

Study of Offshore Monopile Behaviour Due to Ocean Waves

Miao Li*, Hong Zhang and Hong Guan

Griffith School of Engineering, Griffith University, QLD, 4222, Australia

*Corresponding author. Tel: +61 7 555 28607. E-mail address: m.li@griffith.edu.au

Abstract

Offshore monopile foundations are one of the most commonly used foundation structures in offshore renewable energy, especially in areas with relatively shallow water. They are characterised by relatively large geometric dimensions compared with other offshore pile foundations and differ from onshore piles in that they suffer from more dynamic ocean environments during their lifetime. One of the most significant aspects is associated with the wave effect on the structural behaviour of monopile foundations. In this study, a three-dimensional scaled boundary finite element model (SBFEM) is proposed to investigate structural responses of monopile foundations when exposed to ocean waves. Unlike other numerical techniques, SBFEM provides an analytical solution in the radial direction with numerical approximation along the discretised faces of the monopile foundation. The SBFEM model is validated by an equivalent finite element model, by which favourable computational efficiency and reliable accuracy are demonstrated. Subsequently, a parametric study is carried out focussing on various wave properties to gain an insight into monopile behaviour. Results show that the lateral displacement of the

monopile foundation increases as wave numbers, wave amplitudes or water depths increase. This study aims at improving the design of offshore monopile foundations, when wave load is a dominant factor.

Key words: Offshore monopile foundations; ocean waves; structural behaviour; scaled boundary finite element model; three-dimensional analysis

1. Introduction

Offshore monopiles, as an important foundation concept, have been receiving increasing attention of scientists and engineers since offshore renewable energy gained global popularity in the last few decades. Generally, embedded in shallow waters with a depth of no more than 50 metres, offshore monopiles typically have a much larger diameter than those of other pile foundations. Serving as a supporting element connecting the turbine tower and the seabed, monopiles are key elements in offshore wind farm design and construction as they transfer all the loads acting on the turbine above sea level to the seabed and are exposed to harsh ocean environment itself. Their stability and structural behaviour are of great significance to an offshore wind farm project. Therefore, scientists and engineers have been engaged in exploring and investigating offshore monopile behaviour as it develops. Previous theoretical and practical experiences from conventional geotechnical pile foundations and onshore wind farms have been reviewed and referred to during the course of this research. However, the obvious difference in the sitting environment between offshore monopiles and their onshore counterparts, or conventional geotechnical piles, poses challenges and requires special consideration when conducting the analysis. One of the most significant aspects is associated with

ocean waves, to which monopiles are inevitably exposed. Ocean waves impose cyclic and detrimental loads on monopiles during their entire operational lifetime, and therefore, are considered as predominant factors when analysing monopile behaviour.

Unfortunately, there is no established technical guideline for offshore monopile design and construction that can be universally applied due to its environmental-dependent nature. Pioneers in Northern Europe and other regions have been consistently documenting their experiences so that offshore renewable energy can be expected to contribute to sustainable development thereby benefitting human kind. Kellezi and Hansen (2003) developed both static and dynamic models, based on the Finite Element Method (FEM), to investigate monopile-seabed interactions. Static calculations were carried out for extreme static horizontal loads and rotational moment. The nonlinearity of the pile-seabed interaction was taken into consideration by using a ‘contact pair’ along the monopile-seabed interface, where elastic-plastic behaviour was enforced. Within this static analysis, a maximum horizontal deformation of 35-40 mm for a 22 m height pile at the seabed level was examined. The preliminary dynamic investigation of the seabed-monopile interaction was presented thereafter, which was carried out in time domain with an absorbent boundary modelling the unbounded seabed. Johansen *et al* (2008) proposed two solutions: ‘Fins Structure’ and ‘Diversion Fence Structure’ to prevent scour from occurring around monopile foundations for the sake of environmental protection and cost reduction. A computational fluid dynamics model was employed to numerically analyse the benefits of the proposed scour protection solutions in their work. Physical experiments were also carried out to simulate the installation and maintenance operations. A rough cost analysis was provided to illustrate the economical-feasibility of the

proposed solutions. Achmus (2009) applied the results of drained cyclic triaxial tests on cohesionless seabed soil in a ‘degradation stiffness model’ to evaluate the accumulated displacement of the monopile head under cyclic lateral loads. Parametric studies to identify the effects of geometric configurations, subsoil properties and loading conditions on monopile behaviour were carried out. The results showed that the displacement accumulation rate is closely dependent upon the loading level, whereas that for a given load is mainly governed by the embedded length of a pile. Achmus’ analysis is expected to fulfil the preliminary design which involves long-term cyclic loading.

