

# Full-order nonlinear observer-based excitation controller design for interconnected power systems via exact linearization approach

M. A. Mahmud<sup>a,\*</sup>, H. R. Pota<sup>a</sup>, and M. J. Hossain<sup>b</sup>

<sup>a</sup>*School of Engineering & Information Technology (SEIT), The University of New South Wales at Australian Defence Force Academy (UNSW@ADFA), Canberra, ACT 2600, Australia*

<sup>b</sup>*School of Information Technology & Electrical Engineering (ITEE), University of New Queensland, St Lucia, Brisbane, QLD 4072, Australia*

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## Abstract

This paper presents a novel full-order nonlinear observer-based excitation controller design for interconnected power systems. Exact linearization approach of feedback linearization is used to design the nonlinear observer when the power system is fully linearized. The excitation control law is derived for the exactly linearized power system model. The observed states of power system are directly used as the input to the controller where the control law does not need to be expressed in terms of all measured variables. A single machine infinite bus (SMIB) system is used as test system and all the states are observed for SMIB system. To validate the effectiveness of the proposed control scheme on a large system, a benchmark 3 machine 11 bus system is also simulated. Simulation results show the accuracy and performance of observer-based nonlinear excitation controller by comparing with the exact linearizing controller where the control law is expressed in terms of all measured variables.

*Keywords:* Excitation controller, exact linearization, nonlinear observer, nonlinear gain, power system

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\*Corresponding author: Tel: +61262688199

Email: Md.Mahmud@student.adfa.edu.au (M. A. Mahmud)

## 1. Introduction

Stable operation of poorly damped nonlinear power systems can be ensured by using high-performance controllers which regulate the systems under diversity of operating conditions. Many techniques (linear and nonlinear) have been proposed to assess the stability of the power systems. In [1, 2], there is an extensive description of power system stabilizers (PSSs) which are now widely used in the power industries all over the world. In [1, 2], power system models are linearized based on the conventional Taylor series expansion method. Some improved methodologies of PSS design are proposed in [3-5] which have large disturbance rejection capability. The conventional PSS design technique is proposed in [3] with the exception that the high voltage bus is considered as reference bus instead of infinite bus where all the PSS parameters are obtained from the local measurements. In [6], the controller gains are calculated in such a way that the controller can include the effect of unmodelled dynamics of power systems. A minimax LQG controller is proposed in [7] where the mean-value theorem is used to linearize the power system model with dynamic load. In [7], the power system is reformulated with linear and nonlinear terms where the nonlinear terms are modelled as uncertainties in the design of robust control. Recently, a coordinated PSS design approach is proposed in [8] within frequency-domain framework to achieve near-optimal overall power system stability. The main problem associated with the linear control techniques is the small operating region around equilibrium point as the system is linearized over a fixed set of operating points but the variations of operating regions in power systems are wide following major disturbances. The limitations of linear controllers with variation of loads are clearly discussed in [9].

Nonlinear controllers can ensure the stability of power systems in large operating regions and in the presence of large disturbances [10, 11]. A Fourier-based sliding method is considered in [12] for secure operation of a power system with large disturbances. Feedback

linearization which algebraically transforms nonlinear system dynamics into a (fully or partly) linear one so that the linear control technique can be applied, is widely used in power system [13] as well as for other applications. Feedback linearizing excitation controllers for multimachine power systems are proposed in [14, 15]. Another feedback linearizing excitation controller is proposed in [16] where nonlinear coordinate transformation is done by choosing linearly independent vector field but the choice of independent vector field is a difficult task. A simple form of feedback linearization, called direct feedback linearization (DFL) is proposed in [17-19] and is widely used to design controllers for large power systems [20].

In the design of linear and nonlinear controllers, it is essential to measure all or some of the state variables or all the state variables need to be converted in terms of measured variables which are difficult in practice. The difficulties associated with unmeasured states can be solved through the state estimation process. A reduced-order observer-based variable structure controller is proposed in [21] where linearized power system model is used to estimate the states. A graphical observer design technique is proposed in [22] for large-scale power systems which monitors state estimation and fault-detection as well as isolation. But in [22], it is essential to identify an unknown input to achieve the specific monitoring tasks which is very difficult for a practical system.

Kalman filters are increasingly used in many applications including power systems, to estimate the states of dynamical systems. A Kalman filter can be viewed as a weighted least square (WLS) method [23] in which state estimation is done through a single set of measurements. WLS can estimate the states accurately within acceptable limits but it cannot predict the future operating states of power systems [23]. To overcome this limitation, an iterative reweighed least square (IRLS) method is proposed in [24]. An extended Kalman filter (EKF) for linearized power system model is proposed in [25]. In linear approximation

of nonlinear power system, the nonlinear terms are neglected which reduces the accuracy of estimation.

An iterative Kalman filter (IKF) can be directly used in nonlinear power system model [26] but the iterative methods are time consuming. Unscented Kalman filtering (UKF) for state estimation of continuous-time nonlinear system is described in [27] where unscented transformation is combined with Kalman filter theory. Recently, the UKF is applied to power systems in [28] to estimate the states by showing comparison with WLS and EKF. But using UKF, the task of estimation is difficult when the previous state estimation is far from the new operating point. A nonlinear observer for SMIB system is proposed in [29] which use an invariant manifold. In the design of observer through nonlinear manifold it is essential to determine an appropriate mapping which is difficult and is still an open question in the field of nonlinear control theory.

A nonlinear system can be transformed into a fully linearized system using exact linearization [30, 31]. The exact linearizability of a nonlinear system depends on the relative degree of the system which in turns depends on the chosen output function of the system [32]. If the output function of a nonlinear system is selected in such a way that the relative degree of the system equals to the order of the system, the system is fully or exactly linearized [32]. If a nonlinear system is fully linearized, then linear observer can be used to calculate the nonlinear observer gain for the nonlinear system [33].

A Luenberger-like observer is proposed in [34] which has straightforward gain calculation. Feedback linearization technique is used in [34] to linearize the nonlinear system with full relative degree. Feedback linearization is also used in [35] for a nonlinear system with non-full relative degree. In [35] the gain is not clear and the observer is not widely used in practical applications. The existing literature [33-35] on nonlinear observer related to the

feedback linearization provides complex theoretical background but which is sometimes very difficult in practical implementation.

A robust nonlinear observer-based controller is proposed in [40] where the power system is exactly linearized and a sliding mode observer is used to estimate the state of power systems. Recently, a sliding mode observer for SMIB system is designed in [41] based on a time-varying sliding surface. To implement this controller, the selection of a time varying sliding surface is essential which is quite simple for a SMIB system but it is complicated for large systems. Moreover, the observer in [41] is only developed for the damper winding current. An observer-based injection and damping assignment (IDA) controller is proposed in [42] to estimate the state of synchronous generator and to enhance the stability of power systems. But the performance of feedback linearizing controller is much better than IDA controller [43]. Moreover, to implement the feedback linearizing controller proposed in [10-14], the states of power systems are obtained by solving some algebraic equations rather than solving the dynamical equations of power systems.

This paper focuses on the design of observer-based excitation controller for power systems based on the exact linearization of power systems. This paper also presents all the information required for exact linearization of power systems. To implement the control in a practical system, the states of power systems are observed from a dynamic observer rather than solving power-flow equations [10-14]. The performance of the controller is tested on a simple SMIB system as well as on a 3 machine 11 bus system. The accuracy of the proposed control scheme is ensured by comparing the results with exact linearizing controller which is expressed in terms of measured variables.

