

Quantum weak values

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We experimentally determine weak values for a single photon's polarization, obtained via a weak measurement that employs a two-photon entangling operation, and postselection. The weak values cannot be explained by a semiclassical wave theory, due to the two-photon entanglement. We observe the variation in the size of the weak value with measurement strength, obtaining an average measurement of the \mathcal{S}_1 Stokes parameter more than an order of magnitude outside of the operator's spectrum for the smallest measurement strengths.

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It is commonly thought that the mean value of a quantum mechanical measurement must be bounded by the extrema of a spectrum of eigenvalues, a consequence of statistical mathematics and the measurement postulate of quantum mechanics. However, there exist certain measurement outcomes for which this is not the case – these results are called weak values, since they arise as the outcomes of *weak measurements* on certain pre- and postselected quantum systems [1–11]. The canonical example of weak values is the *gedanken experiment* of Aharonov, Albert and Vaidman (AAV) [1], who described how it would be possible to use a weak measurement to measure (say) the σ_z eigenvalue of a spin-1/2 particle, and determine an average value $\langle\sigma_z\rangle = 100$.

Weak values are an important and interesting phenomenon, because they assist us in understanding many counterintuitive results of quantum mechanics. For instance, weak values form a language by which we can resolve certain paradoxes and model strange quantum behaviour. Important examples include: Hardy's paradox [12, 13], in which two particles that always annihilate upon meeting are sometimes paradoxically measured after this annihilation event; the apparent superluminal transport of pulses in optical fibres displaying polarization mode dispersion [9]; apparently superluminal particles travelling in vacuum [8]; and quantifying momentum transfer in twin-slit “which-path” experiments [14–16]. Weak values are useful in simplifying calculations whenever a system is weakly coupled to a monitored environment [7, 9]. They also are an example of a manifestly quantum phenomenon, in that the analysis of weak values can lead to negative (pseudo-) probabilities [12], an effect never observed in analogous classical measurements.

Here we present the first unambiguously quantum-mechanical experimental realization of weak values, where we use a nondeterministic entangling circuit to enable one single photon to make a weak measurement of the polarization of another, subject to certain pre- and postselections. Previous demonstrations of weak values using electromagnetic radiation [17–20] have used coher-

ent states and weak measurements arising from the coupling of two degrees of freedom of the photon. They can thus be explained semiclassically using a wave equation derived from Maxwell's equations [21]. A cavity QED experiment [22] has been performed that was subsequently analyzed in terms of weak values [7], but the continuous spectrum precluded observations of anomalously large average values. By using two single photons, and realizing the weak measurement with a two-particle entangling operation, the weak values we measure (including extra-spectral weak values) are not able to be described in semiclassical terms—a crucial result in the experimental verification and study of the phenomenon.

The observable we measure is the polarization of a single photon in the horizontal-vertical (H - V) basis, i.e. the quantum operator corresponding to the \mathcal{S}_1 Stokes parameter [23], $\hat{\mathcal{S}}_1 = |H\rangle\langle H| - |V\rangle\langle V|$, with expectation value $\langle\hat{\mathcal{S}}_1\rangle = \langle\mu|\hat{\mathcal{S}}_1|\mu\rangle$ for some state $|\mu\rangle$. According to the standard quantum mechanical formulation of measurement [24], $-1 \leq \langle\hat{\mathcal{S}}_1\rangle \leq 1$ for any single photon polarization state. We will find that it is possible, using weak measurements, to obtain average values for \mathcal{S}_1 far outside this range.

By analogy with the scheme of AAV, we prepare the polarization of a single photon in the state

$$|\psi\rangle = \alpha|H\rangle + \beta|V\rangle, \quad (1)$$

where $|\alpha|^2 + |\beta|^2 = 1$. Subsequently, we make a weak, non-destructive measurement on the photon's polarization in the H - V basis. The weak measurement is made using a nondeterministic generalized photon polarization measurement device [25, 26], which is deemed to have worked whenever a single photon is present at each of the signal and meter outputs. The generalized measurement device works by entangling the signal photon polarization with the polarization of a meter photon prepared in the state $\gamma|H\rangle + \bar{\gamma}|V\rangle$, before measuring the meter photon's polarization. Without loss of generality we choose γ to be real; $\gamma^2 + \bar{\gamma}^2 = 1$. The setup we use for our present experiment is shown in Fig. 1.

