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THE EQUIVALENCE OF INCLINED UNIAXIAL AND BIAXIAL ELECTRICAL ANISOTROPY IN INHOMOGENEOUS TWO-DIMENSIONAL MEDIA FOR HOMOGENEOUS TM-TYPE PLANE WAVE PROPAGATION PROBLEMS

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Abstract—For a homogeneous TM-type wave propagating in a two-dimensional half space with both vertical and horizontal inhomogeneities, where the TM-type wave is aligned with one of the elements of the conductivity tensor, it is shown using exact solutions to boundary value problems that the shearing term in the homogeneous Helmholtz equation for inclined uniaxial anisotropic media unequivocally vanishes and solutions need only be sought to the homogeneous Helmholtz equation for fundamental biaxial anisotropic media. This implies that those problems posed with an inclined uniaxial conductivity tensor can be identically stated with a fundamental biaxial conductivity tensor, provided that the conductivity values are the reciprocal of the diagonal terms from the Euler rotated resistivity tensor. The applications of this for numerical methods of solving arbitrary two-dimensional problems for a homogeneous TM-type wave is that they need only to approximate the homogeneous Helmholtz equation and neglect the corresponding shearing term.

1. INTRODUCTION

The exact one-dimensional surface impedance expression for homogeneous plane wave incidence above a horizontally stratified (vertically inhomogeneous) earth has been well documented. Exact and approximate solutions for the surface impedance anomaly above a horizontally inhomogeneous half space with isotropic conductivity have been previously investigated for both TE- and TM-type waves [1–15]. The issue of a sloping contact has been considered theoretically by Dmitriev and Zakharov [16] and Geyer [17], and numerically by Reddy and Rankin [18]. However, with the exception of the fundamental uniaxial anisotropy solution of Obukhov [19], and the review of d'Erceville and Kunetz's [2] exact solution for inclined uniaxial anisotropic conductivity by Grubert [20], no significant attention has been given to the problem of exactly solving for the surface impedance anomaly above a vertical contact between two conductive media that have inclined anisotropic conductivity. It is the purpose of this paper to present the exact quasi-static solution for the surface impedance response of a conducting layer with inclined anisotropic conductivity, with lateral inhomogeneities. In a similar way that the solutions of Weaver et al. [14, 15] were developed as control models for the COMMEMI project [21], it is suggested that the development of an exact solution for a two-dimensional control model with inclined uniaxial anisotropy will serve as a benchmark for other approximate methods of solution.

The method used is an extension of the Fourier series method presented by previous authors for isotropic and fundamental anisotropic media. The formulation by Rankin [6] is used as the basis for the formulation presented here. It is assumed that the σ_{xx} element of the conductivity tensor is parallel to the strike of the inclusion and the linearly polarised H_x field, as this then allows one to solve for the linearly polarised TM-type homogeneous plane wave, as the TE-type homogeneous plane wave is uncoupled and will be propagated independently. The task is simplified by assuming that the lateral inhomogeneities has infinite strike length. The problem reduces to a two-dimensional one and it becomes only necessary to solve Maxwell's equations in the region $z \geq 0$. These issues are explained further in the derivation.

2. EXACT FORMULATION: INCLINED ANISOTROPY

2.1. Homogeneous Half Space

The quasi-static Maxwell curl equations can be expanded into source field vector components as:

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = J_x, \tag{1}$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = J_y, \tag{2}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = J_z, \tag{3}$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x, \tag{4}$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y, \tag{5}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z. \tag{6}$$

Assuming a full conductivity tensor, one can write the current density components from Ohm's Law as:

$$J_x = \sigma_{xx}E_x + \sigma_{xy}E_y + \sigma_{xz}E_z, \tag{7}$$

$$J_y = \sigma_{yx}E_x + \sigma_{yy}E_y + \sigma_{yz}E_z, \tag{8}$$

$$J_z = \sigma_{zx}E_x + \sigma_{zy}E_y + \sigma_{zz}E_z. \tag{9}$$

For a homogeneous half space excited by a homogeneous plane wave, all partial derivatives with respect to x and y are equal to zero. From (1) to (6), it follows that:

$$-\frac{\partial H_y}{\partial z} = \sigma_{xx}E_x + \sigma_{xy}E_y + \sigma_{xz}E_z, \tag{10}$$

$$\frac{\partial H_x}{\partial z} = \sigma_{yx}E_x + \sigma_{yy}E_y + \sigma_{yz}E_z, \tag{11}$$

$$\frac{\partial E_y}{\partial z} = j\omega\mu H_x, \tag{12}$$

$$\frac{\partial E_x}{\partial z} = -j\omega\mu H_y, \tag{13}$$

and by implication:

$$H_z = 0, \tag{14}$$

and

$$\sigma_{zx}E_x + \sigma_{zy}E_y + \sigma_{zz}E_z = 0. \tag{15}$$

(14) and (15) simply state that the vertical magnetic field and vertical current density in the half space are equal to zero when the source field is a homogeneous plane wave [22]. By differentiating (12) and (13) with respect to z and substituting the results into (10) and (11), one obtains:

$$0 = \frac{\partial^2 E_x}{\partial z^2} - j\omega\mu(\sigma_{xx}E_x + \sigma_{xy}E_y + \sigma_{xz}E_z), \tag{16}$$

$$0 = \frac{\partial^2 E_y}{\partial z^2} - j\omega\mu(\sigma_{yx}E_x + \sigma_{yy}E_y + \sigma_{yz}E_z). \tag{17}$$

From (15), it is possible to write:

$$E_z = -\frac{\sigma_{zx}}{\sigma_{zz}}E_x - \frac{\sigma_{zy}}{\sigma_{zz}}E_y. \tag{18}$$