Given the limited studies of wave impacts on monopile behaviour, Li *et al* (2010b) proposed a semi-analytical numerical model based on the Scaled Boundary Finite Element Method (SBFEM), to study the monopile behaviour due to ocean wave loads and examine the effect of wave numbers on the structure response. SBFEM inherits the advantages of both FEM and Boundary Element Method (BEM), the two numerical techniques from which SBFEM was derived. Featuring at discretising the boundary only and satisfying the boundary condition exactly at infinity when dealing with problems involving unbounded domain, SBFEM does not necessitate any singular integral or fundamental solution. These favourable features bring about a wide application of SBFEM into various fields of engineering, such as structural engineering (Yang, 2006; Yang, 2007), ocean engineering (Li, 2007; Tao, 2007), geotechnical engineering (Khani, 2007), hydraulic engineering (Wang, 2010) and electromagnetic engineering (Liu, 2010).

In this study, the SBFEM is employed to develop a semi-analytical numerical model to further explore the monopile response to ocean waves. The model is non-dimensionalised

for the benefit of subsequent parametric analysis. Discussions based on variation of wave properties are expected to contribute to the fundamental understanding of monopile responses to ocean waves. The remaining part of this paper is organised as follows: the physical problem description is introduced in Section 2, followed by presenting the SBFEM model of the monopile and its non-dimensionalisation in Section 3. Subsequently, the monopile behaviour is investigated and detailed in Section 4 and a parametric study in terms of how wave properties affect monopile behaviour is presented in Section 5.

2. Problem Formulation

A typical monopile-supported wind turbine is associated with three physical aspects according to the spatial division by the medium surrounding it, i.e., aerodynamically with the wind, hydrodynamically with the sea water, and geotechnically with the seabed. The wind exerts aerodynamic forces on the turbine rotor during the wind turbine operation. There are also static axial loads transferred from the turbine tower and act on the monopile. These aspects are not considered due to the intensive objective of this study, which mainly focuses on the wave loads and the resulting structural response of the monopile. On the other hand, the embedded part of the monopile foundation exhibits relative motions with respect to the seabed, horizontal deflection and rotation for instance, the effect of which to the monopile behaviour are out of the scope of the current research. Considering the above assumptions, it is worth mentioning that, passive monopile foundations, such as those for oil rig installations, are also applications of the proposed formulation.

A free standing monopile foundation is shown in Fig. 1, with its bottom fixed at the seabed, i.e., the penetration depth into the seabed is neglected in this study. In Fig. 1, a denotes the monopile radius, h the monopile height, d the mean water depth and A the wave amplitude.

Considering a specific condition when the magnitude of the dynamic wave pressure acts upon the monopile, the governing equations of the monopile behaviour follow those of elastostatics (Gould, 1994; Huang and Bush, 1997):

$$[L]^T \{\sigma\} = 0 \quad (1)$$

with $[L]$ representing the partial differential operator. The stress amplitude $\{\sigma\}$ is related to the strain amplitude $\{\varepsilon\}$ and the elastic matrix $[D]$ as:

$$\{\sigma\} = [D]\{\varepsilon\} \quad (2)$$

The strain amplitude $\{\varepsilon\}$ and displacement amplitude $\{u\}$ are related by $[L]$ in the form of:

$$\{\varepsilon\} = [L]\{u\} \quad (3)$$

Eqs. (1)-(3) describe the structural behaviour of any point within the monopile foundation. They can be solved with the boundary conditions specified at the seabed level, sea water-monopile interface and the faces of monopile above mean water level.

Zero displacements are enforced at the seabed level where the monopile foundation is rested.

$$\{u\} = 0, \quad \text{at } z = 0$$

Wave pressure, acting on the sea water-monopile interface, results from the summation of the dynamic wave pressure and the hydrostatic pressure.

In real ocean environment, short-crested waves are very likely to be generated and therefore, are the most common form of waves resulting from winds blowing over the

surface of the open sea. Since Jeffreys (1924) developed the theory of short-crested waves, many researchers have investigated their kinetic and dynamic properties, as well as the diffraction, reflection and radiation phenomena due to their interactions with coastal and offshore structures. Zhu (1993) systematically studied the loads exerted by short-crested waves on a cylindrical pile of coastal or offshore structures, and presented an analytical expression of the dynamic wave pressure acting at any point on the cylinder from the seabed level up to the free surface elevation, with the magnitude of which being given in Eq. (4) as:

$$p_m(a, \theta, z) = \frac{\rho_w g A \cosh kz'}{2 \cosh kd} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \varepsilon_m \varepsilon_n i^m Q_{mn}(a, \theta) \quad (4)$$

with the origin of the coordinate system being indicated in Fig.1. In Eq. (4), the azimuth angle θ is measured along the monopile circumference in the anti-clockwise direction. p_m is also related to the radius of the pile a , and the properties of the short-crested waves, i.e., the wave amplitude A and the wave number k . ρ_w represents the water density, g the gravitational acceleration, d the water depth and η_θ , the wave elevation along the circumference of the pile, the magnitude of which is expressed as:

$$\eta_\theta = \frac{A}{2} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \varepsilon_m \varepsilon_n i^m Q_{mn}(a, \theta) \quad (5)$$

z' is the stretched coordinate calculated according to Wheeler (1969) as:

$$z' = \frac{zd}{d + \eta_\theta} \quad (6)$$

where z is measured along the height of the monopile.