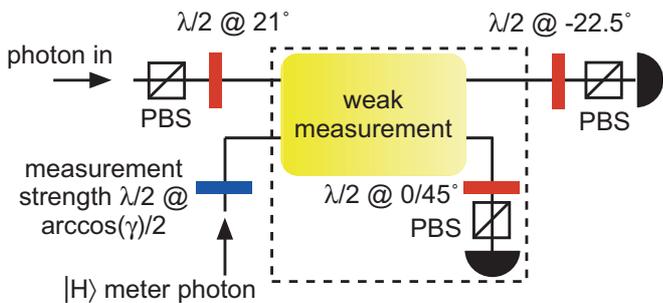


FIG. 1: Conceptual representation of the experiment. A single photon, from a downconversion pair, is input to the top (signal) arm of the apparatus, where it is prepared in a state of form Eq. (1) using a polarizing beam splitter (PBS) and half waveplate ($\lambda/2$). A weak measurement of the polarization is made by interacting the photon with another (meter) photon in a weak measurement device, which operates via measurement-induced nonlinearity (see Ref. [25]). The interaction of the two photons can be controlled using the measurement strength waveplate. The initial single photon is then postselected in the state $|A\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$ using a half waveplate, PBS and photon counter. The value of the weak measurement can be obtained by rotating the output half waveplate and counting photons. A coincidence count flags successful postselection of the signal photon, and weak measurement with an outcome corresponding to the final meter waveplate setting. The signal and meter photons are produced in pairs by spontaneous parametric downconversion from a beta-barium borate (BBO) crystal, pumped by an Ar^+ laser operating at 351.1nm. We deliver the photons to the experiment through single mode optical fibers to provide Gaussian spatial modes for improved spatial mode matching.

When operating with balanced modes [26], the state of the system in two-qubit Hilbert space, after signal and meter photons interact but before either is measured, is:

$$|\phi\rangle = (\alpha\gamma|H\rangle_s + \beta\bar{\gamma}|V\rangle_s)|H\rangle_m + (\alpha\bar{\gamma}|H\rangle_s + \beta\gamma|V\rangle_s)|V\rangle_m \quad (2)$$

where s, m denote the signal and meter photons respectively. It follows that, with measurement of the meter photon in the H - V basis, the weak measurement device implements a POVM $\{\hat{\Pi}_H, \hat{\Pi}_V\}$ on the signal photon, with

$$\begin{aligned} \hat{\Pi}_H &= \frac{1}{2} \left(\hat{1} + (2\gamma^2 - 1) \hat{S}_1 \right), \\ \hat{\Pi}_V &= \frac{1}{2} \left(\hat{1} - (2\gamma^2 - 1) \hat{S}_1 \right). \end{aligned} \quad (3)$$

Eq. (3) gives the measurement strength as $2\gamma^2 - 1$, which is set by the initial state of the meter photon. For a strong, projective measurement, $\gamma = 1$, and weak measurement occurs when γ is close to $1/\sqrt{2}$. A single weak measurement provides little information about the polarization of the signal photon—the result is dominated by the randomness of measuring a meter state close to $(|H\rangle + |V\rangle)/\sqrt{2}$ in the H - V basis. However, for a sufficiently large number of measurements on identically pre-

pared photons, the average signal polarization can be recovered with arbitrary precision. The expectation value for \hat{S}_1 can be written in terms of probabilities of measuring H or V in the meter output:

$$\langle \hat{S}_1 \rangle_W = \frac{\langle \psi | \hat{\Pi}_H | \psi \rangle - \langle \psi | \hat{\Pi}_V | \psi \rangle}{2\gamma^2 - 1} = \frac{P(H) - P(V)}{2\gamma^2 - 1}, \quad (4)$$

where the subscript W refers to the fact that $\langle \hat{S}_1 \rangle$ is obtained from a weak measurement, although the measurement strength in the denominator leads to identical strong and weak values.

