## **2. Methodology**

### *2.1. Objectives*

In the present study, the development of the strut-and-tie model of deep beams with openings is transformed to a topology optimisation problem of continuum structures. In the design of a reinforced concrete member, the location and the amount of reinforcement are unknowns which need to be determined. The designer needs to establish the strut-and-tie layout in a structural concrete member in order to reinforce it. As a result, a plain concrete member is analysed assuming homogeneous continuum behaviour and the nonlinear behaviour of reinforced concrete is not considered. A linear elastic behaviour of cracked concrete is also assumed, and the progressive cracking of concrete is characterised by gradually eliminating concrete from the structural member, eventually leading to a fully cracked stage at the optimum (Liang *et al.* 2000). This is further supported by the fact that the strut-and-tie model representing a truss analogy normally assumes that after concrete cracking the behaviour of a reinforced concrete beam becomes analogous to that of a truss.

The objective of this topology optimisation is to maximise the performance of an initial continuum design domain in terms of material efficiency and overall stiffness. This is achieved by gradually eliminating, from a discretised concrete member, a small number of elements with lowest von Mises stress and lowest displacement sensitivity number.

## 2.2. Topology optimisation with stress and displacement constraints

A finite element analysis of a deep beam occupying an over-design area is first performed, based on which the von Mises stress of each element  $\sigma_{vM}^e$  and the maximum von Mises stress of the structure  $\sigma_{vM,\max}$  are evaluated. A deletion criterion is calculated by multiplying the rejection rate  $RR_i$  and  $\sigma_{vM,\max}$ . This criterion is the stress at which all elements with a lower stress are deemed insignificant. As such, an element is identified if its  $\sigma_{vM}^e$  is less than the deletion criterion (Xie and Steven 1997) or

$$\sigma_{vM}^e \leq RR_i \times \sigma_{vM,\max} \quad (1)$$

A small value of  $RR_i$  (0.1-0.2%) is used in the present study to ensure that only a small number of lowly stressed elements  $N_s$  are identified each time, where  $i$  indicates the iteration number.

For serviceability concerns, the displacement of a structure has to be limited to a prescribed value. This is to ensure that whilst the lowly stressed (redundant) material is removed from the structure, the remaining part of the structure is still stiff enough and its maximum deflection is within the satisfactory limit. When the specified displacement limit is reached, the optimisation procedure will be terminated. As such, a displacement sensitivity number is needed to perform the displacement control.

In the finite element analysis, the stiffness equation of a structure is

$$\mathbf{K} \cdot \mathbf{u} = \mathbf{P} \quad (2)$$

where  $\mathbf{K}$  is the global stiffness matrix of the structure and,  $\mathbf{u}$  and  $\mathbf{P}$  are, respectively, the global nodal displacement and nodal load vectors. Assuming that the element  $i$  ( $i = 1, N$ ) is to be removed from the structure where  $N$  is the total number of elements in the design domain. This induces a change in the stiffness matrix,  $\Delta\mathbf{K}_i$ , as well as a change in the displacement vector  $\Delta\mathbf{u}$ . The stiffness equation then becomes,

$$(\mathbf{K} + \Delta\mathbf{K}_i) \cdot (\mathbf{u} + \Delta\mathbf{u}) = \mathbf{P} \quad (3)$$

Note that the element removal is assumed to have little effect on the load vector  $\mathbf{P}$  (Xie and Steven 1997). Substituting Eqn 2 into 3 and ignoring the higher order term lead to

$$\Delta\mathbf{u} = -\mathbf{K}^{-1} \cdot \Delta\mathbf{K}_i \cdot \mathbf{u} \quad (4)$$

If the displacement at a specific location say the  $j$ th displacement component  $u_j$  is to be limited to a prescribed value,  $u_j^*$ , we have  $|u_j| \leq u_j^*$ , where  $u_j$  can be obtained by introducing a virtual unit load vector  $\mathbf{F}_j$ , in which all its components are zero except that the  $j$ th component is equal to unity. Multiplying Eqn 4 by  $\mathbf{F}_j^T$  results in a change in the displacement  $\Delta u_j$ . Or,

$$\Delta u_j = \mathbf{F}_j^T \cdot \Delta\mathbf{u} = -\mathbf{u}_j^T \cdot \Delta\mathbf{K}_i \cdot \mathbf{u} \quad (5)$$

For the  $i$ th element, the change in displacement is

$$\Delta u_j = \mathbf{u}_{ij}^T \cdot \mathbf{K}_i \cdot \mathbf{u}_i \quad (6)$$

where  $\mathbf{u}_i$  and  $\mathbf{u}_{ij}$  are respectively the displacement vectors of the  $i$ th element due to the real load  $\mathbf{P}$  and the virtual load  $\mathbf{F}_j$ ;  $\mathbf{K}_i$  is the stiffness matrix of the removed  $i$ th element and is equal but opposite to  $\Delta \mathbf{K}_i$ .

A displacement sensitivity number  $\alpha_{d,i}$  indicating the change in the displacement component  $u_j$  due to the removal of the  $i$ th element can then be defined as

$$\alpha_{d,i} = |\alpha_{d,ij}| = |\mathbf{u}_{ij}^T \cdot \mathbf{K}_i \cdot \mathbf{u}_i| \quad (7)$$

where  $\alpha_{d,ij}$  can be positive or negative.

In more general cases when a structure is subjected to multiple load cases  $\mathbf{P}_k$  ( $k = 1, L$ ) with multiple prescribed displacement values  $u_{jk}^*$  ( $j = 1, M$ ), the sensitivity number can be derived as

$$\alpha_{d,i} = \sum_{k=1}^L \sum_{j=1}^M \lambda_{jk} \cdot |\alpha_{d,ijk}| = \sum_{k=1}^L \sum_{j=1}^M \lambda_{jk} \cdot |\mathbf{u}_{ij}^T \cdot \mathbf{K}_i \cdot \mathbf{u}_{ik}| \quad (i = 1, N) \quad (8)$$

in which  $\mathbf{u}_{ik}$  is the displacement vector of the  $i$ th element due to load case  $\mathbf{P}_k$ ;  $L$  and  $M$  are respectively the total number of load cases and that of displacement constraints. In Eqn 8, the weighting parameter  $\lambda_{jk} = |u_{jk}^*| / |u_{jk}^*|$  indicates the contribution of the  $j$ th displacement constraint under the  $k$ th load case.

In the optimisation process, the sensitivity number  $\alpha_{d,i}$  is evaluated for a total number of  $N_s$  elements that satisfy the stress condition as given in Eqn 1. To minimize the change in displacement, a number of elements  $N_d$  with the lowest  $\alpha_{d,i}$  are removed. In the present study,  $N_d = \text{the lesser of } N_s \text{ and } 10$ , has been found to produce satisfactory results.

The finite element analysis followed by systematic removal of lowly stressed elements forms an optimisation cycle where  $RR_i$  remains constant. Such cycle or iteration is continued until no more elements are removed. The following iteration commences with  $RR_{i+1}$ , which equals the sum of  $RR_i$  and  $ER$ , the evolution rate. The repeated cycle of optimisation process continues until a desired topology is obtained. In the present study,  $ER$  is taken as 0.1%.