The rest of the paper is organized as follows. Some preliminaries of nonlinear control theory which are essential in the design of feedback linearizing controller, are given in

Section 2. In Section 3, the mathematical modeling of an SMIB system is given. The exact linearization of power system model with a valid claim is presented in Section 4. Section 5 presents the observer design for exactly linearized power system. The derivation of nonlinear excitation control law is shown in Section 6 and Section 7 includes the simulation results. Concluding remarks and suggestions for future works are collected in Section 8.

## 2. Preliminary Definitions

In this section some definitions related to feedback linearization technique are presented [16, 36, 37].

Let, the nonlinear system can be written as

$$\dot{x} = f(x) + g(x)u \quad (1)$$

$$y = h(x) \quad (2)$$

where  $x \in R^n$  is the state vector;  $u \in R$  is the control vector;  $y \in R$  is the output vector;  $f(x)$  and  $g(x)$  are  $n$ -dimensional vector fields in the state space;  $h(x)$  is the scalar function of  $x$  which needs to be selected for feedback linearization.

In the design of feedback linearizing controller for power system, the definitions of nonlinear coordinate transformation, diffeomorphism, Lie derivative, and relative degree are useful which are given below.

### **Definition 1. (Nonlinear Coordinate Transformation and Diffeomorphism)**

*A nonlinear coordinate transformation for that system which can be written as*

$$z = \phi(x)$$

where  $z$  and  $x$  are the same dimensional vectors and  $\phi$  is the nonlinear function of  $x$  and the following two conditions are satisfied.

Condition 1. There exists an inverse transformation for all  $x$ , i.e.,

$$x = \phi^{-1}(z)$$

Condition 2. Each component of  $\phi$  and  $\phi^{-1}$  has continuous partial derivative up to any order which implies the differentiability of the functions.

**Definition 2. (Lie Derivative)**

For a given differentiable scalar function  $h(x)$  of  $x = [x_1 \ x_2 \ \dots \ x_n]^T$  and a vector field  $f(x) = [f_1 \ f_2 \ \dots \ f_n]^T$ , the new scalar function, denoted by  $L_f h(x)$ , is obtained by the following operation

$$L_f h(x) = \frac{\partial h(x)}{\partial x} f(x) = \sum_{i=1}^n \frac{\partial h(x)}{\partial x_i} f_i(x)$$

which is called Lie derivative of function  $h(x)$  along the vector field  $f(x)$ .

**Definition 3. (Relative Degree)**

If the Lie derivative of the function  $L_f^{r-1} h(x)$  along vector field  $g(x)$  is not equal to zero in a neighbourhood  $\Omega$ , i.e.,

$$L_g L_f^{r-1} h(x) \neq 0$$

then it is said that the system has relative degree  $r$  in  $\Omega$ .

### 3. Power System Model

A single machine infinite bus system is shown in Fig. 1 where a synchronous generator is connected to an infinite bus through a transformer and two parallel transmission lines. Since SMIB system qualitatively exhibits the important aspects of the behaviour of a multimachine system and is relatively simple to study, it is extremely useful in studying general concepts of power system stability [18].

Power system can be modelled at several different levels of complexities, depending on the intended application of the model. With some typical assumptions, the classical third-order dynamical model of a SMIB power system as shown in Fig. 1 can be modelled by the following set of differential equations [2, 16]:

Generator mechanical dynamics:

$$\dot{\delta} = \omega - \omega_0 \quad (3)$$

$$\dot{\omega} = -\frac{D}{2H}(\omega - \omega_0) + \frac{\omega_0}{2H}(P_m - P_e) \quad (4)$$

where  $\delta$  is the power angle of the generator,  $\omega$  is the rotor speed with respect to synchronous reference,  $\omega_0$  is the synchronous speed of the generator,  $H$  is the inertia constant of the generator,  $P_m$  is the mechanical input power to the generator which is assumed to be constant,  $D$  is the damping constant of the generator, and  $P_e$  is the active electrical power delivered by the generator.

Generator electrical dynamics:

$$\dot{E}'_q = \frac{1}{T_{do}}(E_f - E_q) \quad (5)$$

where  $E'_q$  is the quadrature-axis transient voltage of the generator,  $E_q$  is the quadrature-axis voltage of the generator,  $T_{do}$  is the direct-axis open-circuit transient time constant of the generator, and  $E_f$  is the equivalent voltage in the excitation coil.

The algebraic electrical equations of synchronous generator are given below:

$$E_q = \frac{x_{d\Sigma}}{x'_{d\Sigma}} E'_q - (x_d - x'_d) I_d$$

$$I_d = \frac{E'_q}{x'_{d\Sigma}} - \frac{V_s}{x'_{d\Sigma}} \cos \delta$$

$$I_q = \frac{V_s}{x'_{d\Sigma}} \sin \delta$$

$$P_e = \frac{V_s E'_q}{x'_{d\Sigma}} \sin \delta$$

$$Q_e = \frac{V_s E'_q}{x'_{d\Sigma}} \cos \delta - \frac{V_s^2}{x_{d\Sigma}}$$

$$V_t = \sqrt{(E'_q - x'_d I_d)^2 + (x'_d I_q)^2}$$

where  $x_{d\Sigma} = x_d + x_T + x_L$ ,  $x'_{d\Sigma} = x'_d + x_T + x_L$ ,  $x_d$  is the direct-axis synchronous reactance,  $x'_d$  is the direct axis transient reactance,  $x_T$  is the reactance of the transformer,  $x_L$  is the reactance of the transmission line,  $I_d$  and  $I_q$  are direct and quadrature axis currents of the generator respectively,  $V_s$  is the infinite bus voltage,  $Q_e$  is the generator reactive power delivered to the infinite bus, and  $V_t$  is the terminal voltage of the generator.

Substituting the electrical equations into the mechanical and electrical dynamics equation (3)-(5) of the system, the complete mathematical model of SMIB system can be written as follows:

$$\dot{\delta} = \omega - \omega_0 \quad (6)$$

$$\dot{\omega} = -\frac{D}{2H}(\omega - \omega_0) + \frac{\omega_0}{2H}P_m - \frac{\omega_0}{2H}\frac{V_s E'_q}{x'_{d\Sigma}} \sin \delta \quad (7)$$

$$\dot{E}'_q = -\frac{1}{T'_d}E'_q + \frac{1}{T_{do}}\frac{x_d - x'_d}{x'_{d\Sigma}}V_s \cos \delta + \frac{1}{T_{do}}E_f \quad (8)$$

where  $T'_d = \frac{x'_{d\Sigma}}{x_{d\Sigma}}T_{do}$  is the time constant of the field winding.

The SMIB system as shown in Fig. 1, represented by (6)-(8) can be expressed as

$$\dot{x} = f(x) + g(x)u$$

where

$$x = [\delta \quad \omega \quad E'_q]^T$$

$$f(x) = \begin{bmatrix} \omega - \omega_0 \\ -\frac{D}{2H}(\omega - \omega_0) + \frac{\omega_0}{2H}P_m - \frac{\omega_0}{2H}\frac{V_s E'_q}{x'_{d\Sigma}} \sin \delta \\ -\frac{1}{T'_d}E'_q + \frac{1}{T_{do}}\frac{x_d - x'_d}{x'_{d\Sigma}}V_s \cos \delta \end{bmatrix}$$

$$g(x) = [0 \quad 0 \quad \frac{1}{T_{do}}]^T$$

and

$$u = E_f$$

The mathematical model of a multimachine power system is very similar to a SMIB system with the exception that the current and power equations which contain the information of other machines [20]. The transformation of nonlinear power system model into a linear one is shown in the following section.