In Eqs (4) and (5), ε_m , ε_n and $Q_{mn}(a, \theta)$ are related to the series terms and are defined as:

$$\varepsilon_m, \varepsilon_n = \begin{cases} 1 & m, n = 0 \\ 2 & m, n \neq 0 \end{cases}$$

$$\begin{aligned} Q_{mn}(a, \theta) = & \left[J_m(k_x a) J_{2n}(k_y a) - A_{mn} H_{m+2n}(ka) \right] \cos(m+2n)\theta \\ & + \left[J_m(k_x a) J_{2n}(k_y a) - B_{mn} H_{|m-2n|}(ka) \right] \cos(m-2n)\theta \end{aligned} \quad (7)$$

where

$$A_{mn} = \frac{k_x J'_m(k_x a) J_{2n}(k_y a) + k_y J_m(k_x a) J'_{2n}(k_y a)}{k H_{m+2n}(ka)} \quad (8)$$

$$B_{mn} = \frac{k_x J'_m(k_x a) J_{2n}(k_y a) + k_y J_m(k_x a) J'_{2n}(k_y a)}{k H_{|m-2n|}(ka)} \quad (9)$$

In Eqs. (7)-(9), J represents the Bessel function of the first kind and H represents the Hankel function. k_x and k_y explain the periodic property of the short-crested waves at the free-surface level in the x and y directions, respectively. They are related to the incident wave angle α by:

$$k_x = k \cos \alpha, k_y = k \sin \alpha \quad (10)$$

Generally, the wave period T for wind waves in ocean environments ranges from 5 seconds to 20 seconds. With this condition, the wave number k in Eq. (4) can be evaluated using the wave dispersion equation $(2\pi/T)^2 = gk \tanh(kd)$, given the water depth d .

The hydrostatic pressure p_h is calculated as:

$$p_h = \rho_w g (d - z') \quad (11)$$

3. SBFEM Model

3.1 Brief introduction of SBFEM

SBFEM, based on BEM and FEM from the nomenclature, was proposed by Wolf and Song (1996). The main framework of SBFEM is based on a local coordinate system, the scaled boundary coordinate system (ζ, η, ζ) (see Fig. 2), which is defined by a scaling centre O , a radial direction ζ starting from the scaling centre to the boundary and circumferential directions, η and ζ , on the boundary. Same as BEM when dealing with problems involving unbounded domains, SBFEM does not necessitate a fundamental solution, which consequently generalises its application in many fields. Applying the geometric transformation from a conventional coordinate system (Cartesian coordinate $(\hat{x}, \hat{y}, \hat{z})$ for example) to the scaled boundary coordinate system (ζ, η, ζ) , and employing the discretisation concept of FEM only on the boundary of the study domain, the governing partial differential equation (PDE) is transformed into a matrix-form ordinary differential equation (ODE). Therefore, the behaviour of the physical problem is investigated numerically in the FEM sense along the boundary, and analytically in the radial direction by solving the matrix-form ODEs mathematically. The solution procedure of SBFEM is illustrated concisely in Fig. 3. Galerkin's weighted residual concept is employed in the circumferential direction when deriving the weak form scaled boundary finite element equation. The coefficient matrices are assembled from the boundary discretisation in the same way as that in FEM. After the nodal function is obtained, interpolation using shape functions and specification of the scaled boundary coordinates lead to the semi-analytical solution in the entire domain. Apart from the above-mentioned advantages, the radiation condition at infinity can be satisfied exactly in SBFEM for unbounded domain problems. However, SBFEM shows its disadvantages when dealing with problems involving nonlinearity and material inhomogeneity.

3.2 SBFEM model formulation

The three-dimensional local scaled boundary coordinate system (ξ, η, ζ) introduced for the monopile foundation is shown in Fig. 4. Due to symmetry of the physical problem, only half of the monopile is analysed. The scaling centre $O(x_0, y_0, z_0)$ is selected to coincide with the geometric centre of the monopile. The radial coordinate ξ ranges from 0 at O to 1 at Γ , which represents the external surface of the symmetrical half of the monopile (see Fig. 4). The other two local coordinates η and ζ , with magnitudes ranging from -1 to 1 for each element, rely on the tangential directions of Γ . The two coordinate systems are geometrically related by the following expressions:

$$\begin{aligned}\hat{x}(\xi, \eta, \zeta) &= \xi [N(\eta, \zeta)] \{x\} + x_0 \\ \hat{y}(\xi, \eta, \zeta) &= \xi [N(\eta, \zeta)] \{y\} + y_0 \\ \hat{z}(\xi, \eta, \zeta) &= \xi [N(\eta, \zeta)] \{z\} + z_0\end{aligned}\tag{12}$$

where $\{x\}$, $\{y\}$ and $\{z\}$ represent the coordinates of the discretised nodes on Γ . Eight-node quadratic quadrilateral elements are used for boundary discretisation with a shape function $[N(\eta, \zeta)]$ being expressed as:

$$\begin{aligned}N_1(\eta, \zeta) &= \frac{1}{4}(1-\eta)(\zeta-1)(\eta+\zeta+1) & N_2(\eta, \zeta) &= \frac{1}{2}(\eta^2-1)(\zeta-1) \\ N_3(\eta, \zeta) &= \frac{1}{4}(\zeta-1)(1+\eta)(1-\eta+\zeta) & N_4(\eta, \zeta) &= \frac{1}{2}(\eta+1)(1-\zeta^2) \\ N_5(\eta, \zeta) &= \frac{1}{4}(\eta+1)(\zeta+1)(\eta+\zeta-1) & N_6(\eta, \zeta) &= \frac{1}{2}(1-\eta^2)(\zeta+1) \\ N_7(\eta, \zeta) &= \frac{1}{4}(\eta-1)(\zeta+1)(\eta-\zeta+1) & N_8(\eta, \zeta) &= \frac{1}{2}(\eta-1)(\zeta^2-1)\end{aligned}$$

With this geometric mapping, the differential operator $[L]$ is reformulated in the scaled boundary coordinate system using ξ , η and ζ as:

$$[L] = [b^1(\eta, \zeta)] \frac{\partial}{\partial \xi} + \frac{1}{\xi} ([b^2(\eta, \zeta)] \frac{\partial}{\partial \eta} + [b^3(\eta, \zeta)] \frac{\partial}{\partial \zeta}) \quad (13)$$

in which, $[b_1(\eta, \zeta)]$, $[b_2(\eta, \zeta)]$ and $[b_3(\eta, \zeta)]$ only depend on the boundary discretisation, and are independent of the radial coordinate ξ .

Using the same shape function $[N(\eta, \zeta)]$ as for boundary discretisation, the displacement amplitude is expressed as:

$$\{u(\xi, \eta, \zeta)\} = [N(\eta, \zeta)] \{u(\xi)\} \quad (14)$$

where $\{u(\xi)\}$ represents the displacement variation with the radial coordinate ξ . Once it is solved, the displacement field within the monopile foundation can be obtained using Eq.(14) with the specified scaled boundary coordinates ξ , η and ζ , and subsequently the stress and strain fields can be calculated as:

$$\{\varepsilon\} = \{\varepsilon(\xi, \eta, \zeta)\} = [B^1] \{u(\xi)\}_{,\xi} + \frac{1}{\xi} [B^2] \{u(\xi)\}$$

$$\{\sigma\} = \{\sigma(\xi, \eta, \zeta)\} = [D] \left([B^1] \{u(\xi)\}_{,\xi} + \frac{1}{\xi} [B^2] \{u(\xi)\} \right)$$

in which $[B^1]$ and $[B^2]$ are formulated as :

$$[B^1(\eta, \zeta)] = [b^1(\eta, \zeta)] [N(\eta, \zeta)], \quad [B^2(\eta, \zeta)] = [b^2(\eta, \zeta)] [N(\eta, \zeta)]_{,\eta} + [b^3(\eta, \zeta)] [N(\eta, \zeta)]_{,\zeta}$$

Applying the weighted residual technique and Green's theorem, and through a series of manipulations, the governing PDEs (1)-(3) are transformed into the second-order matrix-form Euler-Cauchy ODEs with respect to the nodal displacement function $\{u(\xi)\}$:

$$[E^0] \xi^2 \{u(\xi)\}_{,\xi\xi} + (2[E^0] + [E^1]^T - [E^1]) \xi \{u(\xi)\}_{,\xi} + ([E^1]^T - [E^2]) \{u(\xi)\} = 0 \quad (15)$$

Eq.(15) is termed as the scaled boundary finite element equation. In Eq.(15), only the radial coordinate ξ appears. The other two coordinates η and ζ are incorporated in the coefficient matrices in the form of:

$$\begin{aligned}
[E^0] &= \int_{-1}^1 \int_{-1}^1 [B^1(\eta, \zeta)]^T [D][B^1(\eta, \zeta)] |J(\eta, \zeta)| d\eta d\zeta \\
[E^1] &= \int_{-1}^1 \int_{-1}^1 [B^2(\eta, \zeta)]^T [D][B^1(\eta, \zeta)] |J(\eta, \zeta)| d\eta d\zeta \\
[E^2] &= \int_{-1}^1 \int_{-1}^1 [B^2(\eta, \zeta)]^T [D][B^2(\eta, \zeta)] |J(\eta, \zeta)| d\eta d\zeta
\end{aligned} \tag{16}$$

where $[D]$ is the elastic matrix representing the physical property of the monopile foundation. The three matrices $[E^0]$, $[E^1]$ and $[E^2]$ in Eq.(16) are first formulated for each individual element discretised on Γ and then assembled in the same way as in FEM.

3.3 Non-dimensionalisation of SBFEM model

In order to investigate monopile responses to various ocean wave loads, the variables and coefficient matrices appeared in Eq.(15) are nondimensionalised using a set of reference variables, which are listed in Table 1. Therefore, all computed quantities are of relatively similar magnitude regardless of the unit systems used to measure the variables.