### 2.3. Selection of optimal topology – strut-and-tie model

As the optimisation cycle progresses, the resulting topology improves with increase in iteration. To identify the final topology which can be translated to the optimal strut-and-tie model, a performance index  $PI_d$ , which is regarded as the objective function, can be derived based on the scaling design concept (Liang *et al.* 2000) where the actual design variable such as the element thickness is scaled with respect to the design constraint. The topology optimisation of a continuum structure can be presented in the following form:

$$\begin{aligned}
 & \text{minimize} && V = \sum_{e=1}^N V_e \\
 & \text{subject to} && |u_{jk}| \leq u_{jk}^* \quad (j = 1, M; k = 1, L)
 \end{aligned} \tag{9}$$

where  $V$  and  $V_e$  are respectively the volume of the total design domain and that of the element  $e$ . Note in the present study that minimising the volume is equivalent to minimising the weight, because a single type of material, i.e. plain concrete, is assumed to occupy the entire design domain. For linear elastic plane stress problems, the structural stiffness matrix is a linear function of the design variable such as the thickness or the volume of the structure. To produce the most efficient topology with minimum weight, the volume of the design domain can be scaled with respect to the displacement constraint, with the aim that the  $j$ th displacement component under the  $k$ th load case ( $u_{jk}$ ) reaches the prescribed displacement value  $u_{jk}^*$ . As a result, the relative volume of the initial (original) design domain,  $V_o'$ , can be expressed as

$$V_o' = V_o \cdot \left( |u_{jk}|_o / u_{jk}^* \right) \quad (10)$$

in which  $V_o$  and  $|u_{jk}|_o$  are respectively the volume of, and the  $|u_{jk}|$ , in the original design domain. In an iterative optimisation process, the relative volume of the current design (at the  $i$ th iteration),  $V_i'$ , can also be scaled in a similar manner. Or,

$$V_i' = V_i \cdot \left( |u_{jk}|_i / u_{jk}^* \right) \quad (11)$$

where  $V_i$  and  $|u_{jk}|_i$  are respectively the volume of, and the  $|u_{jk}|$ , in the current design domain.

At the  $i$ th iteration, the performance index  $PI_d$ , a function of the displacement components, can then be determined as

$$PI_d = \frac{V'_o}{V'_i} = \frac{|u_{jk}|_o \cdot V_o}{|u_{jk}|_i \cdot V_i} \quad (12)$$

During the optimisation process,  $PI_d$  measures the efficiency of the progressive topologies. As the optimisation procedure continues, the number of iterations increases while  $PI_d$  is maximised, until a certain point where the efficiency or performance of the topology declines. The maximum value of  $PI_d$  corresponds to the most efficient topology, which can be translated to the optimal strut-and-tie model.

### 3. Numerical investigation

#### 3.1. Details of deep beams

The effect of sizes and positions of web openings on the strut-and-tie models of deep beams is examined herein. In all, eight simply supported deep beams representing a vast range of deep beams with varying sizes and positions of web openings are selected from Kong *et al.*'s test (1978). Each beam had an over-all depth  $D$  of 750 mm and a width  $b$  of 100 mm. The span length  $L$  is 1125 mm making the  $L/D$  ratio of 1.5. The clear shear span  $x$  is 225 mm giving the  $x/D$  ratio of 0.3. Figure 1 shows the configuration of the symmetrical half of the deep beam with indicative size and position of the web openings. The designation and details of the deep beams are given in Table 1. All the beams are subjected to two concentrated loads applied symmetrically about the centre line (see Figure 1). The ultimate load for each beam is also summarised in Table 1. The first deep beam (O-0.3/0) had no web opening. For the remaining seven beams (O-0.3/1 to O-0.3/8), the size of an opening is given by  $(a_1x) \times (a_2D)$

where the height factor  $a_2$  was kept constant at 0.2 but the breadth factor  $a_1$  varied between 0.3 and 1.2.

All the deep beams are optimised under both stress and displacement constraints. By virtue of symmetry, only half of each model is analysed. Note in the laboratory test that, both the loads and support reactions were applied through embedded steel bearing blocks (100×100×30 mm) to the concrete. This is reflected in the numerical model by applying a uniform load and the support restraint respectively over a 100 mm length. The displacement limit is imposed under the point load. The rejection ratio  $RR_i$  is taken as 0.2% and 0.15% at the initial stages, then reduces to 0.1% at the later stages. Also a local refinement around the opening is made for some of the beams to minimise extensive checkerboard patterns.

### 3.2. *Optimal strut-and-tie models*

The progressive topologies are recorded for each beam at typical iterations and they are presented in Figures 2 to 9 for beams O-0.3/0 to O-0.3/8, respectively. Also included in each of these figures is the final topology, which is determined when the performance index  $PI_d$  reaches the maximum value. The optimisation history showing the  $PI_d$  and the volume reduction ( $V_i/V_0$ ) versus iteration is also presented in the figures, where  $PI_d$  increases from unity and the  $V_i/V_0$  decreases from 100% correspondingly. The increase in  $PI_d$  indicates the improvement of the progressive topologies while inefficient material is gradually removed from an initially over-designed area. It can also be seen from Figures 2 to 9 that the optimal strut-and-tie models are translated from the final topologies in which the solid lines represent the tension ties whereas the dash lines denote the compression struts.

In general, when there is no web opening on the load path, the compression transfer takes the shortest route which is the natural load path from the loaded area to the support, as indicated in Figure 2(e) for beam O-0.3/0. For the remaining seven beams where the opening falls in the compression transfer path, the natural load path in the strut-and-tie model is interrupted, as indicated in Figures 3(e) to 9(e). The compression struts pass the opening on the left- and right-hand sides, thus introducing tension ties that connect the compression zones above and below the openings. For all the beams under two concentrated loads, an overall trapezoidal-shaped strut-and-tie model forms. The left and right (re-routed) compression transfers form the sides of the trapezium, together with the horizontal strut between the loaded points as the top and the horizontal tension tie between the supports as the base.

### *3.3. Comparison between strut-and-models and experimental crack patterns*

The optimal strut-and-tie models and the experimental crack patterns are compared in Table 2 where similarities are evident between the experimental observation and the computer-generated results. It is generally accepted that the first cracks appear at the top and bottom corners of the opening provided that the opening is in the compression transfer zone. As the load increases these cracks propagate from the top of the opening towards the loaded area and from the bottom of the opening towards the closest support. In addition to this diagonal or shear cracking, flexural cracking can occur at the bottom of the beams between the supports. However flexural cracks are generally considered to be minor in relation to the crack space and crack width at failure, whereas shear related cracks are more predominant. Therefore it is safe to assume that the top and bottom of the opening are under high-tensile stresses where the first major cracks appear. It should also be noted that the bottom of the beam also experiences reasonably high-tensile forces. Therefore it is logical to place steel reinforcement in these

sections to counteract the low-tension capacity of concrete. As the strut-and-tie model is used to determine reinforcement layout it would be expected that tension ties would form in these places, which is generally what happens for all the beams, as seen in Table 2. Thus it can be concluded that the computer generated strut-and-tie model does accurately predict the placement of the steel reinforcement.

#### *3.4. Effect of sizes and positions of web openings*

A prediction method for the ultimate shear strength of a deep beam with web openings has been suggested by Kong and Sharp (1977) using the structural idealisation of Figure 10. It has been found, in general, the applied load is transmitted to the support mainly by a lower load path ABC and secondarily by an upper load path AEC. Such structural idealisation suggests that the effectiveness of the lower path should increase with the angle  $\phi$ , whilst that of the upper path should increase with  $\phi'$ .