After substituting (18) into (16) and (17), two coupled second order differential equations for the horizontal electric fields are obtained:

$$0 = \frac{\partial^2 E_x}{\partial z^2} - j\omega\mu\left(\sigma_{xx} - \frac{\sigma_{xz}\sigma_{zx}}{\sigma_{zz}}\right)E_x - j\omega\mu\left(\sigma_{xy} - \frac{\sigma_{xz}\sigma_{zy}}{\sigma_{zz}}\right)E_y, \tag{19}$$

$$0 = \frac{\partial^2 E_y}{\partial z^2} - j\omega\mu\left(\sigma_{yx} - \frac{\sigma_{yz}\sigma_{zx}}{\sigma_{zz}}\right)E_x - j\omega\mu\left(\sigma_{yy} - \frac{\sigma_{yz}\sigma_{zy}}{\sigma_{zz}}\right)E_y. \tag{20}$$

Let us now assume that problem is posed with the inclined conductivity tensor only being rotated about the x-axis by α such that the conductivity tensor can be written as:

$$\hat{\sigma} = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{yy} & \sigma_{zy} \\ 0 & \sigma_{yz} & \sigma_{zz} \end{bmatrix}, \tag{21}$$

where the elements of the conductivity tensor have the form:

$$\sigma_{xx} = \sigma_t, \tag{22}$$

$$\sigma_{yy} = \sigma_t \cos^2\alpha + \sigma_n \sin^2\alpha, \tag{23}$$

$$\sigma_{zz} = \sigma_t \sin^2\alpha + \sigma_n \cos^2\alpha, \tag{24}$$

$$\sigma_{yz} = \sigma_{zy} = (\sigma_t - \sigma_n)\sin\alpha\cos\alpha. \tag{25}$$

With such conditions on the conductivity tensor, (19) and (20) take the forms of the uncoupled second order ordinary differential equations:

$$0 = \frac{\partial^2 E_x}{\partial z^2} - j\omega\mu\sigma_{xx}E_x, \tag{26}$$

$$0 = \frac{\partial^2 E_y}{\partial z^2} - j\omega\mu\left(\sigma_{yy} - \frac{\sigma_{yz}\sigma_{zy}}{\sigma_{zz}}\right)E_y. \tag{27}$$

If one now considers a solution for (27) of the form of the down-going homogeneous plane wave, then one can substitute (23) to (25) into (27), where it follows that:

$$\sigma_{yy} - \frac{\sigma_{yz}\sigma_{zy}}{\sigma_{zz}} = \left(\frac{\cos^2\alpha}{\sigma_t} + \frac{\sin^2\alpha}{\sigma_n}\right)^{-1} = \rho_{yy}^{-1}, \tag{28}$$

implying that the ρ_{yy} term of the rotated resistivity tensor can be written in terms of σ_{yy}, σ_{yz}, σ_{zy} and ρ_{zz}, and vice versa. It then follows simply that the wave number is:

$$k = \sqrt{j\omega\mu\left(\frac{\cos^2\alpha}{\sigma_t} + \frac{\sin^2\alpha}{\sigma_n}\right)^{-1}}, \tag{29}$$

provided $\operatorname{Re} k > 0$ to prevent an exponentially divergent solution in E_y The surface impedance can then be written as:

$$Z_{y,x} = \sqrt{j\omega\mu\left(\frac{\cos^2\alpha}{\sigma_t} + \frac{\sin^2\alpha}{\sigma_n}\right)}, \tag{30}$$

which corresponds identically to Chetaev [23]. It is also noticed that:

$$\sigma_{zz} - \frac{\sigma_{yz}\sigma_{zy}}{\sigma_{yy}} = \left(\frac{\sin^2\alpha}{\sigma_t} + \frac{\cos^2\alpha}{\sigma_n}\right)^{-1} = \rho_{zz}^{-1}. \tag{31}$$

2.2. Preliminary Considerations for an Inhomogeneous Half Space

Consider a homogeneous rectangular prism with inclined conductivity anisotropy extending infinitely into the x-direction, embedded in an otherwise homogeneous layer, which also exhibits inclined anisotropic conductivity (see Figure 1). The common depth of the inclusion and the layer is h and they are both underlain by a basement with isotropic conductivity σ_b. By considering a homogeneous plane wave as the source field, then all partial derivatives with respect to x are equal to zero. In the local inclined coordinate system characterised by angle of inclination α_1 about the x-axis, the conductivity of the inclusion is represented with the uniaxial conductivity tensor:

$$\hat{\sigma}_1 = \begin{bmatrix} \sigma_{t,1} & 0 & 0 \\ 0 & \sigma_{t,1} & 0 \\ 0 & 0 & \sigma_{n,1} \end{bmatrix}. \tag{32}$$

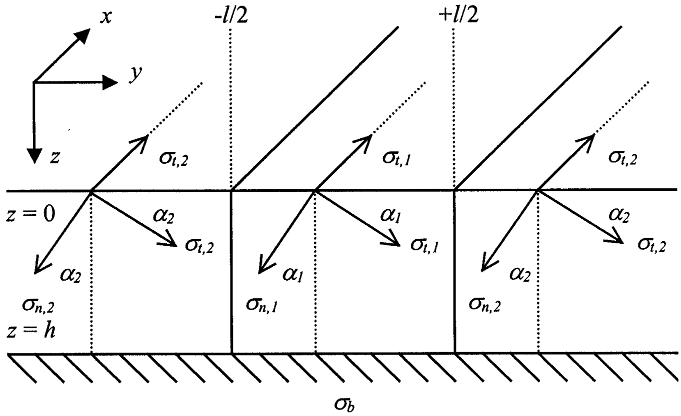

Figure 1. Geometry and parameters of the inclusion embedded in a homogeneous layer.