To derive the non-dimensional version of Eq.(15), the following definitions are introduced based on the fact that, different from conventional coordinates, \hat{x} , \hat{y} and \hat{z} in Cartesian coordinate for example, the scaled boundary coordinates ξ , η and ζ are dimensionless:

$$\begin{aligned}
[E^0]^* &= \frac{[E^0]}{\rho_w g a}, \quad [E^1]^* = \frac{[E^1]}{\rho_w g a}, \quad [E^2]^* = \frac{[E^2]}{\rho_w g a}, \quad \{u(\xi)\}^* = \frac{\{u(\xi)\}}{a}, \quad \{u(\xi)\}_{,\xi}^* = \frac{\{u(\xi)\}_{,\xi}}{a}, \\
\{u(\xi)\}_{,\xi\xi}^* &= \frac{\{u(\xi)\}_{,\xi\xi}}{a}
\end{aligned}$$

where, asterisks denote dimensionless variables. Rearrange the above equations in terms of the dimensional variables and substitute into Eq.(15), a constant coefficient $\rho_w g a^2$

appears for each term throughout the equation and can be cancelled out. Therefore, the non-dimensionalised form of Eq.(15) can be obtained as:

$$\left[E^0 \right]^* \xi^2 \{u(\xi)\}_{,\xi\xi}^* + \left(2 \left[E^0 \right]^* + \left[E^1 \right]^{*T} - \left[E^1 \right]^* \right) \xi \{u(\xi)\}_{,\xi}^* + \left(\left[E^1 \right]^{*T} - \left[E^2 \right]^* \right) \{u(\xi)\}^* = 0 \quad (17)$$

Following discussions are based on this non-dimensionalised analysis. For ease of presentation, all asterisks are removed from the mathematical expressions thereafter.

To solve the scaled boundary finite element equation Eq.(17), a new variable

$$\{X(\xi)\} = \begin{Bmatrix} \xi^{+0.5} \{u(\xi)\} \\ \xi^{-0.5} \{Q(\xi)\} \end{Bmatrix}$$

is introduced to incorporate the nodal displacement function $\{u(\xi)\}$ and the nodal force function $\{Q(\xi)\}$, which is expressed as: $\{Q(\xi)\} = \left[E^0 \right] \xi^2 \{u(\xi)\}_{,\xi} + \left[E^1 \right]^T \xi \{u(\xi)\}$.

By introducing $\{X(\xi)\}$ and employing a Hamiltonian matrix $[Z]$, which is formulated by coefficient matrices of Eq.(17) and the identity matrix $[I]$ as:

$$[Z] = \begin{bmatrix} \left[E^0 \right]^{-1} \left[E^1 \right]^T - 0.5[I] & -\left[E^0 \right]^{-1} \\ -\left[E^2 \right] + \left[E^1 \right] \left[E^0 \right]^{-1} \left[E^1 \right]^T & -\left[E^1 \right] \left[E^0 \right]^{-1} + 0.5[I] \end{bmatrix},$$

the number of DOFs of the problem is doubled, however, the order of the matrix-form ODE (15) is reduced from two to one, as can be examined from the resulting homogeneous linear ODE (16):

$$\xi \{X(\xi)\}_{,\xi} + [Z] \{X(\xi)\} = 0 \quad (18)$$

The Schur decomposition has been proven to be a qualified and efficient method to solve Eq.(18), following the solution procedure presented in Song (2004) and Li *et al* (2010a).

The nodal displacement function is expressed as:

$$\{u(\xi)\} = \xi^{-0.5} \left(\left[\Psi_{u1} \right] \xi^{-[S_n]} \{C_1\} + \left[\Psi_{u2} \right] \xi^{-[S_n]} \{C_2\} \right) \quad (19)$$

with $[\Psi_{u1}]$, $[\Psi_{u2}]$ and $[S_n]$ being determined from the Schur decomposition of $[Z]$. $\{C_1\}$ and $\{C_2\}$ in Eq.(19) are two sets of constants to be determined according to the

displacement and stress boundary conditions on monopile boundaries Γ .

4. Structural Response

4.1 Convergence test and model verification

To validate the proposed three-dimensional SBFEM model of the monopile and its numerical performance, a case is studied as a benchmark with a monopile foundation subjected to a hydrostatic pressure being expressed as: $p_h = \rho_w g (d - z)$. The geometric properties and other quantities are listed in Table 2. To examine the convergence, the displacements in the z direction at the monopile head level are plotted in Fig. 5 (a) for several discretisation schemes, which are shown in Table 3. ‘Mesh1’ corresponds to ‘1-3-4’, meaning that 1 element along the height of the monopile above the mean water level, 3 elements below and 4 elements each for both monopile radius and a quarter of the monopile circumference. Fig. 5 (a) shows a satisfactory convergence of the proposed SBFEM model when ‘Mesh4’ is employed, which is thus adopted for the analysis thereafter.