The comparisons of the strut-and-tie models for the five beams (O-0.3/1 to O-0.3/5) with varying sizes of web openings are presented in Table 3. It is noticed that when the size of opening increases up to and more than the clear shear span  $x$ , the bottom tension ties are interrupted by the lower compression transfer. Also the upper and lower compression transfers converge to the support point whereas only the upper transfer extends vertically down to the support when the breadth of the opening is smaller than  $x$ . Considering the structural idealisation, the angle  $\phi$  of the five beams remains unchanged because  $k_1 (=1.0)$  remains constant. Therefore the lower load paths are equally effective for all the five beams. However, with an increase in  $a_1$  ( $=0.3, 0.5, 0.7, 1.0, 1.2$  for O-0.3/1 to O-0.3/5),  $\phi'$  progressively decreases which reduces the effectiveness of the upper path. As a result, there

is a reduction in the ultimate shear strength of the beams, as given Table 1 (from 460 kN for O-0.3/1 down to 200 kN for O-0.3/5).

Table 4 shows the influence on the strut-and-tie model in relation to the different positions of web openings. Position 1 refers to the opening immediately adjacent to the loaded area, and Position 2 is for the opening in line with the support. Four beams are compared herein which makes two pairs. In each pair, two deep beams each contain an opening of identical size. The first pair compares O-0.3/1 and O-0.3/7 and the second pair, O-0.3/2 and O-0.3/8. For Position 1 (O-0.3/1 and O-0.3/2), the lower compression transfer starts with a vertical strut followed by inclined struts directed towards the support. An additional vertical strut also forms between the lower compression transfer and the bottom tension tie. For Position 2 (O-0.3/7 and O-0.3/8), on the other hand, a more simplified strut-and-tie model is generated where the upper and lower compression transfers separate from the load point and unite at the support after detouring around the opening. Moreover, a single horizontal tension tie is generated linking the two supports, which differs from the formation of a number of tension ties in the case of Position 1. For the two deep beams O-0.3/2 and O-0.3/8, due to the identical size of the openings which also located at the mid-depth of the beam, the decrease in angle  $\phi'$  from O-0.3/2 to O-0.3/8 is compensated by the increase in angle  $\phi$ . Therefore the overall effectiveness of the load paths are similar in two beams. This is also seen in Table 1 where the ultimate shear strengths of the two beams (i.e. 390 kN and 380 kN respectively) are similar. On the other hand, the ultimate shear strength of beam O-0.3/1 (460 kN) is 40 kN more than that of O-0.3/7. This is because in beam O-0.3/1 the opening is small in size and is so located as not to interfere significantly with the natural load path (Kong and Sharp 1977). The optimal strut-and-tie model of beam O-0.3/1, as shown in Figure 3 and Table 4, also indicates that the upper load transfer is almost a straight line.

#### 4. Conclusions

In this study, topology optimisation of eight deep beams with web openings is performed under both stress and displacement constraints, by which the system performance (i.e. overall stiffness) of the structure is satisfied. The final topologies, selected on the basis of maximising the performance index, are interpreted into the optimal strut-and-tie models which compare favorably with published experimental crack patterns. The effect of sizes and positions of web openings on the strut-and-tie models of deep beams are examined, which leads to the following findings:

- (a) when the opening is located in the compression transfer zone, the load transfer path is re-routed around the sides of the opening, thus introducing tension ties that connect the compression zones above and below the opening;
- (b) the strut-and-tie model differs when the size of the opening reaches the threshold of the clear shear span;
- (c) the presence of an opening immediately adjacent to the loaded area complicates the strut-and-tie model;
- (d) based on the structural idealisation of a deep beam with openings, the optimal strut-and-tie model can be used to examine the effectiveness of the upper and lower load transfer paths and the associated ultimate shear strength of the deep beams.

In summary, the development of the optimal strut-and-tie models has confirmed the effectiveness of the proposed optimisation technique. The study has provided some insights into various parameters that affect the load transfer mechanisms in concrete deep beams under ultimate load. Considering the proven simplicity and effectiveness of strut-and-tie models for

the design of reinforced concrete deep beams with and without web openings, the current development has further enhanced the design methodology for such structural elements.

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## Notation

The following symbols are used in this paper:

$a_1$	=	breadth factor
$a_2$	=	height factor
$b$	=	width of deep beam
$D$	=	depth of deep beam
$ER$	=	evolution rate
$\mathbf{K}$	=	global stiffness matrix of structure
$L$	=	span length of deep beam
$N$	=	total number of elements in design domain
$N_d$	=	number of elements with lowest $\alpha_{d,i}$
$N_s$	=	total number of elements
$\mathbf{P}$	=	global nodal load vector
$PI_d$	=	performance index
$\mathbf{P}_k$	=	multiple load cases
$RR_i$	=	rejection rate
$\mathbf{u}$	=	global nodal displacement vector
$\mathbf{u}_{ik}$	=	displacement vector of $i$ th element due to load case $\mathbf{P}_k$
$u_j$	=	$j$ th displacement component
$V$	=	volume of total design domain
$V_e$	=	volume of element $e$
$x$	=	clear shear span of deep beam

$W$	=	ultimate load of deep beam
$ u_{jk} _i$	=	$ u_{jk} $ in current design domain
$ u_{jk} _o$	=	$ u_{jk} $ in original design domain
$\Delta u_j$	=	change in displacement
$\alpha_{d,i}$	=	displacement sensitivity number
$u_i$	=	displacement vector of $i$ th element due to real load $P$
$u_{ij}$	=	displacement vector of $i$ th element due to virtual load $F_j$
$\sigma_{vM}^e$	=	element von Mises stress
$\sigma_{vM,max}$	=	maximum von Mises stress of structure
$u_{jk}^*$	=	multiple prescribed displacement values
$u_j^*$	=	prescribed displacement value
$V_i'$	=	relative volume of current design domain
$V_o'$	=	relative volume of original design domain
$K_i$	=	stiffness matrix of removed $i$ th element
$F_j$	=	virtual unit load vector
$V_i$	=	volume of current design domain
$V_o$	=	volume of original design domain
$\lambda_{jk}$	=	weighting parameter
$\Delta K_i$	=	change in stiffness matrix
$\Delta u$	=	change in displacement vector

## **List of Tables**

- Table 1. Details of deep beams
- Table 2. Comparison between strut-and-tie models and experimental crack patterns
- Table 3. Effect of sizes of web openings on strut-and-tie models of deep beams
- Table 4. Effect of positions of web openings on strut-and-tie models of deep beams

## **List of Figures**

- Figure 1 Configuration of the deep beam
- Figure 2 Development of strut-and-tie model for deep beam O-0.3/0
- Figure 3 Development of strut-and-tie model for deep beam O-0.3/1
- Figure 4 Development of strut-and-tie model for deep beam O-0.3/2
- Figure 5 Development of strut-and-tie model for deep beam O-0.3/3
- Figure 6 Development of strut-and-tie model for deep beam O-0.3/4
- Figure 7 Development of strut-and-tie model for deep beam O-0.3/5
- Figure 8 Development of strut-and-tie model for deep beam O-0.3/7
- Figure 9 Development of strut-and-tie model for deep beam O-0.3/8
- Figure 10 Structural idealisation of deep beam with web opening