Similarly, in the local inclined coordinate system characterised by angle of inclination α_2 about the x-axis, the conductivity of the host layer is represented with the uniaxial conductivity tensor:

$$\hat{\sigma}_2 = \begin{bmatrix} \sigma_{t,2} & 0 & 0 \\ 0 & \sigma_{t,2} & 0 \\ 0 & 0 & \sigma_{n,2} \end{bmatrix}. \tag{33}$$

t subscripts denote the conductivity parallel to the inclined horizontal axis of the medium, and n subscripts denote the conductivity normal to the inclined horizontal axis of the medium. When general solutions to Maxwell's equations are considered in this section, subscript m is introduced to designate the medium number, where $m = 1$ for the inclusion and $m = 2$ for the layer. The equations presented here can be considered to satisfy an arbitrary inclined coordinate system $\{x', y', z'\}$ rotated through an angle α_m about the x-axis. From the Maxwell equation:

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}, \tag{34}$$

it follows that $H_{y,m} = H_{z,m} = 0$ for a linearly polarised TM-type homogeneous plane wave where $H_{x,m}$ is parallel to the σ_{xx} component of $\hat{\sigma}_m$. From Maxwell's curl equations, we obtain the family of equations:

$$-j\omega\mu H_{x',m} = \frac{\partial E_{z',m}}{\partial y'} - \frac{\partial E_{y',m}}{\partial z'}, \tag{35}$$

$$E_{z',m} = \frac{-1}{\sigma_{t,m}} - \frac{\partial H_{x',m}}{\partial y'}, \tag{36}$$

$$E_{y',m} = \frac{1}{\sigma_{n,m}} \frac{\partial H_{x',m}}{\partial z'}. \tag{37}$$

By differentiating (36) and (37) with respect to y' and z' respectively, and by substituting the results into (35), one obtains the homogeneous Helmholtz equation for anisotropic media:

$$\frac{1}{\sigma_{n,m}} \frac{\partial^2 H_{x',m}}{\partial y'^2} + \frac{1}{\sigma_{t,m}} \frac{\partial^2 H_{x',m}}{\partial z'^2} - j\omega\mu H_{x',m} = 0. \tag{38}$$

The coordinate rotations for transforming $\{x', y', z'\}$ coordinates to $\{x, y, z\}$ coordinates can be written as:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_m & \sin\alpha_m \\ 0 & -\sin\alpha_m & \cos\alpha_m \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{R}(-\alpha) \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \tag{39}$$

such that the second-order partial derivatives of any function f can be written as:

$$\frac{\partial^2 f}{\partial y'^2} = \frac{\partial^2 f}{\partial y^2} \cos^2\alpha_m + 2\frac{\partial^2 f}{\partial y \partial z} \sin\alpha_m \cos\alpha_m + \frac{\partial^2 f}{\partial z^2} \sin^2\alpha_m, \tag{40}$$

$$\frac{\partial^2 f}{\partial z'^2} = \frac{\partial^2 f}{\partial y^2} \sin^2\alpha_m - 2\frac{\partial^2 f}{\partial y \partial z} \sin\alpha_m \cos\alpha_m + \frac{\partial^2 f}{\partial z^2} \cos^2\alpha_m. \tag{41}$$

(38) can now be written in $\{x, y, z\}$ coordinates as:

$$\left(\frac{\cos^2\alpha_m}{\sigma_{n,m}} + \frac{\sin^2\alpha_m}{\sigma_{t,m}}\right) \frac{\partial^2 H_{x,m}}{\partial y^2} + \left(\frac{\sin^2\alpha_m}{\sigma_{n,m}} + \frac{\cos^2\alpha_m}{\sigma_{t,m}}\right) \frac{\partial^2 H_{x,m}}{\partial z^2}$$
$$+2\left(\frac{1}{\sigma_{n,m}} - \frac{1}{\sigma_{t,m}}\right) \sin\alpha_m \cos\alpha_m \frac{\partial^2 H_{x,m}}{\partial y \partial z} - j\omega\mu H_{x,m} = 0. \tag{42}$$

The presence of the lateral inhomogeneities will generate anomalous magnetic fields across the strike of tho inclusion. At an infinite distance from the dyke, the anomalous fields must reduce to zero and the solution for the magnetic field will be identical to that of a horizontally homogeneous medium. Near the inclusion, the general solution for the total magnetic $H_{x,m}(y, z)$ can be written as the sum of the background (b) and anomalous (a) fields:

$$H_{x,m}(y, z) = H^b_{x,m}(z) + H^a_{x,m}(y, z), \tag{43}$$

where $H^a_{x,m}(y, z)$ is the anomalous field that exists due to the lateral inhomogeneities. Since $H_{x,m}(y, z)$ satisfies (42), then $H^b_{x,m}(z)$ and

$H^a_{x,m}(y,z)$ must also satisfy a form of (42) as linearly independent solutions of (42). In this particular problem, the anisotropic media is bound by a perfectly insulating layer (i.e., air) at the upper boundary ($z = 0$), and a basement with isotropic conductivity σ_b at the lower boundary ($z = h$). A general solution for the background magnetic field $H^b_{x,m}(z)$, is a homogeneous plane wave with both up-going and down-going components of the form:

$$H^b_{x,m}(z) = A_m \exp(-k_m z) + B_m \exp(k_m z), \tag{44}$$

which satisfies (42) provided that the z-directed wave number is given by:

$$k_m = \sqrt{j\omega\mu \left(\frac{\sin^2 \alpha_m}{\sigma_{n,m}} + \frac{\cos^2 \alpha_m}{\sigma_{t,m}} \right)^{-1}}, \tag{45}$$

and $\operatorname{Re} k_m > 0$ to prevent an exponentially divergent solution in $H^b_{x,m}(z)$. It should be noted that A_m and B_m in (44) are independent of $\{x, y, z, t\}$ and are not related to components of the vector potential $\mathbf{A}$ or magnetic flux density $\mathbf{B}$. The solution for $H^b_{x,m}(z)$ is easily identified as the solution for the horizontally homogeneous problem.