#### 4. Exact Linearization of Power System Model

To implement the exact linearization approach on power system for designing a nonlinear observer, the relative degree ( $r$ ) of the system needs to be equal to the order ( $n$ ) of the system. In this paper, a third order model ( $n=3$ ) is considered. The exact linearizability of power system is presented in Appendix A, through a valid claim with proof.

Exact linearization and its implementation on SMIB system can be described through the following steps:

##### *Step 1: Selection of output function*

The output function  $y=h(x)$  should be selected in such a way that  $r=n$ . For the considered SMIB system, the output function is chosen as  $y=h(x) = \delta - \delta_0$ .

##### *Step 2: Calculation of relative degree*

In this step, the relative degree of the system needs to be calculated using the definition presented in Section 2. For  $n^{th}$  order system,  $r$  will equal to  $n$  and the following expressions have to be true.

$$L_g L_f^{1-1} h(x) = L_g L_f^{2-1} h(x) = \dots = L_g L_f^{n-2} h(x) = 0$$

$$L_g L_f^{n-1} h(x) \neq 0$$

From Claim 1, it is clear that for SMIB system,  $r=n=3$ ,  $L_g L_f^{1-1} h(x) = L_g L_f^{2-1} h(x) = 0$  and  $L_g L_f^{3-1} h(x) \neq 0$ .

**Step 3: Nonlinear coordinate transformation and formulation of fully linearized system**

In this step, the original  $x$  states are transformed into  $z$  states through nonlinear coordinate transformation by choosing

$$z_1 = h(x) = L_f^{1-1} h(x)$$

Then we can write

$$\dot{z}_1 = \frac{\partial h(x)}{\partial x} \dot{x}$$

Substituting equation (1) into the above equation for  $\dot{x}$ , we get

$$\dot{z}_1 = \frac{\partial h(x)}{\partial x} f(x) + \frac{\partial h(x)}{\partial x} g(x)u = L_f h(x) + L_g L_f^{1-1} h(x)u$$

As  $L_g L_f^{1-1} h(x) = 0$ , from the above equation it can be written that

$$\dot{z}_1 = L_f^{2-1} h(x) = z_2$$

Therefore,

$$\dot{z}_1 = z_2 = L_f^{2-1} h(x)$$

$$\dot{z}_2 = z_3 = L_f^{3-1} h(x)$$

⋮

$$\dot{z}_{n-1} = z_n = L_f^{n-1} h(x)$$

$$\dot{z}_n = v = L_f^n h(\phi^{-1}(z)) + L_g L_f^{n-1} h(\phi^{-1}(z))u \quad (9)$$

The fully linearized system with new coordinates  $z = [z_1 \ z_2 \ \dots \ z_n]^T$  can be written as

$$\dot{z} = Az + Bv \quad (10)$$

where,

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}$$

$$B = [0 \ 0 \ \dots \ 1]^T$$

and  $v$  is the control input for exactly linearized system.

For SMIB system,

$$z_1 = h(x) = \delta - \delta_0 = \int_0^t \Delta\omega \, dt$$

$$z_2 = L_f^{2-1}h(x) = \omega - \omega_0 = \Delta\omega$$

$$z_3 = L_f^{3-1}h(x) = \Delta\dot{\omega}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B = [0 \ 0 \ 1]^T$$

$$v = L_f^3 h(x) + L_g L_f^2 h(x)u \quad (11)$$

$$L_f^3 h(x) = -\omega_0 \frac{\Delta\omega}{2H} \frac{V_s E'_q}{x'_{d\Sigma}} \cos \delta - \frac{D}{2H} \Delta\dot{\omega} - \frac{\omega_0}{2H} \frac{V_s}{x'_{d\Sigma}} E'_q \sin \delta$$

$$L_g L_f^2 h(x) = -\frac{\omega_0}{2H} \frac{V_s}{x'_{d\Sigma}} \sin \delta \frac{1}{T_{do}}$$

The nonlinear observer can be obtained from this exactly linearized model which is shown in the following section.

## 5. Nonlinear Observer Design for Power System

We know that for any linear system

$$\dot{z} = Az + Bv$$

$$y = Cz$$

where  $C$  is the output matrix of the system. If the pair  $(A, C)$ , observable, the linear observer of the above linear system can be written as [38]

$$\dot{\hat{z}} = A\hat{z} + Bv + G(y - \hat{y})$$

$$\hat{y} = C\hat{z}$$

where  $G$  is the linear gain which can be calculated by using any linear observer gain calculation methods as discussed in the literature [17-19].

By substituting  $v$  from (9), the above equation can be written as

$$\dot{\hat{z}} = A\hat{z} + B[a(\phi^{-1}(\hat{z})) + b(\phi^{-1}(\hat{z}))u] + G(y - h(\phi^{-1}(\hat{z}))) \quad (12)$$

where  $a(\phi^{-1}(\hat{z})) = L_f^n h(\phi^{-1}(z))$  and  $b = L_g L_f^{n-1} h(\phi^{-1}(z))$ .

Using the analogy of linear observer, the nonlinear observer can be written as

$$\dot{\hat{x}} = f(\hat{x}) + g(\hat{x})u + l(\hat{x})(y - h(\hat{x})) \quad (13)$$

where  $l$  is the nonlinear gain which is

$$l(\hat{x}) = (J_\phi(\hat{x}))^{-1}G \quad (14)$$

Here,  $J_\phi$  is the Jacobian matrix of coordinates obtained from nonlinear coordinate transformation, i.e.,

$$J_\phi(\hat{x}) = \begin{bmatrix} dh(\hat{x}) \\ dL_f h(\hat{x}) \\ \vdots \\ dL_f^{n-1} h(\hat{x}) \end{bmatrix}$$

The formulation of equation (13) and (14) from (12) will be clarified from a theorem and its proof as presented in Appendix B.

For SMIB system, the nonlinear gain is

$$l = (J_\phi(\hat{x}))^{-1}(G) = \begin{bmatrix} dh(\hat{x}) \\ dL_f h(\hat{x}) \\ dL_f^2 h(\hat{x}) \end{bmatrix}^{-1} \begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix}$$

The Jacobian matrix ( $J_\phi$ ) can be calculated as follows:

$$J_\phi(\hat{x}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{2H} \frac{V_s E'_{q0}}{x'_{d\Sigma}} \cos \hat{\delta} & -\frac{D}{2H} & -\frac{1}{2H} \frac{V_s}{x'_{d\Sigma}} \sin \delta_0 \end{bmatrix}$$

and the inverse of  $J_\phi$  is

$$(J_{\phi}(\hat{x}))^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{E'_{q0} \cos \hat{\delta}}{\sin \delta_0} & -\frac{Dx'_{d\Sigma}}{V_s \sin \delta_0} & -\frac{2Hx'_{d\Sigma}}{V_s \sin \delta_0} \end{bmatrix}$$

$G = [2 \ 2 \ 1]^T$  is calculated by using Kalman filtering [39]. Using (14), the nonlinear gain  $l$  can be written as

$$l = \begin{bmatrix} 2 \\ 2 \\ \frac{x'_{d\Sigma}}{V_s \sin \delta_0} \left( \frac{E'_{q0} V_s}{x'_{d\Sigma}} \cos \hat{\delta} - D - 2H \right) \end{bmatrix}$$

Therefore, the full-order nonlinear observer for SMIB system can be written as

$$\begin{bmatrix} \dot{\hat{\delta}} \\ \dot{\hat{\omega}} \\ \dot{\hat{E}'_q} \end{bmatrix} = \begin{bmatrix} \hat{\omega} - \omega_0 \\ -\frac{D}{2H}(\hat{\omega} - \omega_0) + \frac{\omega_0}{2H} P_m - \frac{\omega_0}{2H} \frac{V_s \hat{E}'_q}{x'_{d\Sigma}} \sin \hat{\delta} \\ -\frac{1}{T'_d} \hat{E}'_q + \frac{1}{T_{do}} \frac{x_d - x'_d}{x'_{d\Sigma}} V_s \cos \hat{\delta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_{do}} \end{bmatrix} E_f$$

$$+ \begin{bmatrix} 2 \\ 2 \\ \frac{x'_{d\Sigma}}{V_s \sin \delta_0} \left( \frac{E'_{q0} V_s}{x'_{d\Sigma}} \cos \hat{\delta} - D - 2H \right) \end{bmatrix} (\delta - \hat{\delta})$$

The observed states obtained from the designed observers, are used as the input of exact linearizing excitation controller. The derivation of exact linearizing control law is shown in the next section.