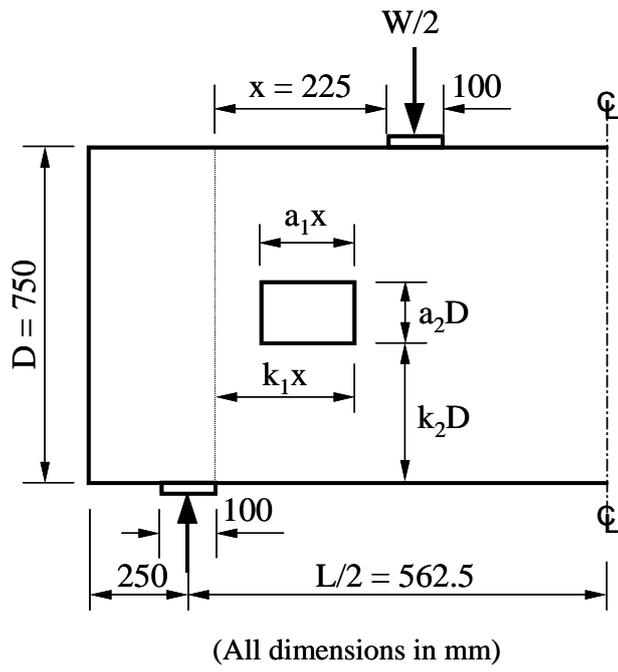


Figure 1 Configuration of the deep beam

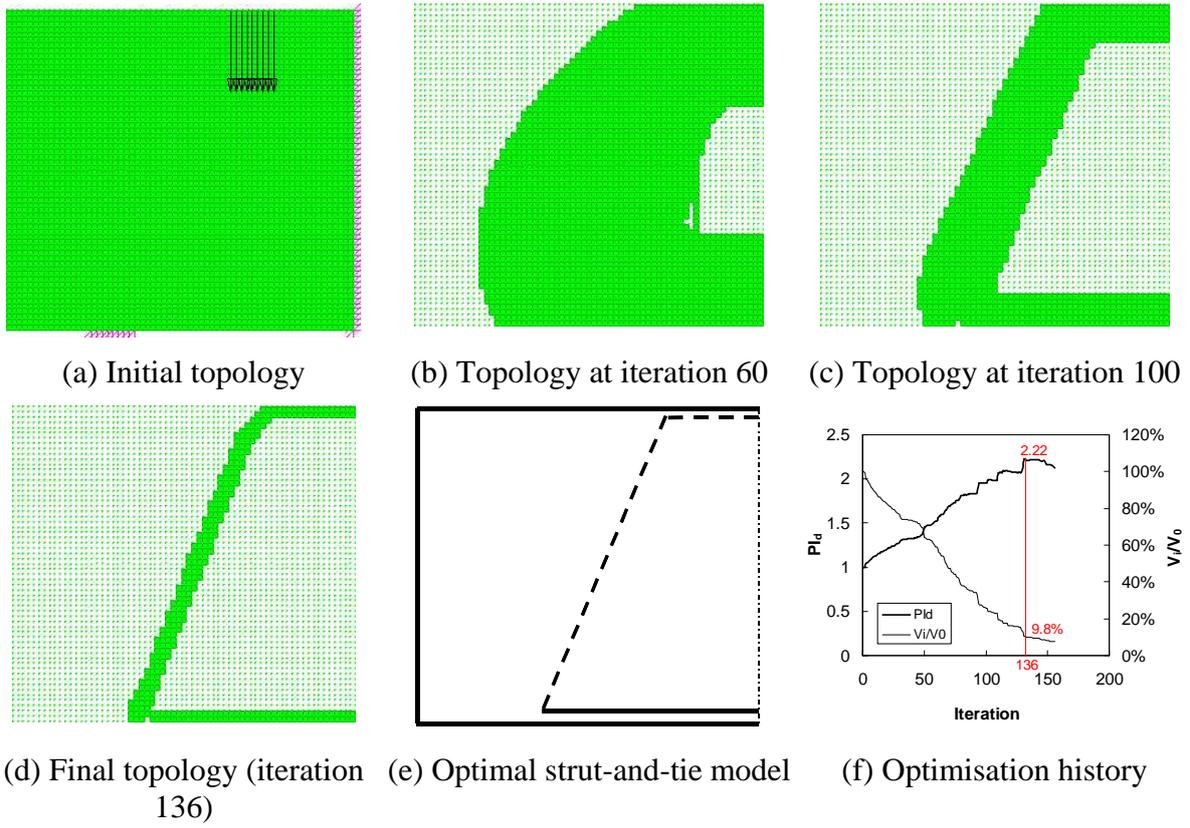


Figure 2 Development of strut-and-tie model for deep beam O-0.3/0

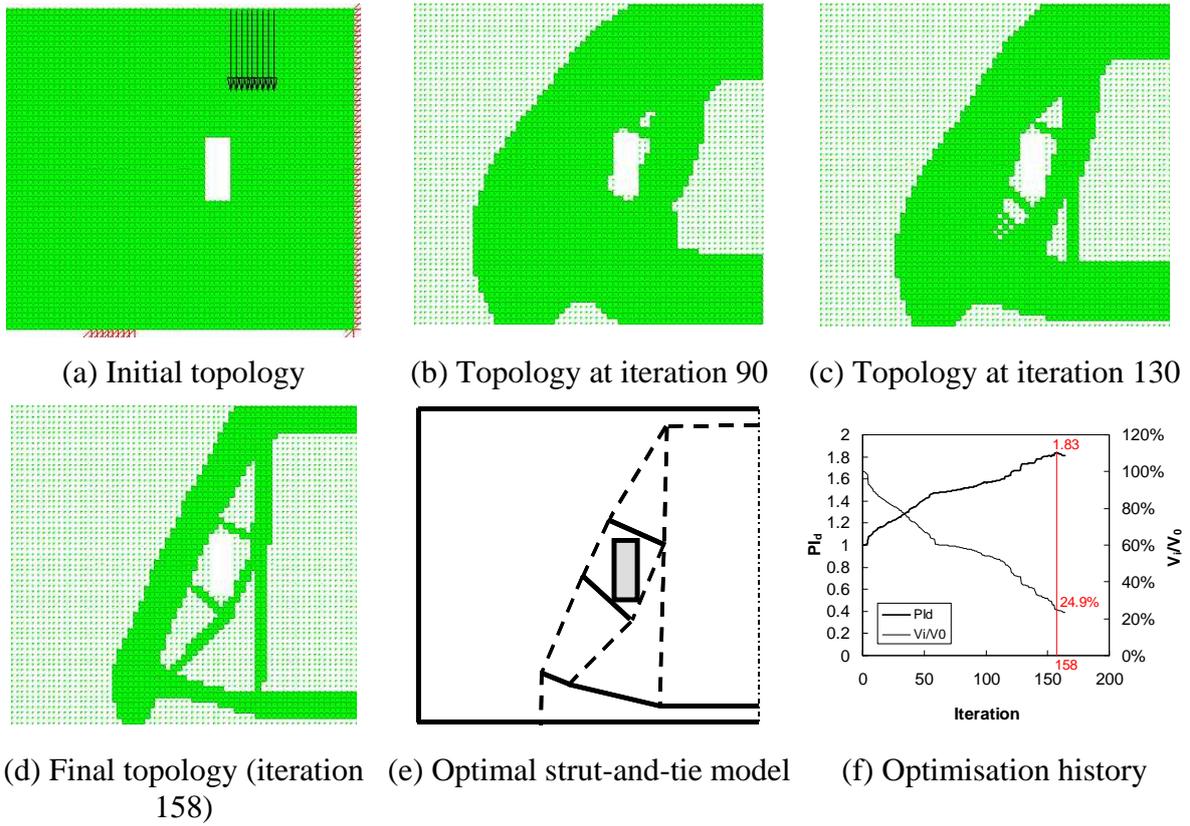


Figure 3 Development of strut-and-tie model for deep beam O-0.3/1

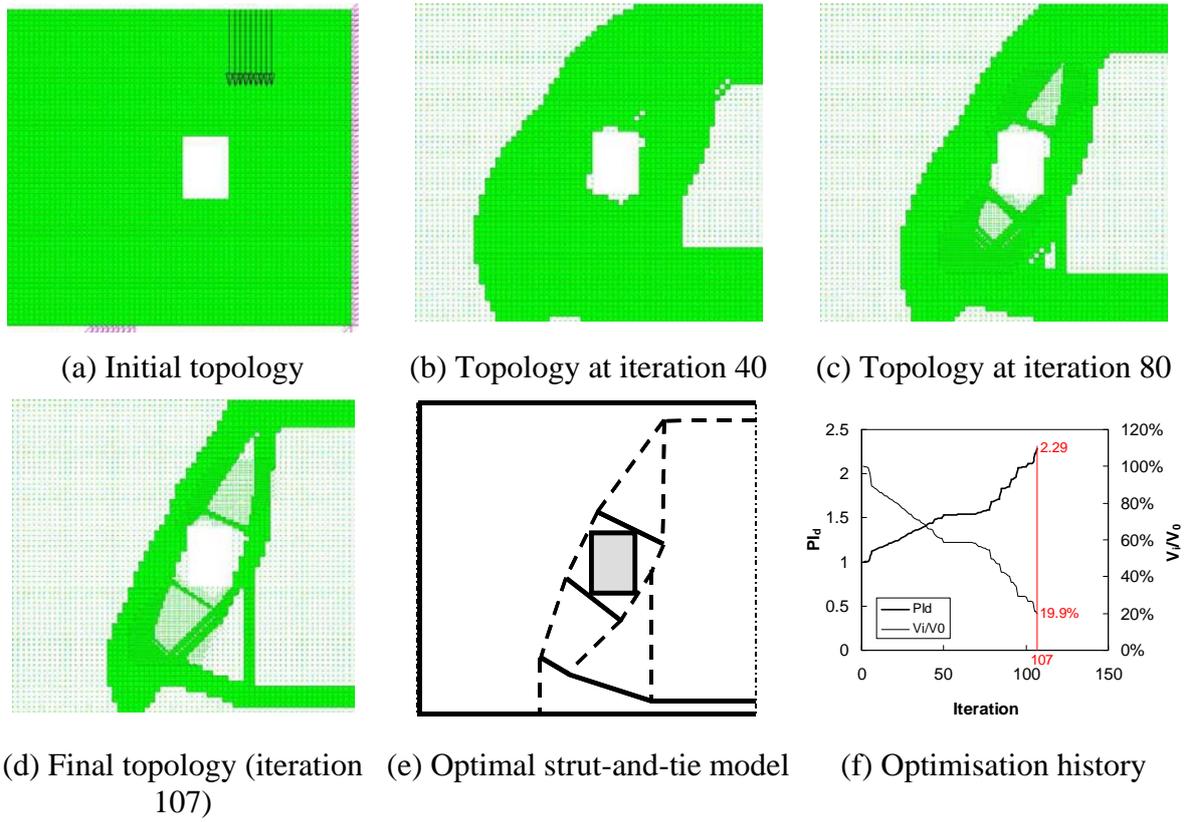
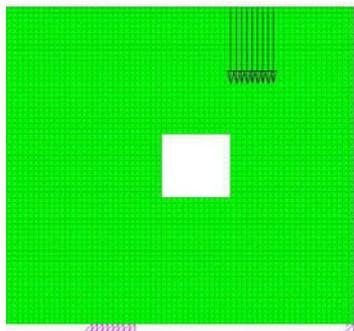