2.3. Perfectly Insulating Basement Solution

If $\sigma_b = 0$, then $H_{x,m}(y,z) = 0$ at $z = h$ and this boundary condition is equivalent to the top of the basement being a perfect magnetic conductor [11]. It follows that the coefficients for (44) are given by:

$$A_m = \frac{H_0 \exp(k_m h)}{2\sinh(k_m h)}, \tag{46}$$

$$B_m = \frac{-H_0 \exp(-k_m h)}{2\sinh(k_m h)}, \tag{47}$$

where H_0 is the magnetic field magnitude at $z = 0$, and is a constant which may be complex. (44) can then be written as:

$$H^b_{x,m}(z) = \frac{H_0 \sinh k_m (h - z)}{\sinh(k_m h)}. \tag{48}$$

It is easily observed from (48) that $H^b_{x,m}(z) = 0$ when $z = h$. Similarly, $H^a_{x,m}(z) = 0$ when $z = h$. This also implies that $H^a_{x,m}(z) = 0$ when $z = 0$ since $H_{x,m}(y,z) = H_0$. By separation of variables, the anomalous magnetic field will be written as the product of two independent functions, $f_m(y)$ and $g_m(z)$:

$$H^a_{x,m}(y,z) = f_m(y)g_m(z), \tag{49}$$

where $g_m(z)$ can be expressed as a Fourier series of sine terms with an argument of $\frac{n\pi z}{h}$:

$$g_m(z) = \sum_{n=1}^{\infty} A_{m,n} \sin\left(\frac{n\pi z}{h}\right), \tag{50}$$

where n is the mode number $(1, 2, 3, \ldots, \infty)$ and where $A_{m,n}$ are the Fourier series coefficients, and should not be confused with A_m in (44), a coefficient of the magnetic field wave equations. For convenience, we will include $A_{m,n}$ in the values of $f_m(y)$ at each n. (49) can then be written as:

$$H^a_{x,m}(y,z) = \sum_{n=1}^{\infty} f_m(y) \sin\left(\frac{n\pi z}{h}\right). \tag{51}$$

For $n = 1, 2, 3, \ldots, \infty$, each term of (51) must satisfy a form of (42) as a linear sum of solutions. For each term from (51), we have the partial derivatives:

$$\frac{\partial^2 H^a_{x,m,n}(y,z)}{\partial y^2} = \frac{\partial^2 f_{m,n}(y)}{\partial y^2} \sin\left(\frac{n\pi z}{h}\right), \tag{52}$$

$$\frac{\partial^2 H^a_{x,m,n}(y,z)}{\partial z^2} = -\frac{n^2\pi^2}{h^2} f_{m,n}(y) \sin\left(\frac{n\pi z}{h}\right), \tag{53}$$

$$\frac{\partial^2 H^a_{x,m,n}(y,z)}{\partial y \partial z} = \frac{n\pi}{h} \frac{\partial f_{m,n}(y)}{\partial y} \cos\left(\frac{n\pi z}{h}\right). \tag{54}$$

The form of (42) that the anomalous fields must satisfy is then written as:

$$\begin{aligned}
&\left(\frac{\cos^2\alpha_m}{\sigma_{n,m}} + \frac{\sin^2\alpha_m}{\sigma_{t,m}}\right) \frac{\partial^2 f_{m,n}(y)}{\partial y^2} \sin\left(\frac{n\pi z}{h}\right) \\
&- \left(\frac{\sin^2\alpha_m}{\sigma_{n,m}} + \frac{\cos^2\alpha_m}{\sigma_{t,m}}\right) \frac{n^2\pi^2}{h^2} f_{m,n}(y) \sin\left(\frac{n\pi z}{h}\right) \\
&+2\left(\frac{1}{\sigma_{n,m}} - \frac{1}{\sigma_{t,m}}\right) \sin\alpha_m \cos\alpha_m \frac{n\pi}{h} \frac{\partial f_{m,n}(y)}{\partial y} \cos\left(\frac{n\pi z}{h}\right) \\
&-j\omega\mu f_{m,n}(y) \sin\left(\frac{n\pi z}{h}\right) = 0.
\end{aligned} \tag{55}$$

At the $z = 0$ and $z = h$ boundaries, $\sin\left(\frac{n\pi z}{h}\right) = 0$ and the shearing term:

$$2\left(\frac{1}{\sigma_{n,m}} - \frac{1}{\sigma_{t,m}}\right)\sin\alpha_m\cos\alpha_m\frac{n\pi}{h}\frac{\partial f_{m,n}(y)}{\partial y}\cos\left(\frac{n\pi z}{h}\right) = 0. \quad (56)$$

At $z = 0$, $\cos\left(\frac{n\pi z}{h}\right) = 1$, so (56) can only vanish for three possible cases:
(a) if:

$$\frac{1}{\sigma_{n,m}} - \frac{1}{\sigma_{t,m}} = 0, \quad (57)$$

which is the special case for an isotropic solution ($\sigma_{t,m} = \sigma_{n,m}$); or
(b) if:

$$\sin\alpha_m\cos\alpha_m = 0, \quad (58)$$

which is only the special case of either $\alpha_m = 0$ or $\alpha_m = 90°$, corresponding to fundamental anisotropic solutions; or else,
(c) if:

$$\frac{\partial f_{m,n}(y)}{\partial y} = 0 \ \forall \ n. \quad (59)$$

As a general solution for the inclined anisotropic problem is sought, (57) and (58) are trivial (as they are special conditions) implying that (59) must hold true in all cases. This means that (55) can be reduced to:

$$\left(\frac{\cos^2\alpha_m}{\sigma_{n,m}} + \frac{\sin^2\alpha_m}{\sigma_{t,m}}\right)\frac{\partial^2 f_{m,n}(y)}{\partial y^2}\sin\left(\frac{n\pi z}{h}\right) - \left(\frac{\sin^2\alpha_m}{\sigma_{n,m}} + \frac{\cos^2\alpha_m}{\sigma_{t,m}}\right)\frac{n^2\pi^2}{h^2}f_{m,n}(y)\sin\left(\frac{n\pi z}{h}\right) - j\omega\mu f_{m,n}(y)\sin\left(\frac{n\pi z}{h}\right) = 0. \quad (60)$$