After making the weak measurement, we postselect in a basis mutually unbiased with respect to H - V (specifically, on the state $|A\rangle = \frac{1}{\sqrt{2}}(|H\rangle - \frac{1}{\sqrt{2}}|V\rangle)$). It is the selection of a subensemble of measurement results that can lead to the strange results of weak values. This leads to an expression for the postselected weak value of \hat{S}_1 :

$${}_A \langle \hat{S}_1 \rangle_W = \frac{P(H|A) - P(V|A)}{2\gamma^2 - 1}, \quad (5)$$

where, for example, $P(H|A)$ denotes the probability of measuring H in the meter output given that postselection on signal state $|A\rangle$ was successful. Using Eq. (2), it can be shown that if $\gamma \rightarrow 1/\sqrt{2}$, then

$${}_A \langle \hat{S}_1 \rangle_W = \text{Re} \frac{\alpha + \beta}{\alpha - \beta}, \quad (6)$$

so that when $\alpha - \beta \approx 0$, the weak value of \hat{S}_1 can be arbitrarily large. In practice, it is necessary to operate with nonzero measurement strength and postselection probability, so that a precise experimental value for $\langle \hat{S}_1 \rangle$ can be obtained in a finite acquisition time. In this case, the expression for the expected weak value reduces to

$${}_A \langle \hat{S}_1 \rangle_W = \frac{\alpha^* \alpha - \beta^* \beta}{1 - 4\gamma\bar{\gamma}\text{Re}[\alpha\beta]}. \quad (7)$$

More detail on the theory of qubit weak values can be found in Ref. [27].

We measured the weak value of the single photon polarization for a range of measurement strengths, with a nominal input state $|\psi\rangle = \cos(42^\circ)|H\rangle + \sin(42^\circ)|V\rangle \approx 0.743|H\rangle + 0.669|V\rangle$. In principle, the experimental value of γ can be determined from the meter input waveplate settings. However, since the calculated values of $\langle \hat{S}_1 \rangle_W$ are very sensitive to γ , it is desirable to obtain the actual measurement strength from additional coincidence measurements, to deal with errors in the input waveplate setting and the remainder of the optical setup. The measurement strength is identical to the *knowledge* of the generalized measurement device [25], $K = P_{HH} + P_{VV} - P_{HV} - P_{VH} = 2\gamma^2 - 1$, where, e.g., P_{HV} is the probability of observing a horizontally and a vertically polarized photon at the signal and meter outputs of the device respectively, and where these probabilities are

measured with a signal input state $|D\rangle = (|H\rangle + |V\rangle)/\sqrt{2}$, and without postselecting the state $|A\rangle$. Due to Poissonian counting statistics in the measurement of K , the relative size of the error bar is quite large when the measurement strength is close to zero.

The weak values for \hat{S}_1 were determined using Eq. (5) over a range of measurement strengths (Fig. 2). $P(H|A)$ and $P(V|A)$ were obtained from experimental coincidence measurements. For the smallest measurement strength, $2\gamma^2 - 1 = 0.006$, we observed $\langle \hat{S}_1 \rangle_W = 47$, which is much larger than would be expected for a strong QND measurement followed by postselection on $|A\rangle$, i.e. ${}_A\langle \hat{S}_1 \rangle = \alpha^*\alpha - \beta^*\beta \approx 0.1$, and also well outside the spectrum of \hat{S}_1 . The errors in $K = 2\gamma^2 - 1$ of approximately ± 0.015 lead to substantial error bars on largest weak values due to Eq. (5). In fact, for the smallest measurement strengths, the uncertainty in $2\gamma^2 - 1$ encompasses $2\gamma^2 - 1 = 0$, and the error in ${}_A\langle \hat{S}_1 \rangle_W$ is unbounded above. However, the lower bounds of the absolute value are well above 1 over a range of measurement strengths, as shown by the triangles in Fig. 2. In principle, the errors, which are all derived from Poissonian photon counting statistics, could be reduced arbitrarily by collecting larger samples of data. However, the low probability of the postselection, along with the very small correlation between the signal and meter photons, leads to very long collection times—a practical restriction on the size of the data set [28]. As the strength of the measurement is increased, we observed that the weak value of \hat{S}_1 is decreased until it is no longer greater than the strong value $|\alpha|^2 - |\beta|^2 \approx 0.1$. As noted in Ref. [25], the generalized measurement device does not exhibit perfect correlations between signal and meter due to imperfect mode matching. In the present case, this leads to a systematic offset in the weak value at larger measurement strengths, so that in fact it drops below this value.