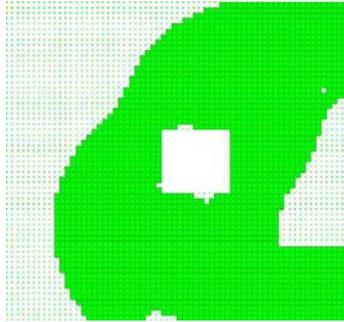


Figure 4 Development of strut-and-tie model for deep beam O-0.3/2



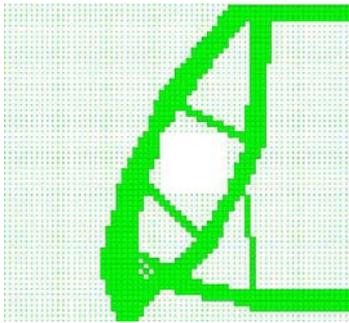
(a) Initial topology



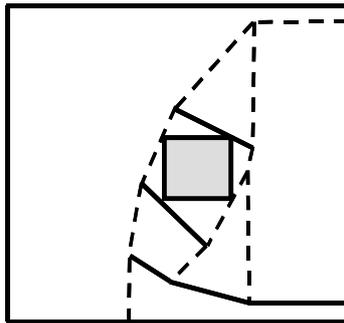
(b) Topology at iteration 80



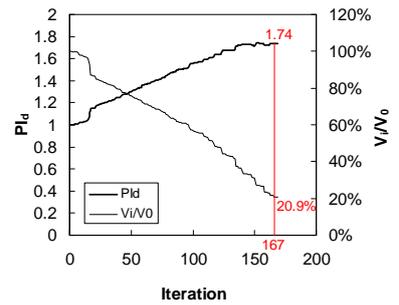
(c) Topology at iteration 140



(d) Final topology (iteration 167)