A solution for $f_{m,n}(y)$ must satisfy both (59) and (60), and a solution can be shown to be of the form:

$$f_{m,n}(y) = a_{m,n}\exp\left(\frac{-q_{m,n}y}{h}\right) + b_{m,n}\exp\left(\frac{q_{m,n}y}{h}\right), \quad (61)$$

provided that:

$$q_{m,n} = \sqrt{k_{z,m}^2 h^2 + n^2\pi^2\left(\frac{\sigma_{t,m}\sin^2\alpha_m + \sigma_{n,m}\cos^2\alpha_m}{\sigma_{t,m}\sin^2\alpha_m + \sigma_{n,m}\cos^2\alpha_m}\right)}, \quad (62)$$

where

$$k_{z,m} = \sqrt{j\omega\mu\left(\frac{\cos^2\alpha_m}{\sigma_{n,m}} + \frac{\sin^2\alpha_m}{\sigma_{t,m}}\right)^{-1}},$$

and $\mathrm{Re}\, q_{m,n} > 0$ for a-type terms and $\mathrm{Re}\, q_{m,n} < 0$ for b-type terms to prevent an exponentially divergent solution in $f_{m,n}(y)$. (61) satisfies the condition that (51) vanish for $|y| = \infty$, only if the a-type terms correspond to $+y$ terms and b-type terms correspond to $-y$ terms for $m = 2$, whilst both positive and negative exponents are permissible in the finite region of $m = 1$. Symmetry conditions at the boundaries then permit:

$$a_{m,n} = b_{m,n}, \tag{63}$$

which will ensure that $H_{x,m}(y,z)$ is an even function about $y = 0$; i.e., $H_{x,m}(y,z) = H_{x,m}(-y,z)$. Employing this boundary condition is equivalent to using one of the boundaries for solving the continuity of the magnetic field components, with the remaining boundary condition to be available for solving the remainder of the coefficients. If the $y = \frac{l}{2}$ boundary is considered, then following from (63), the use of symmetry implies that:

$$\sum_{n=1}^{\infty}\left[2a_{1,n}\cosh\left(\frac{q_{1,n}l}{2h}\right) - a_{2,n}\exp\left(\frac{q_{2,n}l}{2h}\right)\right]\sin\left(\frac{n\pi z}{h}\right) = H_2 - H_1, \tag{64}$$

where

$$H_m \equiv H^b_{x,m} = \frac{H_0 \sinh k_m(h-z)}{\sinh(k_m h)}.$$

The expansion of $H_2 - H_1$ into a sine series of argument $\frac{n\pi z}{h}$ is written as:

$$H_2 - H_1 = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi z}{h}\right), \tag{65}$$

where C_n is a complex constant yet to be determined. At the boundary, term-by-term must be equated, so both equations:

$$2a_{1,n}\cosh\left(\frac{q_{1,n}l}{2h}\right) - a_{2,n}\exp\left(-\frac{q_{2,n}l}{2h}\right) = C_n, \tag{66}$$

$$2a_{1,n}q_{1,n}\rho_{zz,1}\sinh\left(\frac{q_{1,n}l}{2h}\right) + a_{2,n}q_{2,n}\rho_{zz,2}\exp\left(-\frac{q_{2,n}l}{h}\right) = 0, \tag{67}$$

must be satisfied, where (67) is obtained from the Maxwell equation:

$$E_{z,m} = -\rho_{zz,m}\frac{\partial H_{x,m}}{\partial y},$$

stating the continuity of the tangential electric field across the boundary $y = \pm l/2$. Solutions for the a-type coefficients are then:

$$a_{1,n} = \frac{C_n \rho_{zz,2}}{2\rho_{zz,2} \cosh\left(\frac{q_{1,n} l}{2h}\right) + 2\rho_{zz,1} \frac{q_{1,n}}{q_{2,n}} \sinh\left(\frac{q_{1,n} l}{2h}\right)}, \tag{68}$$

$$a_{2,n} = \frac{-C_n \rho_{zz,1} \exp\left(\frac{q_{2,n} l}{2h}\right) \sinh\left(\frac{q_{1,n} l}{2h}\right)}{\rho_{zz,2} \cosh\left(\frac{q_{1,n} l}{2h}\right) + \rho_{zz,1} \sinh\left(\frac{q_{1,n} l}{2h}\right)}. \tag{69}$$

Following the expansion of $H_2 - H_1$ into an odd Fourier series with argument $\frac{n\pi z}{h}$, one obtains:

$$C_n = \frac{2}{h} \int_0^h (H_2 - H_1) \sin\left(\frac{n\pi z}{h}\right) dz, \tag{70}$$

where:

$$H_2 - H_1 = \frac{H_0 \sinh k_2(h-z)}{\sinh(k_2 h)} - \frac{H_0 \sinh k_1(h-z)}{\sinh(k_1 h)}. \tag{71}$$

Following through with (70) using integration by parts, the solution for C_n is:

$$C_n = -\frac{2H_0 n\pi \left(k_2^2 - k_1^2\right)}{h^2 \left(k_2^2 + \frac{n^2\pi^2}{h^2}\right) \left(k_1^2 + \frac{n^2\pi^2}{h^2}\right)}. \tag{72}$$