For convenient interpretation of the results, two representative locations on the monopile foundation as shown in Fig. 6 are specifically examined. One is Line $L-L'$ along the monopile height at $\theta = 0^\circ$; the other one is Line $R-R'$ along the monopile circumference at the monopile head level.

A comparative FEM analysis, using the commercial software package STRAND7 (2010), is also carried out to verify the credibility and numerical competency of the SBFEM

model. The converged displacements from both SBFEM and FEM models are plotted in Fig. 7 against the height of the monopile along $L-L'$. Only a 0.67% discrepancy is observed which guarantees the reassuring agreement of the results from the two models. With respect to the DOFs used by the two models, as compared in Fig. 5 (b) and Table 3, it is easy to see that SBFEM requires a significantly less number of DOFs for the same mesh than that of an equivalent FEM model, through which the favourable numerical efficiency of the SBFEM model is demonstrated.

4.2 Monopile behaviour analysis

With the proposed SBFEM model, the problem described in Section 2, viz the structural response of a monopile foundation to ocean wave loads is studied herein. Parameters of the monopile foundation and the wave condition are tabulated in Table 4.

As mentioned in Section 3.2, when setting up the SBFEM model, the physical problem is symmetric with respect to the incident wave direction, which results in zero displacement in the y direction along $L-L'$. Displacements in the z direction are less significant compared with the x counterparts and are not of major concern. Therefore, displacements in the x direction, reflecting lateral deflections of the monopile when subjected to wave loads, are mainly addressed in the following discussions.

The non-dimensionalised lateral displacement along $L-L'$ in Fig. 8 (a) shows a maximum displacement of 0.8254×10^{-3} at the monopile head level when the wave amplitude, wave number and the water depth are 1, 0.25 and 12, respectively. The displacement variation

along monopile circumference at $z = 16$ is illustrated by the polar plot in Fig. 8 (b). Note that numerics in the polar plot mark the scale of the radial axis, and this is the same for the following polar plots. It is noticed that the maximum displacements at the monopile head level are the same everywhere around the monopile circumference when θ ranges from 0 to π .

To understand how each force component, i.e., the dynamic wave pressure and the hydrostatic pressure, contributes to the monopile behaviour, displacements due to the two components are illustrated separately. It is observed from Fig. 9 that the displacement at the monopile head level caused by the hydrostatic component is greater than that from the dynamic counterpart, with corresponding magnitudes being 0.4354×10^{-3} and 0.3899×10^{-3} , respectively. Propagating in the positive x direction with $A = 1$ and $k = 0.25$, the incident short-crested wave generates a total free-surface elevation η_θ as being depicted in Fig. 10. It is noticed that η_θ at $\theta = 180^\circ$ is greater than that at $\theta = 0^\circ$. This elevation distribution leads to a resultant hydrostatic force acting on the monopile in the incident wave direction and causes a displacement of 0.4354×10^{-3} . Prevailing throughout the entire vertical length from the free surface to the seabed, the hydrostatic pressure contributes more to the monopile deflection than the dynamic pressure, which predominates only around the free surface and decays rapidly into the water. The displacements around the monopile circumference are uniformly distributed for both the hydrostatic and the dynamic components, as read from Fig. 9 (b).

5. Parametric study

Key wave parameters such as wave numbers, wave amplitudes and water depths are of great significance to the monopile behaviour. Considering the real situation where these parameters vary within a certain range, a study on the monopile response to the variation of these parameters is carried out to gain further insight into the functional performance of the monopile foundation. In this study, the analysis mainly focuses on how these parameters affect the wave load on the monopile and accordingly the monopile behaviour.

5.1 Effect of wave number, k

For a water depth of 30 m, and with the wave period ranging from 5 s to 20 s, the wave number varies approximately from 0.02 m^{-1} to 0.18 m^{-1} . Therefore, the non-dimensionalised wave number k is chosen as 0.05, 0.15, 0.25, 0.35 and 0.45 to investigate how it affects the monopile behaviour. Other relevant parameters are listed in Table 4.

Wave numbers influence the wave pressure distribution on the monopile foundation in the vertical direction as well as the horizontal direction. Being dominated by the z component: $\cosh kz' / \cosh kd$ of the pressure formulation Eq.(4), the dynamic wave pressure shows a rather rapid decay with water depth when it is associated with a higher wave number. Superimposed with the hydrostatic pressure, the total wave pressure variation in the vertical direction along $L-L'$ on the monopile foundation for varying k is plotted in Fig. 11 (a). Horizontally, on the other hand, greater wave numbers indicate more frequent waves acting on the monopile. With incident waves propagating in the positive x direction, the wave pressure generated with a relatively small wave number is distributed axisymmetrically around the monopile circumference as shown in Fig. 11 (b)

for $k = 0.05$ and 0.15 . When the wave number k gradually increases to 0.45 by an increment of 0.10 , the wave pressure acting on the upstream face of the monopile (corresponds to $90^\circ < \theta < 180^\circ$) increases, whereas that on the other side ($0^\circ < \theta < 90^\circ$) decreases. This, consequently, results in a substantial increase in the magnitude of the resultant force acting on the monopile in the incident wave direction.