## 6. Excitation Controller Design for Power System

From (9), the control law for exactly linearized system can be written as

$$u = \frac{-L_f^n h(\phi^{-1}(z)) + v}{L_g L_f^{n-1} h(\phi^{-1}(z))} \quad (15)$$

The new control input  $v$ , for linear system (10) can be determined by using linear control techniques. If we use state-feedback, then

$$v = -Kz$$

where  $K$  is the feedback gain. We can write  $v$  as

$$v = -k_1 h(x) - k_2 L_f h(x) - \dots - k_n L_f^{n-1} h(x)$$

Hence, the final control law for the nonlinear system (1)-(2) is

$$u = -\frac{L_f^n h(x) + k_1 h(x) + k_2 L_f h(x) + \dots + k_n L_f^{n-1} h(x)}{L_g L_f^{n-1} h(x)} \quad (16)$$

For the considered power system, the control law can be written as

$$u = -\frac{k_1 h(x) + k_2 L_f h(x) + k_3 L_f^2 h(x) + L_f^3 h(x)}{L_g L_f^2 h(x)} \quad (17)$$

We calculate  $k_1$ ,  $k_2$ , and  $k_3$  using LQR for fully linearized SMIB system with  $Q = \text{diag}(1 \ 1 \ 0)$  and  $R = 1$  as  $k_1 = 1$ ,  $k_2 = 2.29$ , and  $k_3 = 2.14$ . Therefore, the control law for SMIB system can be written as

$$u = \frac{1}{T_{do}} E'_q + \frac{\Delta\delta + \left( k_2 - \frac{\omega_0}{2H} \frac{V_s E'_q}{x'_{d\Sigma}} \cos\delta \right) \Delta\omega + \left( k_3 - \frac{D}{2H} \right) \Delta\dot{\omega}}{\frac{\omega_0}{2H} \frac{V_s}{x'_{d\Sigma}} \sin\delta \frac{1}{T_{do}}} \quad (18)$$

The estimated states obtained from the observers are used to implement this control law. The performance of observer-based exact linearizing excitation controller is discussed and compared with the exact linearizing excitation controller when the control law is expressed in terms of measured variables in Section 7.

### ***6.1. Provisions to implement on a large scale power system***

The exact linearization of multimachine systems using the method proposed in this paper transforms the system into several linear decoupled subsystems based on the number of inputs and outputs. Suppose, if there are  $N$  machines in a system which has  $N$  inputs and  $N$  outputs. In this case, feedback linearization transforms the system into  $N$  linear decoupled subsystems [44] where each subsystem is exactly linearized if the rotor angle is the output of each subsystem. Under these circumstances, the control law contains the states of the other generators but these states can be modified into the real and reactive power of the local generators. The final control law is simplified enough and uses the information from the local generators only. This means that the observer-based controllers of power systems having multiple generators are independent in the sense that they require only local information of the generators to which they are applied.

## **7. Simulation Results**

The block diagram representation of power systems with nonlinear observer-based excitation controller is shown in Fig. 2. The observer-based excitation controller is implemented on two test systems (SMIB and 3 machine 11 bus systems) through the exciter of the synchronous generator (SG). To perform the simulation in this paper, the model of excitation system is considered as IEEE type AC4A (ESAC4A) whose physical limit of the excitation voltage is  $\pm 5$  pu. Moreover, the input mechanical power supplied to the generator is assumed as constant throughout the simulation. Simulations are carried out to investigate

the performances of the proposed controller following large external disturbances within the system. There are mainly two parts of the simulation- one is the implementation of the proposed control scheme on a simple SMIB system and the other is the performance evaluation of the controller on a large power system.

### ***7.1. Performance of nonlinear observer-based excitation controller on SMIB system***

In this case, the test system (SMIB system) consists of a synchronous generators connected to infinite bus through double circuit transmission lines. When major external disturbances occur on SMIB system, the system becomes unstable and the instability problem related to such system is known as transient stability problem. This type of instability causes rotor angle oscillations of small magnitudes and low frequencies which have been persisting for a long term and in some cases, it limits the amount of power transmission. Under this circumstance, the exciter of synchronous generator should be capable of responding quickly to the disturbances by regulating the excitation to enhance the dynamic stability of the system. The proposed observer-based controller does the same which is demonstrated by the simulation results. The numerical values of the parameters used for simulations are given in Appendix C. In this paper, a three-phase fault is applied at the terminal of the synchronous generator for which the following fault sequence is considered:

- Fault occurs at  $t=1$  s
- Fault is cleared at  $t=1.6$  s

Since the generator is supplying constant power, the rotor angle of the synchronous generator should be same to that of pre-fault condition after the occurrence of fault. From Fig. 3, it is seen the proposed controller stabilizes to rotor angle within few cycles of the

three-phase fault by providing additional damping through the exciter of synchronous generator. Synchronous generator operates at synchronous speed, i.e., the speed deviation is zero under normal operating conditions. But the speed is also disturbed, when a fault occurs within the system (especially at the terminal of SG). Fig. 4 shows the speed deviation response (solid line) with the observer-based excitation controller where zero speed deviation is obtained during the post-fault steady-state operation. Since the system is dominated by transient stability, the voltage stability will be unaffected by the external disturbances if the transient stability of the system is ensured. The proposed controller ensures the transient stability of the system. Therefore, the terminal voltage of the generator which is zero during faulted period (1.1 s to 1.6 s), will reset to pre-fault voltage during post-fault operation. Fig. 5 presents the terminal voltage response (solid line) of the proposed controller. Thus, the proposed controller enhances the dynamic stability of power system following large disturbance.

## ***7.2. Performance of nonlinear observer-based excitation controller on 3 machine 11 bus system***

In this subsection, a two area 3 machine 11 bus test system [2] as shown in Fig. 6 is considered to analyze the performance of the proposed excitation controller. Area 1 has two synchronous generators,  $G_1$  and  $G_2$ . In this analysis,  $G_1$  is considered as an infinite bus and  $G_2$  which has a nominal capacity of 2200 MVA is supplying the remote area 2 through five 500 kV parallel lines. Area 2 also contains 1400 MVA local generator  $G_3$  and two aggregated loads. The total load on the system is  $P_L=6655$  MW and  $Q_L=2021$  Mvar. Other details about the system can be found in [2]. In the simulation, transient level generator model is used for all the synchronous generators.