The slight imperfections of the generalized measurement device mean that the theoretical weak value of Eq. (7), which is calculated assuming no mixture, does not completely describe the measurement. Instead, we determine the actual transfer matrix of the device—the process matrix—using quantum process tomography (in the manner of Ref. [30]), and use it to obtain an expression for the theoretical weak value. As with Eq. (7), this expression is parametrized by α , β , γ and $\bar{\gamma}$, although the slight mixture leads to a lengthier form. The theoretical curve for ${}_A\langle \hat{S}_1 \rangle_W$, plotted for our nominal input state of Eq. (1), is shown in Fig. 2.

From a classical point of view, or even a typical quantum measurement point of view, it is quite strange that the measured expectation value of the \hat{S}_1 Stokes operator lies outside the interval $[-1, 1]$. The strangeness is perhaps more dramatic when we consider the results in terms of mean photon number. In the dual-rail picture, we can think of our input state as a superposition over two spatially degenerate modes with orthogonal linear

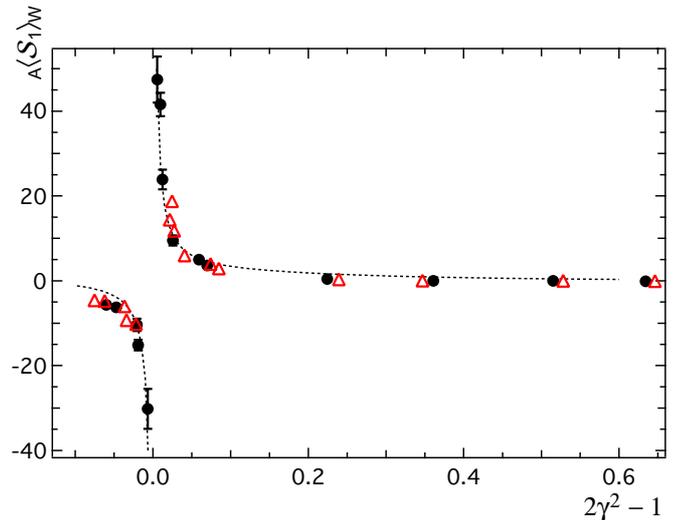


FIG. 2: Variation of experimentally observed weak values of \hat{S}_1 (circles) with measurement strength, $2\gamma^2 - 1$. The error bars plotted arise from the effect of Poissonian counting statistics on $P(H|A)$ and $P(V|A)$. Bars not shown are smaller than the marker dimensions. In addition, errors in $2\gamma^2 - 1$ of approximately ± 0.015 lead to correlated errors in ${}_A\langle \hat{S}_1 \rangle_W$ via Eq. (5)—i.e. a displacement in $2\gamma^2 - 1$ due to error leads to a displacement in the weak value such that a given data point moves along a hyperbola. The triangles show the effect of this error—they represent the bounds obtained when we apply a pointwise displacement of the measured value of $2\gamma^2 - 1$ by the error magnitude, in a direction that minimizes the magnitude of the weak value. The dashed line is the theoretical prediction, based on a model of the generalized measurement device obtained from quantum process tomography.