Fig. 12 (a) shows the lateral displacement of the monopile along $L-L'$ for varying wave numbers at $A = 1$ and $d = 12$. With increasing wave numbers from 0.05 , 0.15 , 0.25 , 0.35 to 0.45 , the maximum displacement at the monopile head level increases from 0.0076×10^{-3} , 0.1878×10^{-3} , 0.8254×10^{-3} , 1.8805×10^{-3} to 3.3194×10^{-3} . It is noticed from Fig. 12 (b) that for each individual case, the displacement at the monopile head level are the same everywhere around the monopile circumference when θ ranges from 0 to π . Plotting these maximum displacements against the wave numbers k in Fig. 13 and examining the slope of the curve, it can be concluded that as k becomes greater, the increase in the maximum displacement becomes more noticeable.

5.2 Effect of wave amplitude, A

The magnitude of the wave amplitude reflects the kinetic energy associated with the wave motion. In this analysis, the non-dimensionalised wave amplitude A ranges from 0.5 to 2.0 at 0.5 increments, which corresponds to a wave height of 2.5 m , 5.0 m , 7.5 m and 10.0 m , respectively. The wave height of 10.0 m represents a wave condition which may serve as an extreme case for engineering design. Other parameters are shown in Table 4. The total wave pressure at the mean water level, shown in Fig. 14, increases

evenly as the wave amplitude A rises.

The lateral displacement along $L-L'$ of the monopile for each case is plotted in Fig. 15 (a). The corresponding polar plot, illustrating the variation in the lateral displacement with respect to the azimuth θ , is shown in Fig. 15 (b). Similarly, for each case with certain wave amplitude, equal lateral displacement is examined around the monopile circumference although the pressure distribution, shown in Fig.14 (b), is not uniform when θ goes from 0 to π . With the wave amplitude increasing from 0.5 to 2.0, the maximum displacement increases from 0.4353×10^{-3} , 0.8254×10^{-3} , 1.5024×10^{-3} to 2.2067×10^{-3} at the monopile head level. The dependence of the mechanical behaviour of the monopile on wave amplitudes is presented in Fig. 16. Physically, the greater the wave amplitude, the greater the energy associated with the wave motion, accordingly, the greater the displacement of the monopile foundation induced by the wave load.

5.3 Effect of water depth, d

The variation of water depth inevitably affects the hydrostatic pressure, as well as the dynamic pressure according to Eqs (4) and (11). Therefore, it is an important parameter when analysing the monopile response to ocean wave loads. In this study, the non-dimensionalised water depth in shallow water conditions varies from 9 to 13 with an increment of 1. Being linearly related to the water depth, the hydrostatic pressure increases with the water depth. The dynamic wave pressure, on the other hand, is related to the water depth by the hyperbolic cosine function $\cosh kz' / \cosh kd$. The superimposed total wave pressures acting along $L-L'$ for varying d at $k=0.25$ and $A=1$ are plotted in Fig.

17 (a). Those acting upon the monopile foundation around $R-R'$ at the mean water level are the same, and are overlapped as shown in Fig. 17 (b).

As shown in Fig. 18, the corresponding maximum lateral displacement for varying water depth at $k = 0.25$ and $A = 1$ is 0.4827×10^{-3} , 0.5966×10^{-3} , 0.7232×10^{-3} , 0.8254×10^{-3} and 0.8954×10^{-3} when the water depth is 9, 10, 11, 12 and 13, respectively. Similar to Fig. 13, the maximum displacement at the monopile head level is plotted against the water depth in Fig. 19, which illustrates that the deeper the water, the more significant the displacement becomes. Also, same lateral displacement is examined around the monopile circumference for each individual water depth.

6. Conclusions

A three-dimensional SBFEM model is developed herein to study the structural behaviour of a monopile foundation when subjected to varying ocean wave loads. By introducing a local scaled boundary coordinate system, the SBFEM model reduces the PDEs governing the structural behaviour of the monopile foundation to matrix-form ODEs in the radial direction. Only the degrees of freedom associated with the discretised monopile boundaries are involved when formulating the coefficient matrices of the ODEs, which considerably reduces the computational effort. Subsequently, the ODEs are solved analytically for the nodal displacement function, which represents the displacement variation in the radial direction. Adopting the same interpolation concept as that of FEM, the SBFEM model explores the displacement field within the monopile foundation by specifying the radial coordinate in the nodal displacement function and the other two

coordinates in the shape functions. This model therefore demonstrates analytical as well as numerical features in the solution process, and has displayed favourable applicability in modelling monopile behaviour through comparison with an equivalent FEM model.