(e) Optimal strut-and-tie model



(f) Optimisation history

Figure 5 Development of strut-and-tie model for deep beam O-0.3/3

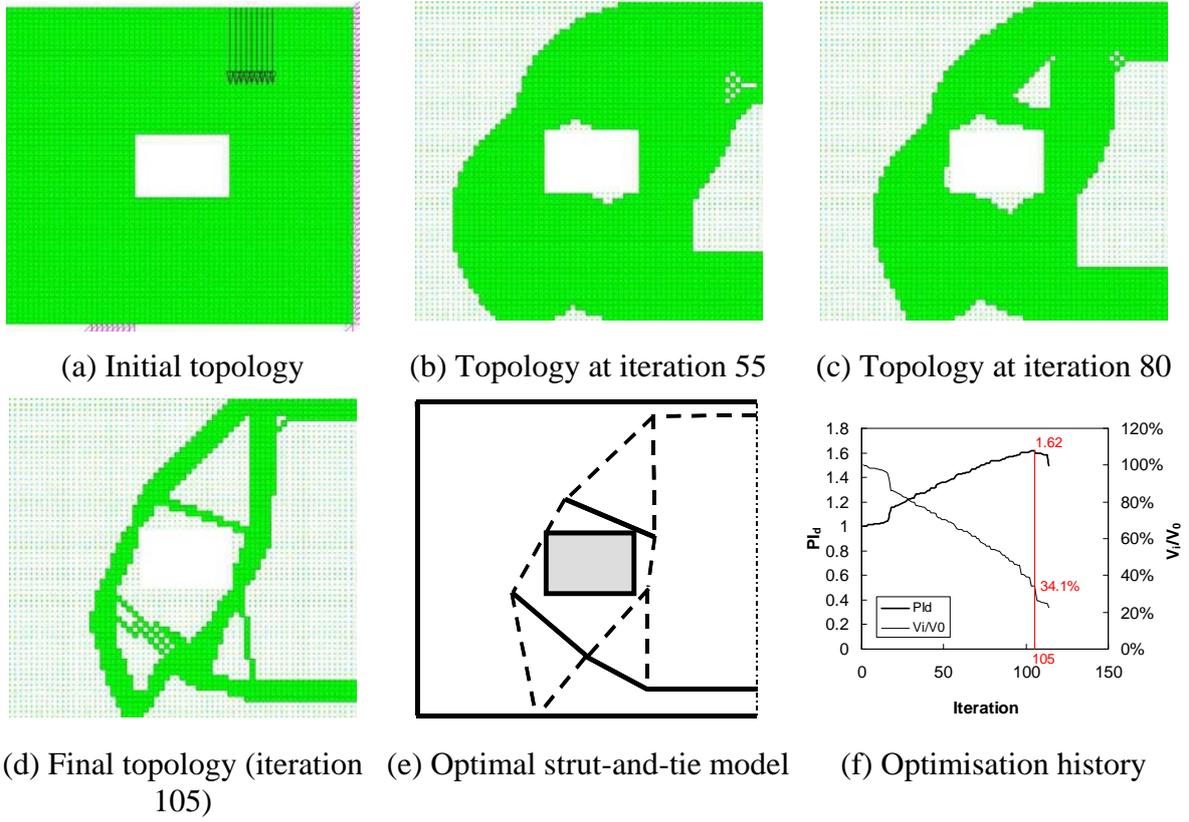


Figure 6 Development of strut-and-tie model for deep beam O-0.3/4

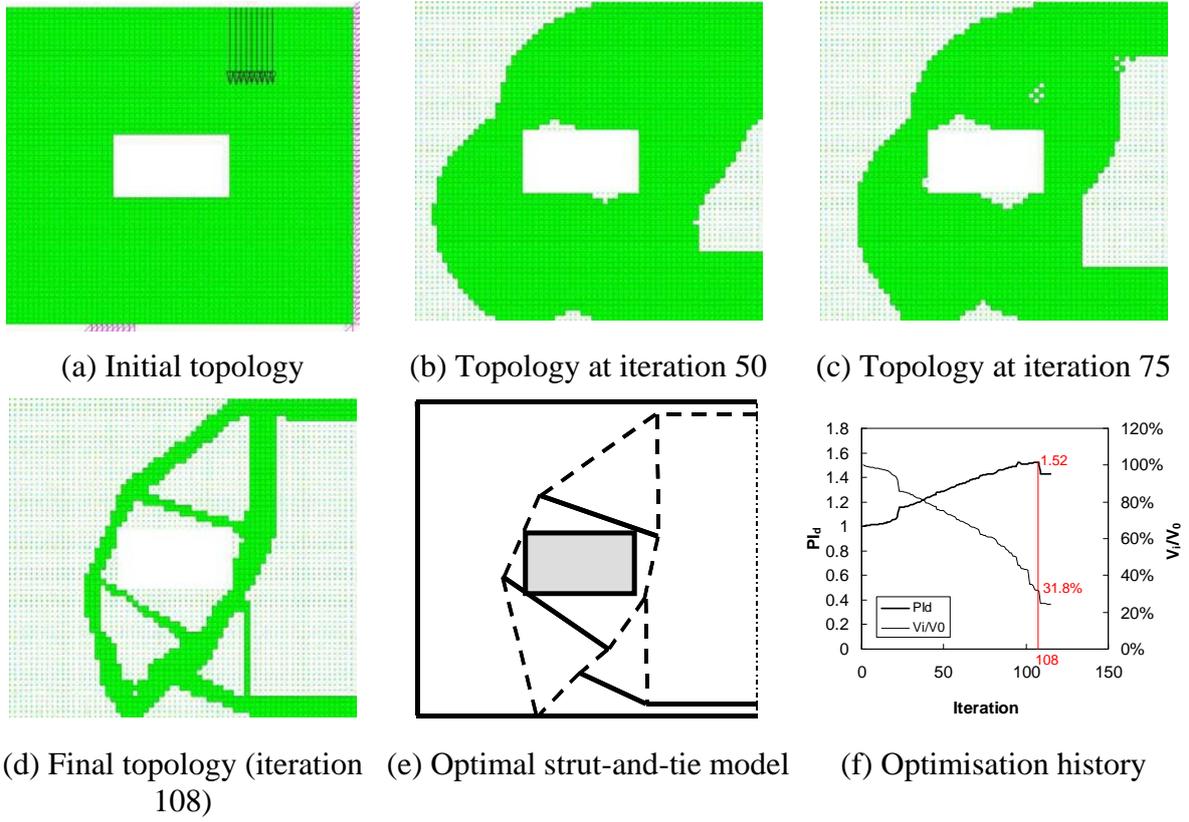
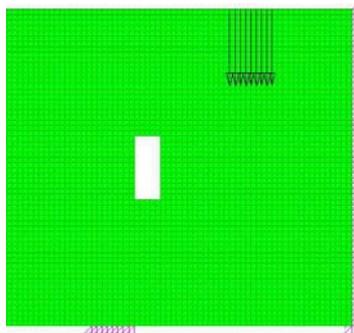
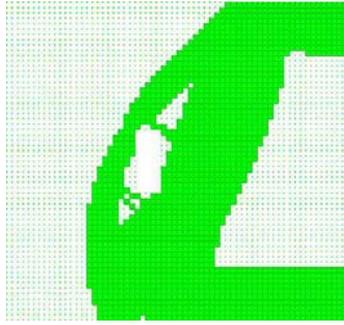


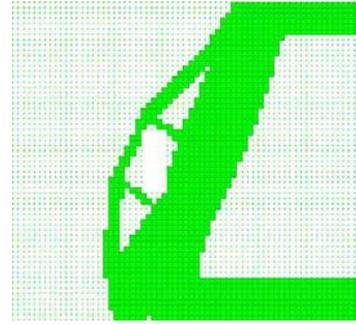
Figure 7 Development of strut-and-tie model for deep beam O-0.3/5



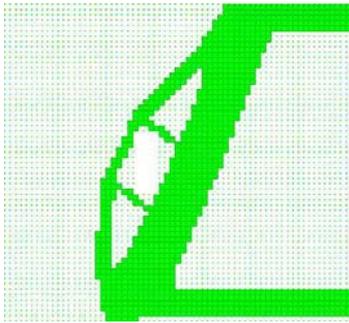
(a) Initial topology



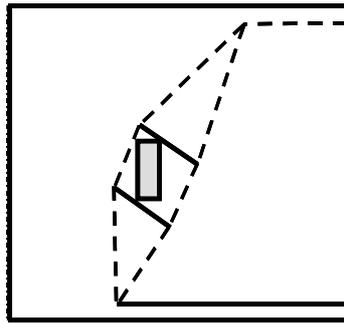
(b) Topology at iteration 50



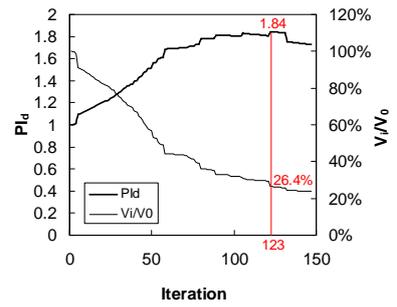
(c) Topology at iteration 75



(d) Final topology (iteration 123)