Before implementing a controller on a multimachine power system, it is essential to perform the modal analysis [2] of the system to identify the dominant modes. Special care should be taken for the generators which dominate the stability of the system. For the 3 machine 11 bus test system, the dominant mode is  $-0.31592 \pm j12.445$  and from the participation factor of the dominant mode it is seen that the speed deviation and rotor angle of  $G_3$  is participating more which indicates that extra care is needed for  $G_3$ . In this paper, the controller is connected to the exciter of  $G_3$ . To evaluate the performance of the controller a three-phase short circuit fault is applied at bus 3 where  $G_3$  is connected. The following fault sequence is considered:

- Fault occurs at  $t=1$  s
- Fault is cleared at  $t=1.2$  s

The rotor angle response of  $G_3$  as presented in Fig. 7 indicates that the proposed observer-based controller damps out the oscillation by settling the rotor angle to the equilibrium point within a few cycles of fault occurrence. The speed deviation of  $G_3$  is also zero when post-fault steady-state is achieved with the proposed controller which is shown in Fig. 8. The terminal voltage as shown in Fig. 9, is also unaffected after clearing the fault. Simulation results clearly investigate that the proposed control scheme is also suitable for large power systems.

The simulation results show the effectiveness of the proposed control scheme on small and large power systems. The solid lines in the figures present performances of the observer-based controller and the dotted lines represent the performances of the exact linearizing controller where the control law is expressed in terms of all measured variables. Simulation results also clarify the accuracy of the proposed control scheme with less computation

burden, i.e., reduces the expense of transforming the state variables in terms of measured variables.

## 8. Conclusion

The nonlinear full-order observer-based nonlinear excitation controller is designed when the power system is exactly linearized which is shown through valid claim. The key point of designing an observer is the calculation of its gain. This paper shows the calculation of nonlinear gain from the analogy of linear observer design which is the main novelty of this paper. The theorem presented in this paper, justifies the considered nonlinear gain of the observer. The estimated states of power systems obtained by using the proposed nonlinear observers are used in the exact linearizing excitation controller to maintain the stability of power systems. The performances of the exact linearizing excitation controller with the estimated states are very similar to the exact linearizing controller which contains all measurable terms. But power system cannot always be fully linearized, it may be partially linearized. The future work will deal with the design of nonlinear observer for partially linearized power system as well as power systems model with uncertainties.

## Appendix

### Appendix A. Exact Linearizability of Power Systems

**Claim.** The SMIB system (6)--(8) is fully linearizable with respect to the output,  $y=h(x) = \delta - \delta_0$ .

**Proof.** We choose, the output function of SMIB system as  $y=h(x) = \delta - \delta_0$ . Now, it is essential to calculate the relative degree  $r$  of the system. If the relative degree  $r$  equals to the order  $n$  of the system, then the system is fully linearizable [16]. We have

$$L_f^{1-1}h(x) = h(x) = \delta - \delta_0 = \Delta\delta$$

and

$$L_g L_f^{1-1}h(x) = L_g h(x) = \frac{\partial h(x)}{\partial x} g(x) = 0$$

Again,

$$L_f^{2-1}h(x) = L_f h(x) = \omega - \omega_0 = \Delta\omega$$

and

$$L_g L_f^{2-1}h(x) = \frac{\partial(L_f h(x))}{\partial x} g(x) = 0$$

Finally,

$$L_f^{3-1}h(x) = L_f^2 h(x) = -\frac{D}{2H} \Delta\omega + \frac{P_m}{2H} \omega_0 - \frac{\omega_0}{2H} \frac{V_s E'_q}{x'_{d\Sigma}} \sin \delta$$

and

$$L_g L_f^{3-1}h(x) = \frac{\partial(L_f^2 h(x))}{\partial x} g(x) = -\frac{\omega_0}{2H} \frac{V_s}{x'_{d\Sigma}} \sin \delta \frac{1}{T_{do}} \neq 0$$

From the above calculation, it is clear the relative degree of the SMIB system is equal to the order of the system which is 3. Thus, the system is fully or exactly linearizable with  $y=h(x) = \delta - \delta_0$ .

## Appendix B. Nonlinear Gain of the Observer

**Theorem.** The system  $\dot{\hat{x}} = f(\hat{x}) + g(\hat{x})u + l(\hat{x})(y - h(\hat{x}))$  can be transformed to  $\dot{\hat{z}} = A\hat{z} + B[a(\phi^{-1}(\hat{z})) + b(\phi^{-1}(\hat{z}))u] + G(y - h(\phi^{-1}(\hat{z})))$  with nonlinear gain  $l(\hat{x}) = (J_\phi(\hat{x}))^{-1}G$ .

**Proof.** The Jacobian matrix  $J_\phi$  can be written as

$$J_\phi(\hat{x}) = \frac{\partial \phi(\hat{x})}{\partial \hat{x}}$$

Multiplying both sides of (13) by  $J_\phi$ , we get

$$\frac{\partial \phi(\hat{x})}{\partial \hat{x}} \dot{\hat{x}} = \frac{\partial \phi(\hat{x})}{\partial \hat{x}} f(\hat{x}) + \frac{\partial \phi(\hat{x})}{\partial \hat{x}} g(\hat{x})u + G(y - h(\hat{x}))$$

For very first step if we put  $\phi(\hat{x}) = h(\hat{x})$ , then from the above equation we can write

$$\frac{\partial h(\hat{x})}{\partial \hat{x}} \dot{\hat{x}} = \frac{\partial h(\hat{x})}{\partial \hat{x}} f(\hat{x}) + \frac{\partial h(\hat{x})}{\partial \hat{x}} g(\hat{x})u + G(y - h(\hat{x}))$$

which implies that

$$\dot{\hat{z}}_1 = L_f h(\hat{x}) + L_g h(\hat{x})u + G_1(y - h(\hat{x}))$$

But  $L_g h(\hat{x}) = 0$  and  $L_f h(\hat{x}) = \hat{z}_2$ , therefore,

$$\dot{\hat{z}}_1 = \hat{z}_2 + G_1(y - h(\hat{x}))$$

In a similar way,

$$\dot{\hat{z}}_2 = \hat{z}_3 + G_2(y - h(\hat{x}))$$

⋮

$$\dot{\hat{z}}_{n-1} = \hat{z}_n + G_{n-1}(y - h(\hat{x}))$$

and as  $L_g L_f^{n-1} h(\hat{x}) \neq 0$ ; finally, we can write

$$\dot{\hat{z}}_n = a(\phi^{-1}(\hat{z})) + b(\phi^{-1}(\hat{z}))u + G_n(y - h(\hat{x}))$$

Therefore, using these relations we can write

$$\dot{\hat{z}} = A\hat{z} + B[a(\phi^{-1}(\hat{z})) + b(\phi^{-1}(\hat{z}))u] + G(y - h(\phi^{-1}(\hat{z})))$$

which is equation (12). Hence, the proof.

### **Appendix C. Parameters of SMIB system**

The parameters used for the SMIB system are given below:

Synchronous generator parameters:

$$x_d = 2.1 \text{ pu}, x'_d = 0.4 \text{ pu}, H = 3.5 \text{ s}, T_{do} = 8 \text{ s}, D = 4.$$

Transformer parameter:  $x_T = 0.016 \text{ pu}$ .

Transmission line parameter:  $x_L = 0.054 \text{ pu}$ .

Infinite bus voltage:  $V_s = 1 \text{ pu}$ .

Input mechanical power:  $P_m = 0.9 \text{ pu}$ .

### **References**

- [1] Gibbard MJ. Robust design of fixed parameter stabilizers over a wide range of operating conditions. IEEE Transactions on Power Systems 1991; 6(2):794-800.
- [2] Kundur P. Power System Stability and Control. New York: McGraw-Hill, 1994.
- [3] Gurralla G, Sen I. Power system stabilizers design for interconnected power systems. IEEE Transactions on Power Systems 2010; 25(2):1042-1051.