Structural responses of the monopile foundation to wave loads are studied non-dimensionally using the SBFEM model. It is found that:

- The hydrostatic pressure is a more dominant factor contributing to the monopile deflection in the incident wave direction than its dynamic counterpart as it prevails from the free surface to the seabed, whereas the dynamic pressure decays rapidly as it goes into the water.
- The lateral displacement of the monopile increases when wave numbers, wave amplitudes and water depths increase. More specifically, when the wave number increases, the increase of the maximum lateral displacement of the monopile becomes more noticeable. For all cases, equal lateral displacement is obtained around the monopile circumference no matter how the wave pressure is distributed.

The model presented in this study demonstrates the favourable capability of SBFEM in modelling the structural behaviour of the monopile foundation for offshore wind turbines, and is considered to be applicable to other structures, such as piers of coastal bridges and passive pile foundations for oil rig installations. Further analysis, taking the wind and structural load effects into consideration, will be performed during the next step of study to gain further insight into the monopile behaviour.

Acknowledgement

The first author would like to thank Prof. Gao Lin and Mr Yong Zhang, from Dalian University of Technology, for their technical assistance.

References

Achmus, M, Kuo, YS, and Abdel-Rahman, K (2009). "Behavior of monopile foundations under cyclic lateral load," *Computers and Geotechnics*, Vol 36, No 5, pp 725-735.

Gould, PL (1994). "Introduction to linear elasticity," Second edition, Springer-Verlag New York, Inc.

Strand7 Pty Ltd (2010). *Using Strand7: Introduction to the Strand7 Finite Element Analysis System*, Sydney, Australia.

Huang, H and Bush, MB (1997). "Finite element analysis of mechanical properties in discontinuously reinforced metal matrix composites with ultrafine microstructure", *Materials Science & Engineering A*, 232, 63-72.

Jeffreys, H (1924). "On water waves near the coast," *Philosophical Magazine Series 6*, Vol 14, pp 44-48.

Johansen, N, Simon, JM, and Danyluk, R (2008). "Scour prevention devices for large offshore wind turbine monopile foundations," *Creative Offshore Challenge 2008*, International Offshore Competition Final Report, Denmark Technical University, Denmark.

Kellezi, L, and Hansen, PB (2003). "Static and dynamic analysis of an offshore monopile windmill foundation," *Offshore mono-pile*, Lyngby, Denmark, GEO - Danish

Geotechnical Institute.

Khani, MHBM (2007). "Dynamic soil-structure interaction analysis using the scaled boundary finite-element method," The University of New South Wales. Doctor of Philosophy. Sydney, Australia.

Li, BN (2007). "Extending the scaled boundary finite element method to wave diffraction problems," The University of Western Australia. Doctor of Philosophy. Perth, Australia.

Li, M, Song, H, Guan, H, and Zhang, H (2010a). "Schur decomposition in the scaled boundary finite element method in elastostatics," 9th World Congress on Computational Mechanics and 4th Asian Pacific Congress on Computational Mechanics, Sydney, Australia.

Li, M, Song, H, Zhang, H, and Guan, H (2010b). "Structural Response of offshore Monopile Foundations to Ocean Waves," Proceedings of the Ninth (2010) ISOPE Pacific/Asia Offshore Mechanics Symposium, Busan, Korea.

Liu, J, Lin, G, Wang FM, Li, JB (2010). "The scaled boundary finite element method applied to Electromagnetic field problems," 9th World Congress on Computational Mechanics and 4th Asian Pacific Congress on Computational Mechanics, Sydney, Australia.

Song, CM (2004). "A matrix function solution for the scaled boundary finite-element equation in statics," Computer Methods in Applied Mechanics and Engineering, Vol 193, No 23-26, pp 2325-2356.

Tao, LB, Song, H, and Chakrabarti, S (2007). "Scaled boundary FEM solution of short-crested wave diffraction by a vertical cylinder," Comput. Methods Appl. Mech. Engrg., Vol 197, pp 232-242.

Wang, Y, Lin, G, Hu, ZQ (2010). "A coupled FE and Scaled Boundary FE-Approach for

the Earthquake Response Analysis of Arch Dam-Reservoir-Foundation System," 9th World Congress on Computational Mechanics and 4th Asian Pacific Congress on Computational Mechanics, Sydney, Australia.

Wheeler, JD (1969). "Methods for Calculating Forces Produced by Irregular Waves," Offshore Technology Conference, Houston, America.

Wolf, JP, and Song, CM (1996). Finite-element modelling of unbounded media, Chichester, Wiley.

Yang, ZJ (2006). "Fully automatic modelling of mixed-mode crack propagation using scaled boundary finite element method," Engineering Fracture Mechanics, Vol 73, No 12, pp 1711-1731.

Yang, ZJ, and Deeks, AJ (2007). "Fully-automatic modelling of cohesive crack growth using a finite element-scaled boundary finite element coupled method," Engineering Fracture Mechanics, Vol 74, No 16, pp 2547-2573.

Zhu, S (1993). "Diffraction of short-crested waves around a circular cylinder," Ocean Engineering, Vol 20, No 4, pp 389-407.