Substituting (72) into (68) and (69), solutions for the a-type coefficients now take the form:

$$a_{1,n} = \frac{-H_0 n\pi \rho_{zz,2} \left(k_2^2 - k_1^2\right)}{\left(h^2 \left(k_2^2 + \frac{n^2\pi^2}{h^2}\right) \left(k_1^2 + \frac{n^2\pi^2}{h^2}\right) \left[\rho_{zz,2} \cosh\left(\frac{q_{1,n} l}{2h}\right) + \rho_{zz,1} \frac{q_{1,n}}{q_{2,n}} \sinh\left(\frac{q_{1,n} l}{2h}\right)\right] \right)}, \tag{73}$$

$$a_{2,n} = \frac{-2H_0 n\pi \rho_{zz,1} \left(k_2^2 - k_1^2\right) \exp\left(\frac{q_{2,n} l}{2h}\right) \sinh\left(\frac{q_{1,n} l}{2h}\right)}{\left(h^2 \left(k_2^2 + \frac{n^2\pi^2}{h^2}\right) \left(k_1^2 + \frac{n^2\pi^2}{h^2}\right) \left[\rho_{zz,1} \sinh\left(\frac{q_{1,n} l}{2h}\right) + \rho_{zz,2} \frac{q_{2,n}}{q_{1,n}} \cosh\left(\frac{q_{1,n} l}{2h}\right)\right] \right)}. \tag{74}$$

For $-\frac{l}{2} \leq y \leq \frac{l}{2}$, the anomalous magnetic field is written as:

$$H^a_{x,1}(y,z) = \sum_{n=1}^{\infty} 2a_{1,n} \cosh\left(\frac{q_{1,n}y}{h}\right) \sin\left(\frac{n\pi z}{h}\right). \tag{75}$$

The total magnetic field can then be written as:

$$H_{x,1}(y,z) = \frac{H_0 \sinh k_1(h-z)}{\sinh(k_1 h)} + \sum_{n=1}^{\infty} 2a_{1,n} \cosh\left(\frac{q_{1,n}y}{h}\right) \sin\left(\frac{n\pi z}{h}\right). \tag{76}$$

From Maxwell equations (36) and (37) rotated about the x-axis, we have:

$$E_{y,1} = \rho_{yy,1} \frac{\partial H_{x,1}}{\partial z}, \tag{77}$$

so the horizontal electric field can be written as:

$$\begin{aligned} E_{y,1}(y,z) &= \frac{-k_1 \rho_{yy,1} H_0 \cosh k_1(h-z)}{\sinh(k_1 h)} \\ &\quad + \sum_{n=1}^{\infty} 2\rho_{yy,1} \frac{n\pi}{h} a_{1,n} \cosh\left(\frac{q_{1,n}y}{h}\right) \cos\left(\frac{n\pi z}{h}\right). \end{aligned} \tag{78}$$

At the air-half space interface ($z = 0$), (76) reduces to:

$$H_{x,1}(y,0) = H_0, \tag{79}$$

and is constant $\forall\ y$. For $z = 0$, (78) reduces to:

$$E_{y,1}(y,z) = \frac{-k_1 \rho_{yy,1} H_0 \cosh(k_1 h)}{\sinh(k_1 h)} + \sum_{n=1}^{\infty} 2\rho_{yy,1} \frac{n\pi}{h} a_{1,n} \cosh\left(\frac{q_{1,n}y}{h}\right). \tag{80}$$

The surface impedance is defined as:

$$Z_{y,x}(y,0) = k_1 \rho_{yy,1} \coth(k_1 h) - \sum_{n=1}^{\infty} 2\rho_{yy,1} \frac{n\pi}{h} \frac{a_{1,n}}{H_0} \cosh\left(\frac{q_{1,n}y}{h}\right). \tag{81}$$

In (81), the term:

$$k_1 \rho_{yy,1} \coth(k_1 h) = \sqrt{j\omega\mu \left(\frac{\cos^2 \alpha_1}{\sigma_{t,1}} + \frac{\sin^2 \alpha_1}{\sigma_{n,1}}\right)} \coth(k_1 h), \tag{82}$$

is identified as the surface impedance of a laterally homogeneous layer above a perfect magnetic conducting basement. Once (82) is substituted into (81), one obtains:

$$Z_{y,x}(y,0) = \sqrt{j\omega\mu\left(\frac{\cos^2\alpha_1}{\sigma_{t,1}}+\frac{\sin^2\alpha_1}{\sigma_{n,1}}\right)}\coth(k_1h) + \frac{2\pi^2}{h^3}\left(k_2^2-k_1^2\right)\rho_{zz,2}\rho_{zz,1} \times \sum_{n=1}^{\infty}\frac{n^2\cosh\left(\frac{q_{1,n}y}{h}\right)}{\left(\left(k_2^2+\frac{n^2\pi^2}{h^2}\right)\left(k_1^2+\frac{n^2\pi^2}{h^2}\right)\left[\rho_{zz,2}\cosh\left(\frac{q_{1,n}l}{2h}\right)+\rho_{zz,1}\frac{q_{1,n}}{q_{2,n}}\sinh\left(\frac{q_{1,n}l}{2h}\right)\right]\right)}. \quad (83)$$

For $y \geq \frac{l}{2}$ and $y \leq -\frac{l}{2}$, the constant term is the same as for medium 1, however the subscripts are interchanged. One writes the anomalous magnetic field as:

$$H_{x,2}^{a}(y,z) = \sum_{n=1}^{\infty} a_{2,n}\exp\left(-\frac{q_{2,n}|y|}{h}\right)\sin\left(\frac{n\pi z}{h}\right). \quad (84)$$

The total magnetic field can then be written as:

$$H_{x,2}(y,z) = \frac{H_0\sinh k_2(h-z)}{\sinh(k_2h)} + \sum_{n=1}^{\infty} a_{2,n}\exp\left(-\frac{q_{2,n}y}{h}\right)\sin\left(\frac{n\pi z}{h}\right). \quad (85)$$

From Maxwell's equations (36) and (37) rotated about the x-axis, we have:

$$E_{y,2} = \rho_{yy,2}\frac{\partial H_{x,2}}{\partial z}, \quad (86)$$

so the horizontal electric field is written as:

$$E_{y,2}(y,z) = \frac{-k_2\rho_{yy,2}H_0\cosh k_2(h-z)}{\sinh(k_2h)} + \sum_{n=1}^{\infty}\rho_{yy,2}\frac{n\pi}{h}a_{2,n}\exp\left(-\frac{q_{2,n}y}{h}\right)\cos\left(\frac{n\pi z}{h}\right). \quad (87)$$