- [4] Reddy J, Kishore MJ. Real time implementation of  $H^\infty$  loop shaping robust PSS for multimachine power system using dSPACE. International Journal of Electrical Power and Energy Systems 2011; 33(10):1750-1759.
- [5] Folly KA. Performance evaluation of power system stabilizers based on Population-Based Incremental Learning (PBIL) algorithm. International Journal of Electrical Power and Energy Systems 2011; 33(7):1279-1287.
- [6] Ramos RA, Martins ACP, Bretas NG. An improved methodology for the design of power system damping controllers. IEEE Transactions on Power Systems 2005; 20(4):1938-1945.
- [7] Hossain MJ, Pota HR, Ugrinovskii VA, Ramos RA. Voltage mode stabilisation in power systems with dynamic loads. International Journal of Electric Power and Energy Systems 2010; 32(9):911-920.
- [8] Dysko A, Leithead WE, O'Reilly J. Enhanced power system stability by coordinated PSS design. IEEE Transactions on Power Systems 2010; 25(1):413-422.
- [9] Mahmud MA, Hossain MJ, Pota HR. Effects of large dynamic loads on interconnected power systems with power oscillation damping controller (PODC). Proceedings of 20<sup>th</sup> Australasian Universities Power Engineering Conference, Christchurch, New Zealand, 2010.
- [10] Lahdhiri T, Alouani TA. Nonlinear stabilizing controller for a single machine infinite-bus system. Proceedings of 4<sup>th</sup> IEEE Conference on Control Applications, New York, USA, 1995.
- [11] Lahdhiri T, Alouani TA. Design of a robust nonlinear excitation controller for a single machine infinite-bus system. Proceedings of 29th Southeastern Symposium on System Theory, Washington DC, USA, 1997.

- [12] Glickman G, Shea PO, Ledwich G. Estimation of modal damping in power networks. IEEE Transactions on Power Systems 2007; 22(3): 1340-1350.
- [13] Lahdhiri T, Alouani TA. Design of a robust nonlinear excitation controller for a synchronous generator using the concept of exact stochastic feedback linearization. Proceedings of American Control Conference, New Mexico, USA, 1997.
- [14] Chapman JW, Ilic MD, King CA, Eng L, Kaufman H. Stabilizing a multimachine power system via decentralized feedback linearizing excitation control. IEEE Transactions on Power Systems 1993; 8(3):830-839.
- [15] King CA, Chapman JW, Ilic MD. Feedback linearizing excitation control on a full-scale power system model. IEEE Transactions on Power Systems 1994; 9(2):1102-1109.
- [16] Lu Q, Sun Y, Mei S. Nonlinear Control Systems and Power System Dynamics. London: Kluwer Academic Publisher, 2001.
- [17] Wang Y, Guo G, Hill DJ, Gao L. Nonlinear decentralized coordinated control for multimachine power systems. Proceedings of Int. Conf. on Energy Management and Power Delivery, 1995.
- [18] Guo G, Hill DJ, Wang Y. Global transient stability and voltage regulation for power systems. IEEE Transactions on Power Systems 2001; 16(4):678-688.
- [19] Kenné G, Goma R, Nkwawo H, Lamnabhi-Lagarrigue F, Arzandé A, Vannier JC. An improved direct feedback linearization technique for transient stability enhancement and voltage regulation of power generators. International Journal of Electrical Power and Energy Systems 2010; 32(7):809-816.

- [20] Guo G, Hill DJ, Wang Y. Nonlinear output stabilization control for multimachine power systems. *IEEE Transactions on Circuits and Systems-Part 1* 2000; 47(1):46-52.
- [21] Lee SS, Park JK. Design of reduced-order observer-based variable structure power system stabiliser for unmeasurable state variables. *IEE Proc. of Generation, Transmission and Distribution* 1998; 145(5):525-530.
- [22] Scholtz E, Lesieutre BC. Graphical observer design suitable for large-scale DAE power systems. *Proceedings of 47<sup>th</sup> IEEE Conference on Decision and Control, Cancun, Mexico, 2008.*
- [23] Rousseaux P, Cutsem TV, Liacco TED, Ramos RA. Whither dynamic state estimation?. *International Journal of Electric Power and Energy Systems* 1990; 12(2):104-116.
- [24] Pires RC, Costa AS, Mili L. Iteratively reweighed least-squares state estimation through givens rotations. *IEEE Transactions on Power Systems* 1999; 14(3):1499-1507.
- [25] Dash PK, Pradhan AK, Panda G. Frequency estimation of distorted power system signals using extended complex Kalman filter. *IEEE Transactions on Power Delivery* 1999; 14(3):761-766.
- [26] Blood E, Krogh B, Ilic MD. Electric power system static state estimation through Kalman filtering and load forecasting. *Proceedings of IEEE Power and Energy Society General Meeting, Calgary, Canada, 2008.*
- [27] Sarkka S. On unscented Kalman filtering for state estimation of continuous-time nonlinear systems. *IEEE Transactions on Automatic Control* 2007; 52(9):1631-1641.
- [28] Valverde V, Terzija V. Unscented Kalman filter for power system dynamic state estimation. *IET Generation, Transmission and Distribution* 2011; 5(1):29-37.

- [29] Karagiannis D, Astolfi A. Nonlinear observer design using invariant manifolds and applications. Proceedings of 44<sup>th</sup> IEEE Conference on Decision and Control and European Control Conference, Seville, Spain, 2005.
- [30] Kaprielian SR, Clements KA, Turi J. Applications of exact linearization techniques for steady-state stability enhancement in a weak AC/DC system. IEEE Transactions on Power Systems 1992; 7(2):536-543.
- [31] Eriksson R, Knazkins V, Söder L. Coordinated control of multiple HVDC links using input–output exact linearization. Electric Power Systems Research 2010; 80(12):1406-1412.
- [32] Isidori A. Nonlinear Control Systems (2<sup>nd</sup> edn). Berlin: Springer-Verlag, 1989.
- [33] Gauthier JP, Hammouri H, Othman S. A simple observer for nonlinear systems: applications to bioreactors. IEEE Transactions on Automatic Control 1992; 37(6):875-880.
- [34] Ciccarella G, Morra MD, Germani A. A Luenberger-like observer for nonlinear systems. International Journal of Control 1993; 57(3):537-556.
- [35] Jo NH, Seo JH. Input output linearization approach to state observer design for nonlinear system. IEEE Transactions on Automatic Control 2000; 45(12):2388-2393.
- [36] Slotine JJE, Li W. Applied Nonlinear Control. New Jersey: Prentice-Hall, 1991.
- [37] Khalil HK. Nonlinear Systems. New York: Prentice-Hall, 1996.
- [38] Ogata K. Modern Control Engineering (5<sup>th</sup> edn). New York: Prentice-Hall, 2010.
- [39] Grewal MS, Andrews AP. Kalman Filtering: Theory and Practice Using Matlab (3<sup>rd</sup> Edn). New Jersey: Prentice-Hall, 2008.

- [40] Jiang L, Wu QH, Wang J, Zhang C, Zhou XX. Robust observer-based nonlinear control of multimachine power systems. *IEE Proc. Generation, Transmission, and Distribution* 2001; 148(6):623-631.
- [41] Ouassaid M, Maaroufi M, Cherkaoui M. Observer-based nonlinear control of power system using sliding mode control strategy. *Electric Power Systems Research* 2012; 84(1):135-143.
- [42] Maya-Oriz P, Espinosa-PCre G. Observer-based IDA control of synchronous generators. *Proceedings of 42<sup>nd</sup> IEEE Conference on Decision and Control, Cancun, Mexico, 2003.*
- [43] Leon AE, Solsona JA, Valla MI. Observer-based nonlinear control of power system using sliding mode control strategy. *Energy Conversion and Management* 2012; 53(1):55-67.
- [44] Lu Q, Sun Y, Xu Z, Mochizuki T. Decentralized nonlinear optimal excitation control. *IEEE Transactions on Power Systems* 1996; 11(4):1957-1962.