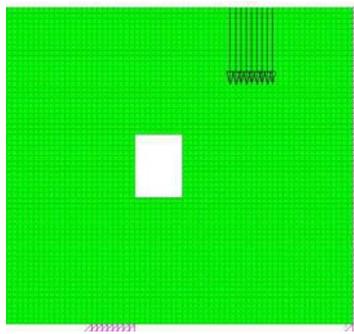


(e) Optimal strut-and-tie model

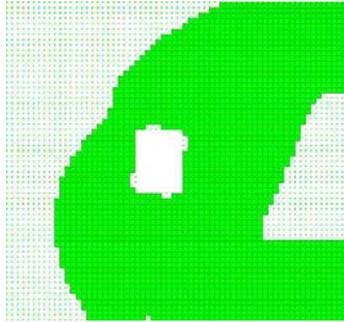


(f) Optimisation history

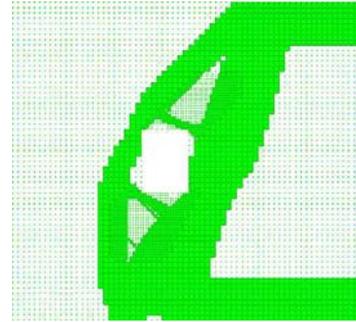
Figure 8 Development of strut-and-tie model for deep beam O-0.3/7



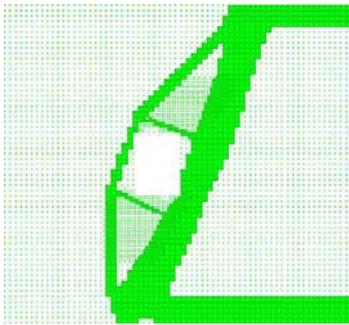
(a) Initial topology



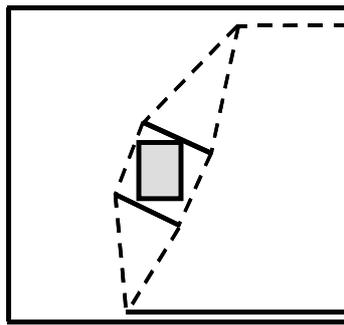
(b) Topology at iteration 45



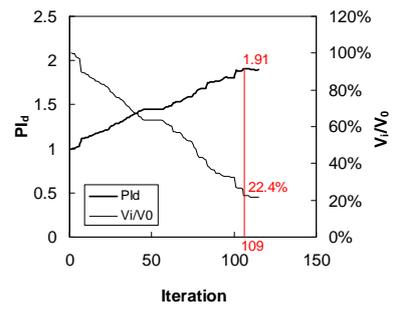
(c) Topology at iteration 85



(d) Final topology (iteration 109)



(e) Optimal strut-and-tie model



(f) Optimisation history

Figure 9 Development of strut-and-tie model for deep beam O-0.3/8

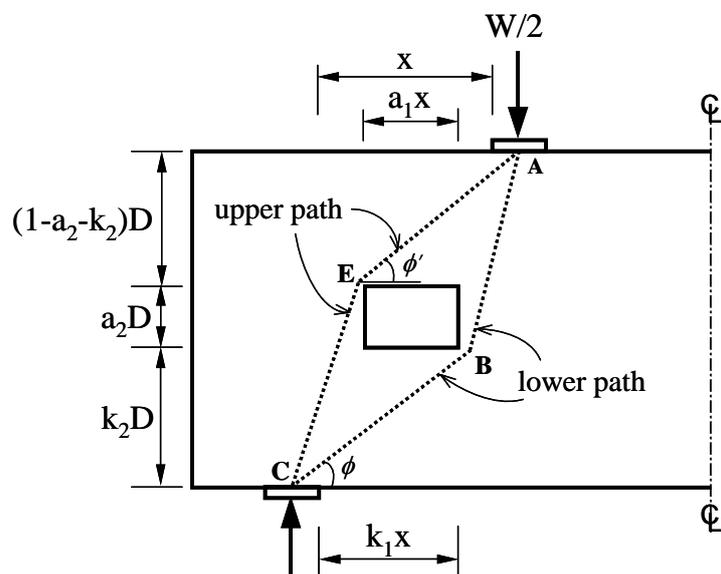
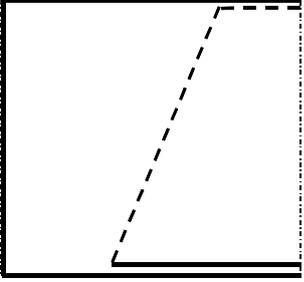
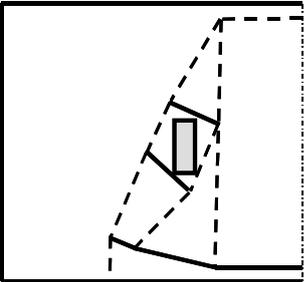
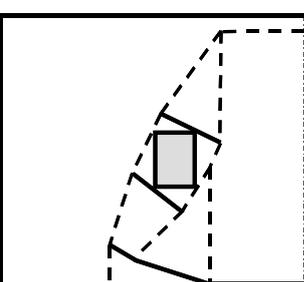
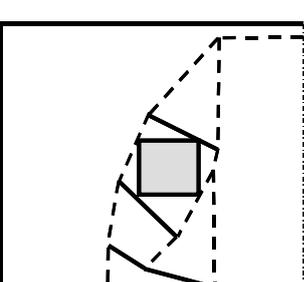
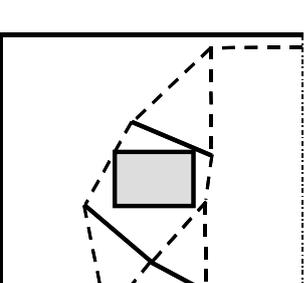
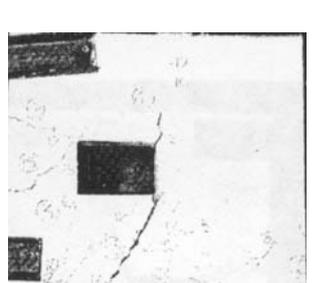


Figure 10 Structural idealisation of deep beam with web opening

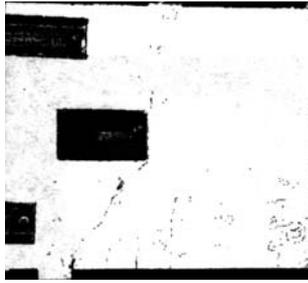
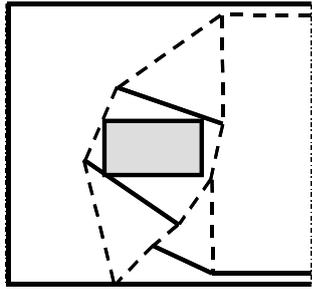
Beam designation	Reference No. of web opening	Size of opening		Position of opening		Ultimate load $W$ (kN)
		$a_1$	$a_2$	$k_1$	$k_2$	
O-0.3/0	0	—	—	—	—	595
O-0.3/1	1	0.3	0.2	1.0	0.4	460
O-0.3/2	2	0.5	0.2	1.0	0.4	390
O-0.3/3	3	0.7	0.2	1.0	0.4	280
O-0.3/4	4	1.0	0.2	1.0	0.4	260
O-0.3/5	5	1.2	0.2	1.0	0.4	200
O-0.3/7	7	0.3	0.2	0.3	0.4	420
O-0.3/8	8	0.5	0.2	0.5	0.4	380

Note: A letter O indicates that there is no web reinforcement; the  $x/D$  ratio is given after the hyphen, followed by the web opening reference number.