polarizations $\{H, V\}$. Then the expectation value of \hat{S}_1 can be thought of as the difference in mode occupation between the H and V modes. For instance, in the case of a strong measurement of a single photon in a superposition of $|H\rangle$ and $|V\rangle$,

$$\begin{aligned} \langle \hat{S}_1 \rangle &= [\alpha^*\langle H| + \beta^*\langle V|] \hat{S}_1 [\alpha|H\rangle + \beta|V\rangle] \\ &= [\alpha^*\langle 1|_H \langle 0|_V + \beta^*\langle 0|_H \langle 1|_V] (\hat{n}_H - \hat{n}_V) \\ &\quad \times [\alpha|1\rangle_H |0\rangle_V + \beta|0\rangle_H |1\rangle_V] \\ &= \langle \hat{n}_H \rangle - \langle \hat{n}_V \rangle. \end{aligned} \quad (8)$$

It follows that in the weak postselected case, ${}_A\langle \hat{n}_H \rangle_W - {}_A\langle \hat{n}_V \rangle_W = {}_A\langle \hat{S}_1 \rangle_W$, for postselection on the state $|\sigma\rangle$. That is to say, we experimentally predict that conditional on preparing a single photon superposed across two polarization modes, and conditional on the measurement of $|A\rangle$ in the signal arm, there is a net difference of as many as 47 photons between the two modes when we measure with the weakest generalized measurement! This seems nonsensical when we know that one photon was sent into the signal mode [31].

The resolution to this problem is to note that the weak values we measure are not to be directly interpreted as the *actual* expectation value for \hat{S}_1 , but rather as the *ap-*

parent expectation value according to the measurement results. The actual expectation value for \hat{S}_1 in terms of ${}_A\langle\hat{S}_1\rangle_W$ can be calculated from Eqs. (2) and (4):

$$\begin{aligned} \langle\hat{S}_1\rangle &= {}_A\langle\hat{S}\rangle_W P(A) + {}_D\langle\hat{S}\rangle_W P(D) \\ &= 2 {}_A\langle\hat{S}_1\rangle_W P(A) \\ &= \frac{2[P(H, A) - P(V, A)]}{2\gamma^2 - 1} \\ &= \alpha\alpha^* - \beta\beta^*, \end{aligned} \quad (9)$$

where e.g. $P(A)$ is the probability of postselecting on the state $|A\rangle$ regardless of the weak measurement outcome, $|D\rangle$ is the state orthogonal to $|A\rangle$, and e.g. $P(H, A)$ is the joint probability of measuring H and postselecting on $|A\rangle$. The factor of 2 arises because, in this particular case, the weak measurement basis and the postselection basis are mutually unbiased—in general, this factor will depend on the Hilbert space angle between the measurement and postselection bases.

Postselected weak values are an important indicator of quantum behaviour, since the bizarre results that we obtain for the weak values of \hat{S}_1 and photon number are not paralleled in the probabilities of analogous classical measurements. Large weak values arise from a quantum interference effect that results from the postselection of the signal photon state. The interference effect is most easily seen by referring to the entangled state in Eq. 2: consider the result when the meter photon is detected in the state $|H\rangle_m$, but no postselection is employed in the signal arm. The probability of this event is given by the expectation value of the projector $\hat{1} \otimes |H\rangle_m\langle H|$, with the value $|\alpha\gamma|^2 + |\beta\bar{\gamma}|^2$. This simply corresponds to the probability of measuring H in the signal and H in the meter, plus the probability of measuring V in the signal and H in the meter—i.e., the probabilities add, and there is no quantum interference. If we postselect on $|A\rangle$ in the signal arm, the probability of measuring H in the meter, conditional on the postselection, is given by $(|\alpha\gamma - \beta\bar{\gamma}|^2)/(|\alpha\gamma - \beta\bar{\gamma}|^2 + |\alpha\bar{\gamma} - \beta\gamma|^2)$. It can be seen in the numerator that now the amplitudes add before squaring, allowing the possibility of a quantum interference effect. Combined with the similar expression for a V measurement result, this leads to Eqs. (6) and (7).

In conclusion, we have demonstrated a completely quantum realization of weak values. The weak measurement step relies on a nonclassical interference between a signal and meter photon, meaning that the results cannot be explained by Maxwell's equations alone. We demonstrate that using this technique, we can observe expectation values of quantum mechanical observables far outside the range generally allowed by quantum measurement theory, including mean values of the single-photon S_1 Stokes parameter of up to 47.

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