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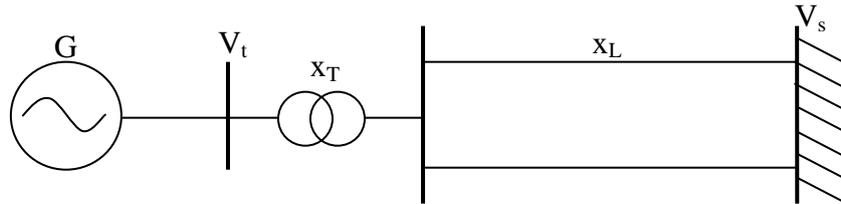


Figure 1: Test system: SMIB system

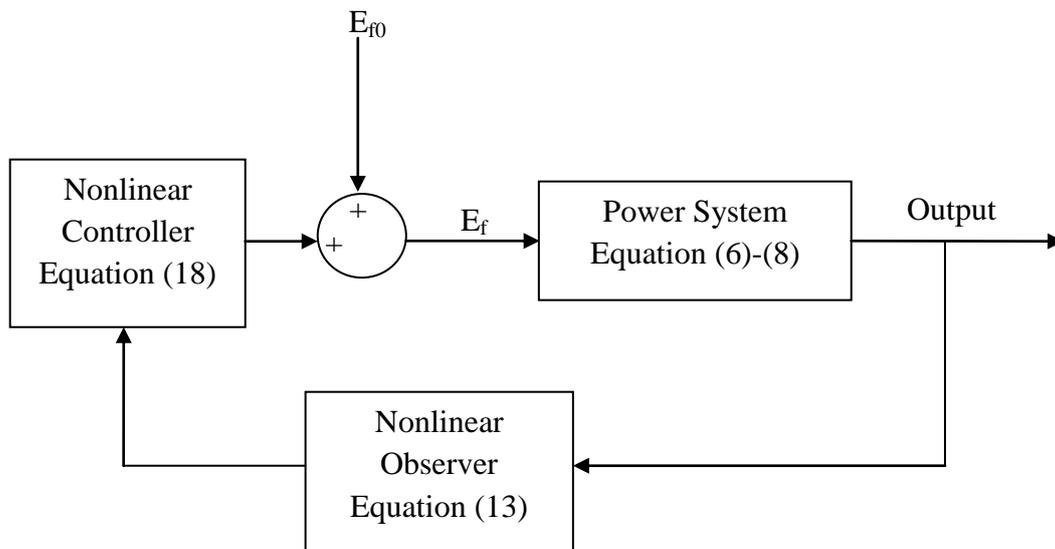


Figure 2: Power System with nonlinear observer-based nonlinear controller

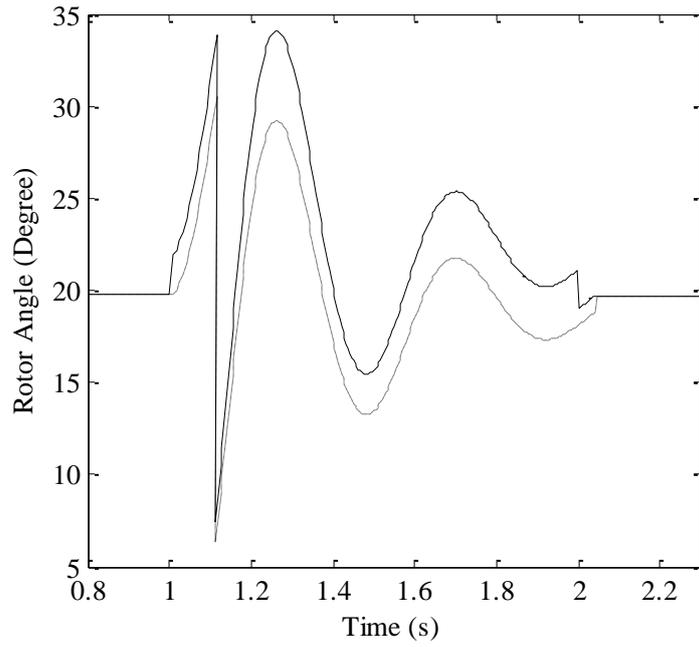


Figure 3: Rotor angle of SG connected to SMIB (Solid line – observer feedback, dotted line – actual state feedback)

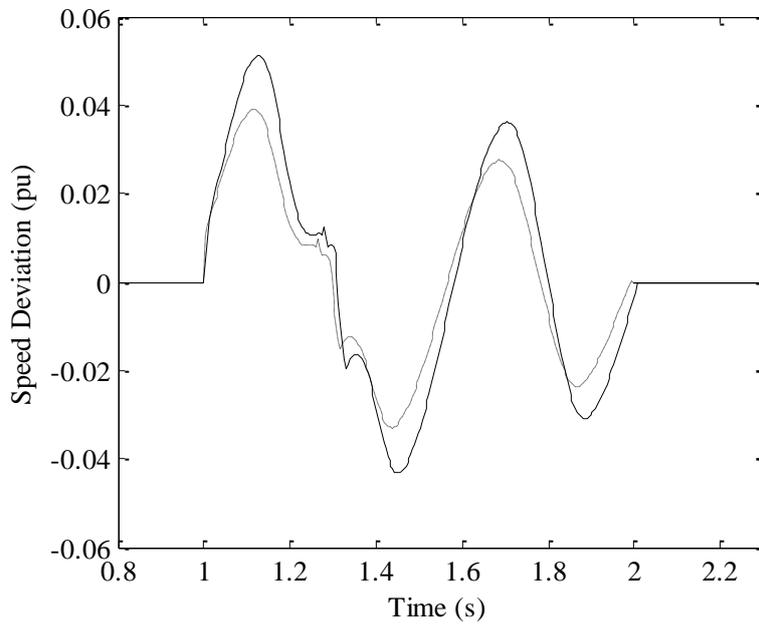


Figure 4: Speed deviation of SG connected to SMIB (Solid line – observer feedback, dotted line – actual state feedback)

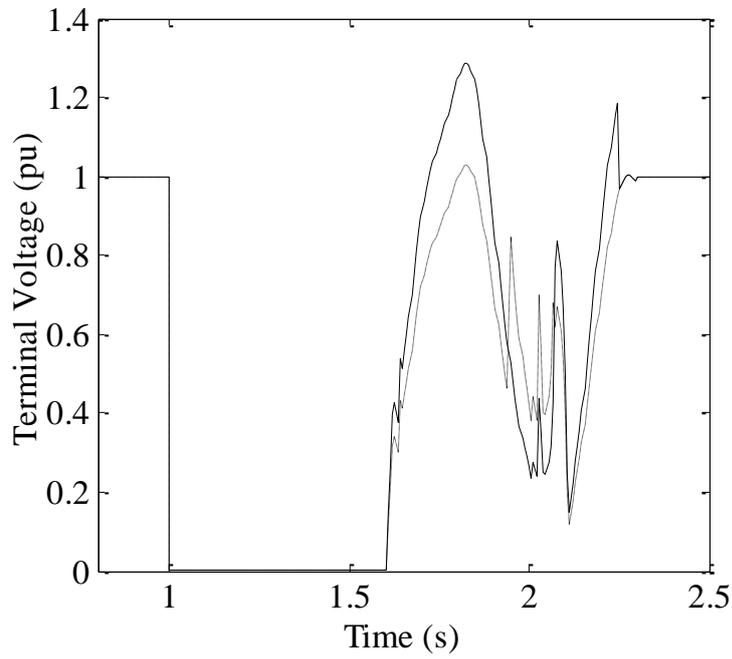


Figure 5: Terminal voltage of SG connected to SMIB (Solid line – observer feedback, dotted line – actual state feedback)