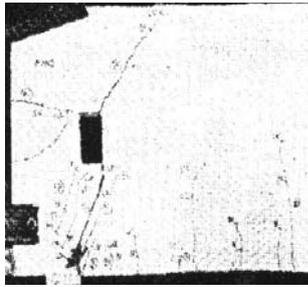
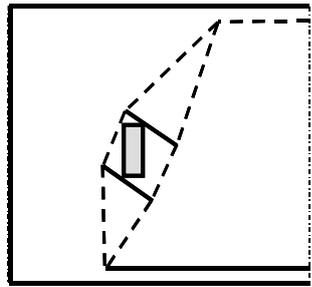
Table 2. Comparison between strut-and-tie models and experimental crack patterns

Beam designation	Optimal strut-and-tie model	Experimental crack pattern
O-0.3/0		
O-0.3/1		
O-0.3/2		
O-0.3/3		
O-0.3/4		

O-0.3/5



O-0.3/7



O-0.3/8

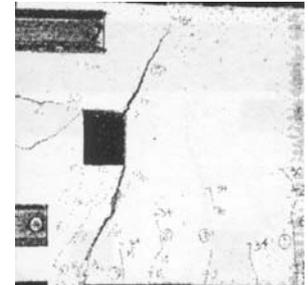
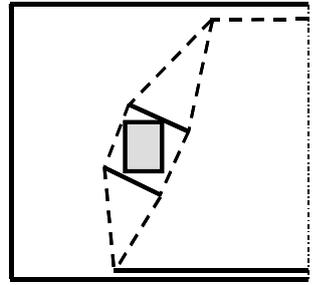


Table 3. Effect of sizes of web openings on strut-and-tie models of deep beams

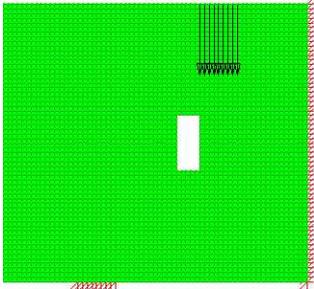
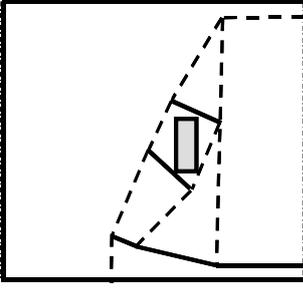
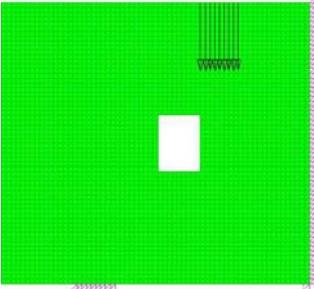
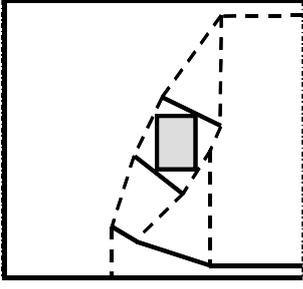
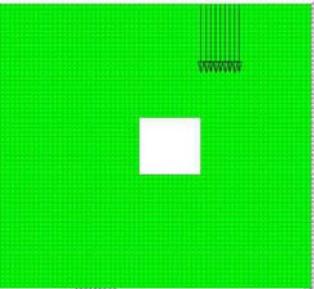
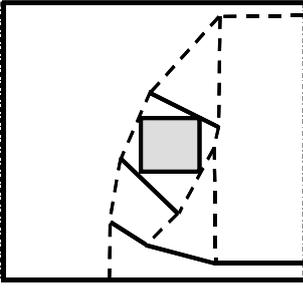
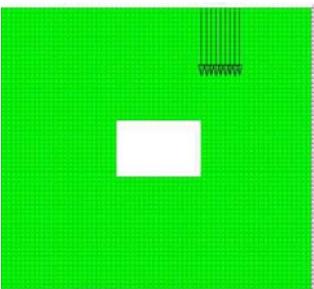
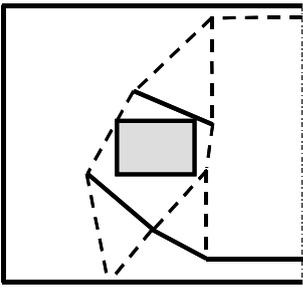
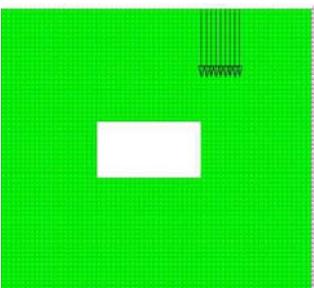
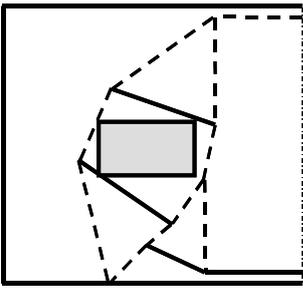
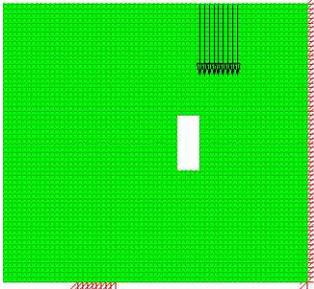
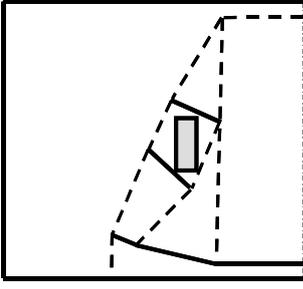
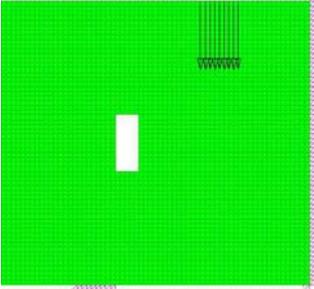
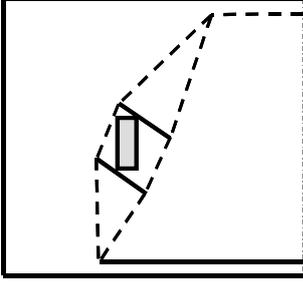
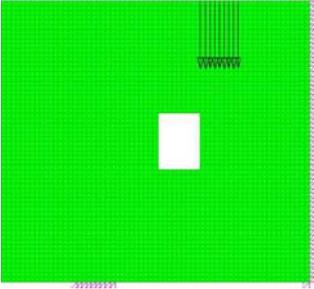
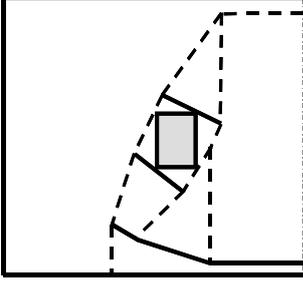
Beam designation	Initial topology	Optimal strut-and-tie model
O-0.3/1		
O-0.3/2		
O-0.3/3		
O-0.3/4		
O-0.3/5		

Table 4. Effect of positions of web openings on strut-and-tie models of deep beams

Beam designation	Initial topology	Optimal strut-and-tie model
O-0.3/1		
O-0.3/7		
O-0.3/2		
O-0.3/8	