At the air-half space interface ($z=0$), (85) reduces to:

$$H_{x,2}(y,0) = H_0, \quad (88)$$

and is constant $\forall\, y$. For $z = 0$, (87) reduces to:

$$E_{y,2}(y,z) = \frac{-k_2\rho_{yy,2}H_0\cosh(k_2h)}{\sinh(k_2h)} + \sum_{n=1}^{\infty}\rho_{yy,2}\frac{n\pi}{h}a_{2,n}\exp\left(-\frac{q_{2,n}y}{h}\right). \tag{89}$$

The surface impedance is defined as:

$$Z_{y,x}(y,0) = k_2\rho_{yy,2}\coth(k_2h) - \sum_{n=1}^{\infty}\rho_{yy,1}\frac{n\pi}{h}\frac{a_{2,n}}{H_0}\exp\left(-\frac{q_{2,n}y}{h}\right). \tag{90}$$

In (90), the term:

$$k_2\rho_{yy,2}\coth(k_2h) = \sqrt{j\omega\mu\left(\frac{\cos^2\alpha_2}{\sigma_{t,2}} + \frac{\sin^2\alpha_2}{\sigma_{n,2}}\right)}\coth(k_2h) \tag{91}$$

is identified as the surface impedance of a laterally homogeneous layer above a perfect magnetic conducting basement. Once (91) is substituted into (90), one obtains:

$$\begin{aligned} Z_{y,x}(y,0) \;=\; & \sqrt{j\omega\mu\left(\frac{\cos^2\alpha_2}{\sigma_{t,2}} + \frac{\sin^2\alpha_2}{\sigma_{n,2}}\right)}\coth(k_2h) \\ & + \frac{2\pi^2}{h^3}\left(k_2^2 - k_1^2\right)\rho_{zz,2}\rho_{yy,1} \\ & \times \sum_{n=1}^{\infty}\frac{n^2\exp\left(\frac{q_{2,n}y}{h}\right)\exp\left(\frac{q_{2,n}l}{2h}\right)\sinh\left(\frac{q_{1,n}l}{2h}\right)}{\left(\left(k_2^2+\frac{n^2\pi^2}{h^2}\right)\left(k_1^2+\frac{n^2\pi^2}{h^2}\right)\left[\rho_{zz,1}\sinh\left(\frac{q_{1,n}l}{2h}\right)+\rho_{zz,2}\frac{q_{2,n}}{q_{1,n}}\cosh\left(\frac{q_{1,n}l}{2h}\right)\right]\right)}. \end{aligned} \tag{92}$$

(83) and (92) are the complete exact solution for the surface impedance of an inclusion with inclined anisotropic conductivity embedded in an otherwise homogeneous layer above a perfect magnetic conductor.

3. EXACT FORMULATION: FUNDAMENTAL ANISOTROPY

The formulations derived in Section 2 will now be reduced to the problem of an inclusion with fundamental uniaxial anisotropy. In this case, it can be assumed that $\alpha_1 = \alpha_2 = 0$. Firstly, it is noted that the wave number for medium m is now given by:

$$k_m = \sqrt{j\omega\mu\sigma_{t,m}}, \tag{93}$$

where $\operatorname{Re} k_m > 0$ to prevent exponentially divergent solutions. Now (58) is satisfied and a solution to (60) is sought. (61) is still a valid solution, and (62) reduces to:

$$q_{m,n} = \sqrt{k_{z,m}^2 h^2 + n^2\pi^2 \frac{\sigma_{n,m}}{\sigma_{t,m}}}, \tag{94}$$

where:

$$k_{z,m} = \sqrt{j\omega\mu\sigma_{n,m}},$$

which we can also write as:

$$q_{m,n} = \sqrt{\frac{k_m^2 h^2 + n^2\pi^2}{\lambda_m^2}}, \tag{95}$$

where λ_m is the coefficient of anisotropy of medium (m) and this is identical to the solution of Obukhov [19]. It follows that:

$$a_{1,n} = \frac{-H_0 n\pi\left(k_2^2 - k_1^2\right)}{\left(\begin{array}{l} h^2\left(k_2^2 + \frac{n^2\pi^2}{h^2}\right)\left(k_1^2 + \frac{n^2\pi^2}{h^2}\right) \\ \qquad \left[\cosh\left(\frac{q_{1,n}l}{2h}\right) + \frac{\sigma_{n,2}}{\sigma_{n,1}}\frac{q_{1,n}}{q_{2,n}}\sinh\left(\frac{q_{1,n}l}{2h}\right)\right] \end{array}\right)}, \tag{96}$$

$$a_{2,n} = \frac{-2H_0 n\pi\left(k_2^2 - k_1^2\right)\exp\left(\frac{q_{2,n}l}{2h}\right)\sinh\left(\frac{q_{1,n}l}{2h}\right)}{\left(\begin{array}{l} h^2\left(k_2^2 + \frac{n^2\pi^2}{h^2}\right)\left(k_1^2 + \frac{n^2\pi^2}{h^2}\right) \\ \qquad \left[\sinh\left(\frac{q_{1,n}l}{2h}\right) + \frac{\sigma_{n,1}}{\sigma_{n,2}}\frac{q_{2,n}}{q_{1,n}}\cosh\left(\frac{q_{1,n}l}{2h}\right)\right] \end{array}\right)}, \tag{97}$$

and that for $-\frac{l}{2} \leq y \leq \frac{l}{2}$, the surface impedance can be written as:

$$Z_{y,x}(y,0) = \sqrt{\frac{j\omega\mu}{\sigma_{t,1}}}\coth(k_1 h) + \frac{2\pi^2}{h^3}\left(k_2^2 - k_1^2\right)\rho_{n,1}$$

$$\times \sum_{n=1}^{\infty} \frac{n^2 \cosh\left(\frac{q_{1,n} y}{h}\right)}{\left(\left(k_2^2 + \frac{n^2\pi^2}{h^2}\right)\left(k_1^2 + \frac{n^2\pi^2}{h^2}\right) \left[\cosh\left(\frac{q_{1,n} l}{2h}\right) + \frac{\sigma_{n,2}}{\sigma_{n,1}} \frac{q_{1,n}}{q_{2,n}} \sinh\left(\frac{q_{1,n} l}{2h}\right)\right]\right)}. \tag{98}$$