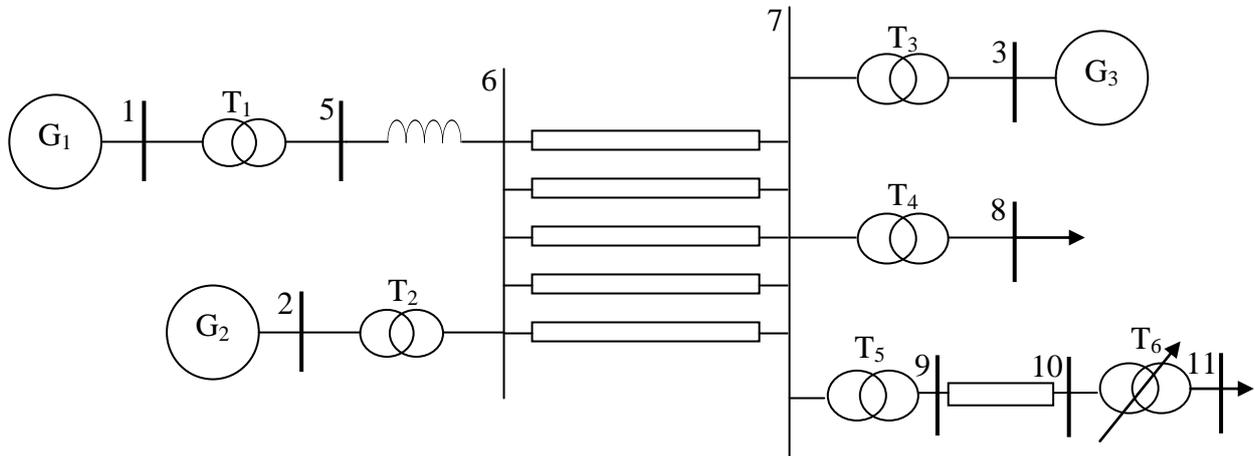


Figure 6: Test system: A benchmark 3 machine 11 bus two area system

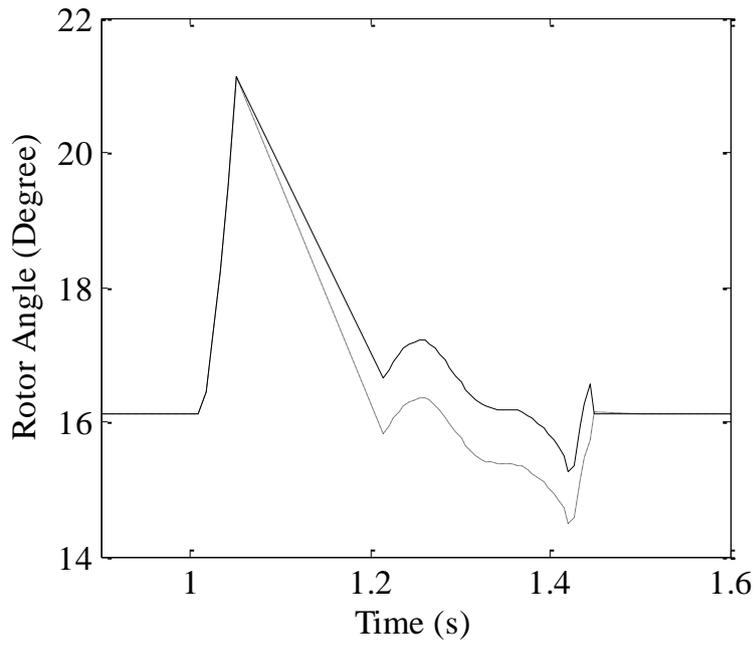


Figure 7: Rotor angle of  $G_3$  (Solid line – observer feedback, dotted line – actual state feedback)

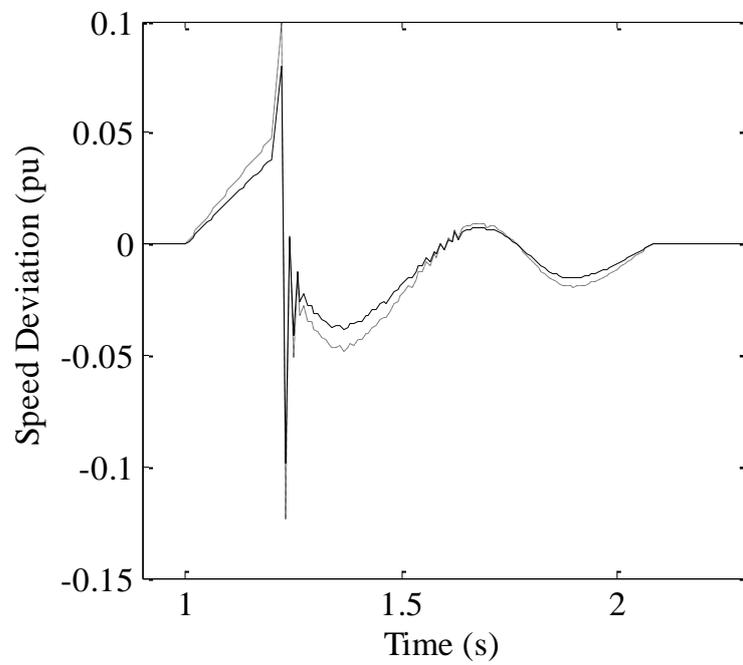


Figure 8: Speed deviation of  $G_3$  (Solid line – observer feedback, dotted line – actual state feedback)

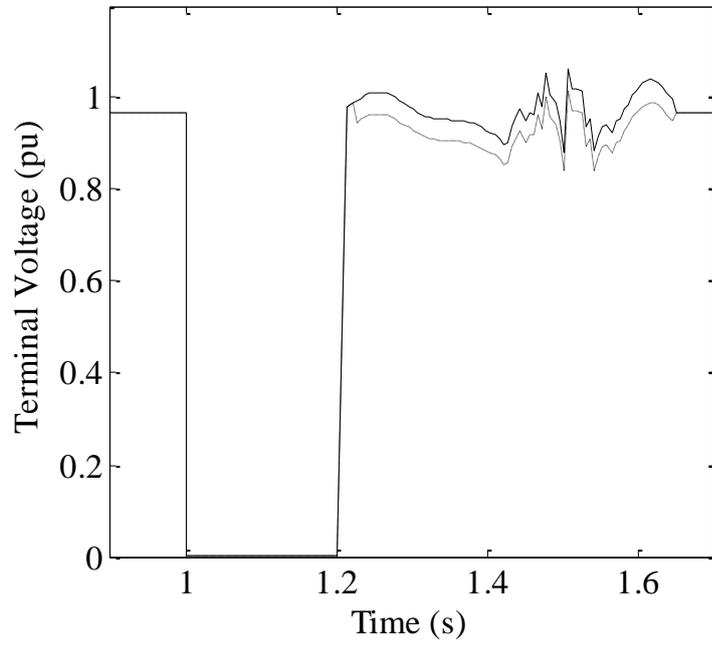


Figure 9: Terminal voltage of  $G_3$  (Solid line – observer feedback, dotted line – actual state feedback)