For $y \geq \frac{l}{2}$ and $y \leq -\frac{l}{2}$, the surface impedance can be written as:

$$\begin{aligned} Z_{y,x}(y,0) &= \sqrt{\frac{j\omega\mu}{\sigma_{t,2}}} \coth(k_2 h) + \frac{2\pi^2}{h^3}\left(k_2^2 - k_1^2\right)\rho_{n,2} \\ &\times \sum_{n=1}^{\infty} \frac{n^2 \exp\left(\frac{q_{2,n} y}{h}\right) \exp\left(\frac{q_{2,n} l}{2h}\right) \sinh\left(\frac{q_{1,n} l}{2h}\right)}{\left(\left(k_2^2 + \frac{n^2\pi^2}{h^2}\right)\left(k_1^2 + \frac{n^2\pi^2}{h^2}\right) \left[\sinh\left(\frac{q_{1,n} l}{2h}\right) + \frac{\sigma_{n,1}}{\sigma_{n,2}} \frac{q_{2,n}}{q_{1,n}} \cosh\left(\frac{q_{1,n} l}{2h}\right)\right]\right)}. \end{aligned} \tag{99}$$

(98) and (99) were identically obtained by Obukhov [19]. Further, it can be observed that when $\sigma_{t,m} = \sigma_{n,m} = \sigma_m$ then for $-\frac{l}{2} \leq y \leq \frac{l}{2}$, the surface impedance is given by:

$$\begin{aligned} Z_{y,x}(y,0) &= \sqrt{\frac{j\omega\mu}{\sigma_1}} \coth(k_1 h) + \frac{2\pi^2}{h^3}\left(k_2^2 - k_1^2\right)\rho_1 \\ &\times \sum_{n=1}^{\infty} \frac{n^2 \cosh\left(\frac{q_{1,n} y}{h}\right)}{\left(\left(k_2^2 + \frac{n^2\pi^2}{h^2}\right)\left(k_1^2 + \frac{n^2\pi^2}{h^2}\right) \left[\cosh\left(\frac{q_{1,n} l}{2h}\right) + \frac{q_{1,n}}{q_{2,n}} \sinh\left(\frac{q_{1,n} l}{2h}\right)\right]\right)}. \end{aligned} \tag{100}$$

For $y \geq \frac{l}{2}$ and $y \leq -\frac{l}{2}$, the surface impedance can be written as:

$$Z_{y,x}(y,0) = \sqrt{\frac{j\omega\mu}{\sigma_{t,2}}} \coth(k_2 h) + \frac{2\pi^2}{h^3}\left(k_2^2 - k_1^2\right)\rho_2$$

$$\times \sum_{n=1}^{\infty} \frac{n^2 \exp\left(\dfrac{q_{2,n}y}{h}\right) \exp\left(\dfrac{q_{2,n}l}{2h}\right) \sinh\left(\dfrac{q_{1,n}l}{2h}\right)}{\left(\begin{array}{c}\left(k_2^2+\dfrac{n^2\pi^2}{h^2}\right)\left(k_1^2+\dfrac{n^2\pi^2}{h^2}\right) \\ \left[\sinh\left(\dfrac{q_{1,n}l}{2h}\right)+\dfrac{q_{2,n}}{q_{1,n}}\cosh\left(\dfrac{q_{1,n}l}{2h}\right)\right]\end{array}\right)}, \tag{101}$$

which are identical equations to those derived by Rankin [6]. It is noted that Rankin's [6] solution was derived using the cgs electromagnetic units (emu) in which μ is dimensionless and equal to unity in free space.

4. CONCLUSION

In this paper, we have considered only the propagation of homogeneous TM-type waves in media with inclined uniaxial anisotropic conductivity. It has been demonstrated that for a half space, and for a horizontally stratified half space, the inclined uniaxial anisotropic conductivity tensor can be written as a corresponding fundamental bianisotropic conductivity tensor where:

$$\sigma_{x,m} = \rho_{xx,m}^{-1} = \sigma_{xx,m} \tag{102}$$

$$\sigma_{y,m} = \left(\frac{\cos^2\alpha_m}{\sigma_{t,m}}+\frac{\sin^2\alpha_m}{\sigma_{n,m}}\right)^{-1} = \rho_{yy,m}^{-1} = \sigma_{yy,m} - \frac{\sigma_{yz,m}\sigma_{zy,m}}{\sigma_{zz,m}}, \tag{103}$$

$$\sigma_{z,m} = \left(\frac{\sin^2\alpha_m}{\sigma_{t,m}}+\frac{\cos^2\alpha_m}{\sigma_{n,m}}\right)^{-1} = \rho_{zz,m}^{-1} = \sigma_{zz,m} - \frac{\sigma_{zy,m}\sigma_{yz,m}}{\sigma_{yy,m}}. \tag{104}$$

It has been demonstrated that the shearing term in the rotated Helmholtz equation vanishes for horizontally inhomogeneous media, as in the case for horizontally homogeneous media. Further, if one compares (83) and (92) with (98) and (99) respectively, it should be noticed that the equations are identical with the exception that in (83) and (92), the conductivity terms are given by (103) and (104). Hence, it is concluded that homogeneous TM-type propagation in inhomogeneous two-dimensional problems with inclined uniaxial anisotropic conductivity can be equivalently described as a two-dimensional problem with fundamental biaxial anisotropic conductivity. This has important applications to approximate methods of solution.

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